Illustrative Mathematics®

Guidance for Accelerating Students in Mathematics

by the team at Illustrative Mathematics

May 2020

The information presented in this document is intended to support schools and districts in making well-informed decisions about mathematics coursework and course placement policies, particularly with respect to accelerating students in mathematics. This piece includes:

1. IM's Recommendations for Accelerated Coursework in K–8 Mathematics
2. The Rationale behind the Recommendations
    a. The Crucial Importance of Understanding K–8 Mathematics
    b. Basing Policies on the Current Standards
    c. Addressing Disproportionate Racial and Ethnic Representation
3. Evaluating Current Acceleration Practices
4. Suggestions for Designing Pathways Leading to Advanced Coursework
5. References

1. IM's Recommendations for Acceleration in K–8 Mathematics

- Commit to cultivating a solid K–8 mathematics foundation by appropriate **implementation of grade-level standards and limiting acceleration options before high school**.
- **Provide students who are ready for more of a challenge throughout K–8 with extension and enrichment opportunities**, aiming for deeper understanding (which is great for future learning), before acceleration is considered (which can undermine future learning). Note that classroom teachers need resources, training, funding, and structures that support this.
- If students are accelerated, **use a curricular approach that *compacts* rather than *skips*** important foundational content. "Compacts" means to cover the same content in less time. To this end, we are creating IM 6–8 Math Accelerated certified by Illustrative Mathematics that can take place over two years.
- If your district is already engaged in accelerating students in K–8 mathematics, we highly recommend **interrogating historical data and assessing the outcomes for students who accelerate**.
- **Use multiple measures to place students in acceleration**, including not just prior math achievement and presumed preparation, but also whether or not it supports their academic and career goals. **Identify possible sources of bias in the selection process and take steps to eliminate them.**
- If the group of students placed in accelerated courses isn't representative of the district population, a likely cause is opportunity gaps earlier in the math pathway. **Work on addressing**

Creating a world where learners know, use, and enjoy mathematics.
© Illustrative Mathematics 2020

opportunity gaps, and also provide other pathways that lead to coursework needed for college admission and STEM careers.

- **Offer alternative pathways** that enable students to achieve their goals **that do not require acceleration in mathematics before high school**.

2. The Rationale Behind the Recommendations

The Crucial Importance of Understanding K–8 Mathematics

The concepts addressed in K–8 mathematics are extremely important, and students need time to learn this content properly. **Acceleration approaches that involve skipping standards put future learning at risk.**

The mathematics that enables the vast majority of students to be successful in first-year college courses is not from high school courses, but rather middle school courses, especially arithmetic, ratio, proportion, expressions, and simple equations (NCEE 2013). A survey conducted by ACT (2012) found that topics from grades 6–9 were more important to college faculty for college preparation than advanced topics.

