IM K–5 Math beta is built upon the problem based design of the IM K–12 Math curriculum:

Lesson structure for grades 2–5 in IM K–5 Math beta:

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SAMPLE LESSON

Grade 2 Unit 1

Lesson 8: Solving Compare Problems with Bar Graphs

Unit 1: Adding and Subtracting with Data	**Lesson 9:** How Many More? How Many Fewer?

Teacher-facing Learning Goals

- Solve compare problems with difference unknown using data presented in a bar graph.
- Analyze counting on and counting back strategies used to solve compare problems.

Addressing CCSS: 2.MD.D.10

Lesson Purpose

The mathematical purpose of this lesson is for students to use the structure of a bar graph to solve compare problems. Students use what they know about the relationship between addition and subtraction and the strategies of counting on and counting back to solve these problems.

Materials Needed

- display image for Activity 1

Cool-down: Second Grade Absences

Second Grade Absences

September	5
October	8
November	17
December	16

1. How many fewer students were absent in October than December?
   Show your thinking using drawings, numbers, or words.

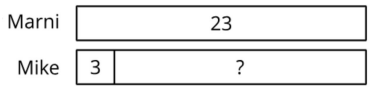
2. Write an equation and show how you solved. Can you write more than one?

Student Responses

1. 8 fewer students were absent in October than December.
   Sample response: First, break 8 into 6 and 2. Then subtract 16 - 6 = 10 and 10 - 2 = 8. I know 8 + 8 = 16, so 16 - 8 = 8.
2. 16 - 8 = 6 or 16 - 8 = _____ or 8 + _____ = 16 or 8 + 8 = 16

Teacher Reflection Question

In future lessons, students will use tape diagrams like the one below to represent compare situations. How does the work with the bar graphs today help build students' understanding of this more abstract diagram?



Lesson Narrative

In grade 1, students studied the relationship between addition and subtraction through compare problems. In prior lessons in this unit, students interpreted bar graphs and solved story problems using the data.