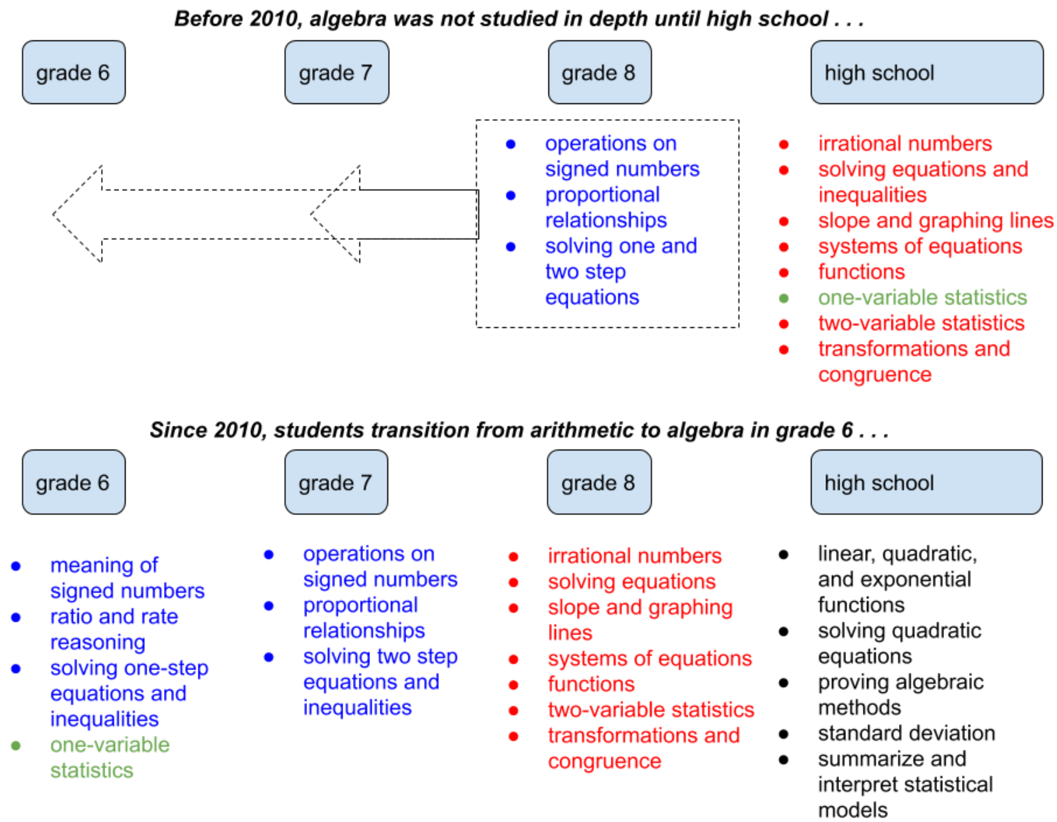
High school teachers often prioritize covering a broad range of advanced topics in mathematics courses, while teachers at the college level think high school students should receive in-depth coverage of more foundational topics like number sense and basic algebra (Chait & Venezia, 2009). Students with a solid understanding of middle school mathematics are better prepared for an advanced course in high school, which improves their success rates in college-level courses (ACT 2009, 2012). The mathematics needed for success in community college math courses is learned in grades 6–8 and Algebra I, and the failure rates in math courses in our community colleges suggest that many students do not know that math well (NCEE 2013). Weak understanding of foundational ideas for algebra readiness often result in significant difficulty reaching proficiency in higher-level high school mathematics (Finkelstein et al. 2012).

Basing Policies on the Current Standards

Prior to 2010, most U.S. math curricula were highly repetitive through K–8 and redundant in middle school. A study of math textbooks from the 80's found that by grade 8 only about 30% of the material was new content (Flanders, 1987). As a result, many districts (reasonably) accelerated students by skipping one or more middle school mathematics courses.

New standards implemented in 2010 shifted much content previously addressed in high school to grade 8, and topics previously addressed in grade 8 cascaded down into lower grades. **These shifts resulted from research showing that all students benefit from access to algebra earlier in their schooling.** This graphic shows how some content was shifted.

Creating a world where learners know, use, and enjoy mathematics.
© Illustrative Mathematics 2020



Around the same time, several studies found correlations between taking a high school algebra course by grade 8 and favorable outcomes in high school and beyond. Specifically, taking algebra in grade 8 was associated with taking more advanced math courses in high school, greater chances of enrolling in a 4-year college or university, and also an increased likelihood of pursuing a STEM degree. (Chen & Weko, 2009; Gamoran & Hannigan, 2000; Ma & Wilkins, 2007; Paul, 2005). As more people (inferring causation from correlation) began to believe that taking high school math in middle school conferred advantages, the percentage of students taking algebra in grade 8 increased dramatically. The percentage of students in the US taking Algebra 1 or higher in grade 8 increased from 27% in 2000 to 48% in 2013 (Loveless, 2016). In California, the percentage increased from 32% in 2003 to 54% in 2009 (Williams et al., 2011).

As a result, multiple factors impeded students' understanding of middle school math: there was a large, nationwide increase in the number of students accelerating in middle school math that coincided with new standards that made middle school courses much less repetitive and much denser with critical content.

The goals of old acceleration policies—access to algebra earlier in their schooling in preparation for standardized tests used for college admission and scholarship decisions—are now achieved by students taking grade-level mathematics through K–8. **As a result, much of the rationale for policies that allow students to skip middle school mathematics courses has disappeared.**

Creating a world where learners know, use, and enjoy mathematics.
© Illustrative Mathematics 2020

There is recent evidence that too many students are being accelerated in middle school mathematics. **When more students were placed into a grade 7 or 8 Algebra 1 course, the pass rates of Algebra 1 declined and the students were significantly less likely to pass Geometry and Algebra 2** (Clotfelter, 2012). Students who were de-tracked in middle school performed better in high school, even the higher achievers (Boaler, 2000). 44% of 8th graders who took Algebra 1 had to repeat it with mixed results in improvement among groups (Fong, et al., 2014). 30% of a sample of accelerated students had to retake Algebra 1 between grades 7 and 12, with very little improvement the second time (Finkelstein, et al., 2012).

Addressing Disproportionate Racial and Ethnic Representation in Accelerated Courses

Babies of all racial and ethnic groups have the same inherent likelihood of growing into adolescents who are interested in and able to study mathematics at an accelerated pace. If students from different racial and ethnic groups don't enroll in and complete accelerated coursework in math at the same rates as their representation in the population, it may be due to bias in the selection process or differences in opportunities to learn that build throughout schooling (Morton & Riegle-Crumb, 2019). Readiness for an accelerated pathway requires both that students are ready to move through the content with less time for practice, and that students do not have unfinished learning that they need time to address. We recommend that schools and districts take concrete steps to rectify the causes of disproportionate representation in accelerated mathematics courses.

It's important to use multiple metrics, including student and parent self-assessment, teacher recommendation, and assessment scores to make placement decisions for accelerated coursework. If that decision process results in disparities in the racial and ethnic composition of the group of students placed in accelerated courses, then **districts should consider both how to address opportunity gaps in elementary schools, as well as how to support students with the interest and aptitude but unfinished learning to enter an accelerated pathway.** Providing accelerated pathways that start in high school, or offering acceleration through extra math courses or summer options, will increase the likelihood that mathematically interested students with unfinished learning have access to the same opportunities going forward as their peers who have had more opportunities from the beginning.

In addition to supporting students who are currently under-represented in accelerated pathways, districts can support populations who are overrepresented and who may be missing out on critical learning opportunities. Ensure that students and families from overrepresented groups are informed about the potential negative consequences of acceleration before high school, and that students and families understand that grade-level mathematics in middle school is important, rigorous, and needn't be rushed. Highlight the availability of pathways that don't require accelerating before high school that still enable students to accomplish their goals. Consider building in a checkpoint with families

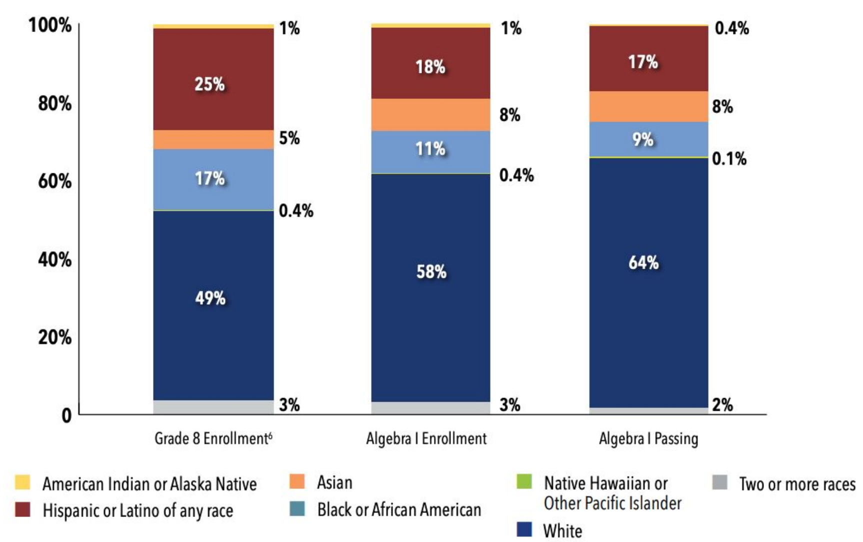
© Illustrative Mathematics 2020

after the first year of acceleration, so that students who might be better served by building a stronger foundation have a well-defined opportunity to not continue on an accelerated path.

Policies that attempt to level the playing field by simply placing all students in a high school course in grade 8 without regard to students' readiness for the topics should be reconsidered. Due to the 2010 changes in standards described earlier, students miss out on crucial learning opportunities with this approach, as they would with any acceleration structure that skips rather than compresses math content.

To provide a sense of the current situation, here is some recent data. Among grade 8 students who are enrolled in Algebra I, white students tend to be overrepresented and black and Latinx students tend to be underrepresented. In data collected by the US Department of Education Office for Civil Rights in 2015–16, white students constituted 49% of the students in schools that offered Algebra I in grade 8, and 58% of students enrolled in Algebra I. Black students constituted 17% of the students in schools that offered Algebra I in grade 8, and 11% of students enrolled. Latinx students constituted 25% of the students in schools that offered Algebra I in grade 8, and 18% of students enrolled.

FIGURE 2: Percentage distribution of Grade 8 students enrolled in and passing Algebra I, by race



NOTE: Data may not add up to 100 percent due to rounding.
SOURCE: U.S. Department of Education, Office for Civil Rights, Civil Rights Data Collection, 2015–16.

(USDoE, 2018).

© Illustrative Mathematics 2020

3. Evaluating Your Current Acceleration Practices

Considering historical practices and data can help show whether acceleration has been good for students in your district.

- What are the explicit and implicit reasons for accelerating students in math?
- What are the criteria for selecting students for an accelerated pathway, and what are the possible sources of bias in the selection process?
- What opportunities do students have to enter an accelerated mathematics pathway later in their schooling, if they are not selected for early opportunities to join an accelerated cohort?
- What do your current data say? Of the students accelerated to high school work in grade 8 or earlier:
    - What proportion repeats one or more high school level courses?
    - What proportion enrolls in and completes a calculus course (or other challenging, advanced math course) in high school?
    - How does their racial and ethnic composition compare to the student population of the whole district?
    - For all of these questions, what proportions are tolerable?

4. Suggestions for Designing Pathways Leading to Advanced Coursework

Reimagining High School Math Pathways

Some college and university admissions policies still advantage applicants who have taken calculus. Until those institutions change their practices, students preparing for a competitive admissions process (and their families) will, understandably, prioritize taking calculus in high school. **However, it is much better for these students to nurture the development of solid mathematical understanding in K–8, and then commit to some extra time and effort in high school.** Many states and organizations have developed roadmaps for organizing high school coursework to enable taking calculus without acceleration before high school, and we have linked to some below.

Over the past few decades, a higher percentage of students has taken calculus in high school compared to previous years. Some believe this path is necessary for a student to succeed in advanced math courses in college. It turns out, only about 20% of those students take Calculus II or higher as their first college math course. (Of the rest, around 30% repeat Calculus I, 20% take no math at all or other post-secondary

Creating a world where learners know, use, and enjoy mathematics.
© Illustrative Mathematics 2020

math courses like business calculus or statistics, and 30% take a course that covers topics from high school like precalculus, college algebra, or remedial math (Bressoud, 2017).)

There is more than one way to get to a STEM career. One approach is to take calculus in high school, but it is also common and reasonable to enter this path later on (Cannady, et al., 2014). **A student's pathway should be determined by considering their goals and choosing courses that best serve them.** With appropriate guidance, students carrying out their own decisions may work harder than students who have been placed into a course or track (Patall et al., 2008).

Suggestions for Designing High School Course Offerings

- Taking a meaningful mathematics course every year in high school is important, but that doesn't mean all students need to take precalculus or calculus in high school. Offer other courses that are aligned with students' interests and plans.
- AP Statistics and AP Computer Science are valuable alternatives to AP Calculus which do not require any acceleration.
- Accelerate students with the interest and motivation to take calculus in the fourth year by:
    - Compacting 4 high school courses into 3 years. (Keep in mind that precalculus is a relatively recent invention, and arguably obsolete due to shifts in standards implemented after 2010.)
    - Compacting 2 high school courses into 1 year, or 3 courses into 2 years.
    - Simultaneously enrolling in two math courses during the same school year.
- Offer summer or semester courses as bridges for students who choose to switch pathways.
- These publications contain examples of ways to structure high school coursework that align with these recommendations:
    - Massachusetts Model Advance Courses http://www.doe.mass.edu/frameworks/math/2017-06.pdf
    - California Mathematics Framework Appendix D: Course Placement and Sequences https://www.cde.ca.gov/ci/ma/cf/documents/mathfw-appendixd.pdf
    - Branching Out: Designing High School Math Pathways for Equity https://justequations.org/wp-content/uploads/Just-Equations-2019-Report-Branching-Out-Digital.pdf
    - Catalyzing Change in High School Mathematics: Initiating Critical Conversations. NCTM (2018)

© Illustrative Mathematics 2020

5. References

ACT. (2012). ACT national curriculum study 2012 mathematics. ACT. Retrieved from: http://www.act.org/research/policymakers/pdf/NCS-Mathematics.pdf.

Boaler, J. (2000). Students experiences of ability grouping-disaffection, polarization, and construction of failure. *British Educational Research Journal.* Vol. 26, No. 5., 631–648.

Bressoud, D. (Ed.). (2017). The role of calculus in the transition from high school to college mathematics. Retrieved from: https://www.maa.org/sites/default/files/ RoleOfCalc_rev.pdf.

Cannady, M., Greenwald, E., Harris, K. (2014). Problematizing the STEM pipeline metaphor: Is the STEM pipeline metaphor serving our students and our workforce? *Science Education*, vol. 99, No. 3, 443–460.

Chait, R., Venezia, A. (2009). Improving academic preparation for college: What we know and how state and federal policy can help. Center for American Progress. Washington, D.C. Retrieved from: https://www.americanprogress.org/wp-content/uploads/issues/2009/01/pdf/academic_prep.pdf

Chen, X., & Weko, T. (2009). Students who study science, technology, engineering, and mathematics (STEM) in postsecondary education (NCES 2009-161). Washington, DC: National Center for Education Statistics, Institute of Education Sciences, U.S. Department of Education. Retrieved from http://nces.ed.gov/pubs2009/2009161.pdf

Clotfelter, C. T., Ladd, H. F., Vigdor, J. L., (2012). The aftermath of accelerating algebra: Evidence from a district policy initiative. Duke University.

Common Core State Standards Initiative. (2010). Common Core State Standards for Mathematics, Appendix A: Designing high school mathematics courses based on the Common Core State Standards. Retrieved from: http://community.ksde.org/LinkClick.aspx?fileticket=8xJH7j3bogI%3d&tabid=5276&mid=13067

Common Core Standards Writing Team. (2013, March 1). Progressions for the Common Core State Standards in Mathematics (draft). Front matter, preface, introduction. Grade 8, High School, Functions. Tucson, AZ: Institute for Mathematics and Education, University of Arizona. Retrieved from http://commoncoretools.me/wpcontent/uploads/2013/07/ccss_progression_frontmatter_2013_07_30.pdf

Daro, P. and Asturias, H. (2019). Branching out: Designing high school math pathways for equity. Retrieved from: https://justequations.org/wp-content/uploads/Just-Equations-2019-Report-Branching-Out-Digital.pdf

Finkelstein, N., Fong, A., Tiffany-Morales, J., Shields, P., Huang, M. (2012). College bound in middle school and high school: How math course sequences matter. The Center for the Future of Teaching and Learning. WestEd.

Fong, A., Jaquet, K., Finkelstein, N. (2014). Who repeats algebra 1, and how does initial performance relate to improvement when the course is repeated? (REL 2015-059). Washington, DC: U.S. Department of Education, Institute of Education Sciences, National Center for Education Evaluation and Regional Assistance, Regional Educational Laboratory West. Retrieved from http://ies.ed.gov/ncee/edlabs

Flanders, J.R. (1987). How much of the content in mathematics textbooks is new? National Council of Teachers of Mathematics. *The Arithmetic Teacher*, Vol. 35, No. 1 pp. 18–23. Retrieved from: http://www.jstor.org/stable/41193199

Gamoran, A., & Hannigan, E. C. (2000). Algebra for everyone? Benefits of college-preparatory mathematics for students with diverse abilities in early secondary school. *Educational Evaluation and Policy Analysis*, 22(3), 241–254. doi:10.3102/01623737022003241

© Illustrative Mathematics 2020

Kansas State Department of Education White Paper (2015). Rethinking mathematics acceleration practices. Retrieved from: https://community.ksde.org/LinkClick.aspx?fileticket=ANMo_s668aY%3d&tabid=6038&mid=15080

Loveless, T. (2016). The 2016 Brown Center report on American education: How well are American students learning? Washington, DC: Brown Center on Education Policy, Brookings Institution. Retrieved from https://www.brookings.edu/research/2016-brown-center-report-on-american-education-how-well-are-american-students-learning/

Ma, X., & Wilkins, J. L. (2007). Mathematics coursework regulates growth in mathematics achievement. *Journal for Research in Mathematics Education*, 38(3), 230–257.

Massachusetts Department of Elementary and Secondary Education. (2012). Making decisions about course sequences and the new model algebra I course. MDESE. Retrieved from: http://www.doe.mass.edu/frameworks/math/2017-06.pdf

Morton, K. and Riegle-Crumb, C. (2019). Who Gets in? Examining inequality in eighth-grade algebra. *NCTM Journal for Research in Mathematics Educatio*n, Vol. 50, Issue 5. Retrieved from https://www.nctm.org/Publications/Journal-for-Research-in-Mathematics-Education/2019/Vol50/Issue5/Who-Gets-in_-Examining-Inequality-in-Eighth-Grade-Algebra/

National Center on Education and the Economy (NCEE) (2013). What does it really mean to be college and career ready? Retrieved from http://ncee.org/wp-content/uploads/2013/05/NCEE_MathReport_May20131.pdf

The National Council of Teachers of Mathematics. (1970). A history of mathematics education in the United States and Canada. National Council of Teachers of Mathematics. Washington, DC.

Patall, E. A., Cooper, H., & Robinson, J. C. (2008). The effects of choice on intrinsic motivation and related outcomes: A meta-analysis of research findings. *Psychological Bulletin*, 134(2), 270-300. Retrieved from: http://dx.doi.org/10.1037/0033-2909.134.2.270

Paul, F. G. (2005). Grouping within Algebra I: A structural sieve with powerful effects for low-income, minority, and immigrant students. *Educational Policy*, 19(2), 262–282. doi:10.1177/0895904804274056

Policy Implications of the ACT National Curriculum Survey Results. (2009). Focusing on the essentials for college and career readiness. ACT. Retrieved from ACT website www.act.org/research/policymakers/pdf/NCS_PolicySummary2009.pdf

Rosenstein, JG. (2014). The rush to calculus. Center for Discrete Mathematics and Theoretical Computer Science. Rutgers University. Retrieved from DIMACS website http://dimacs.rutgers.edu/~joer/The-Rushto-Calculus.pdf

US Department of Education. (2012). An overview of classes taken and credits earned by beginning postsecondary students. NCES. Retrieved from: http://nces.ed.gov/pubs2013/2013151rev.pdf

US Department of Education (2018). STEM course taking: Data highlights on science, technology, engineering, and mathematics course taking in our nation's public schools. Retrieved from https://www2.ed.gov/about/offices/list/ocr/docs/stem-course-taking.pdf

Williams et al. (2011). Improving middle grades math performance: A closer look at district and school policies and practices, course placements, and student outcomes in California. Retrieved from https://www.shastacoe.org/uploaded/Dept/is/County_Curriculum_Leads/study11-mg-math-es.pdf

© Illustrative Mathematics 2020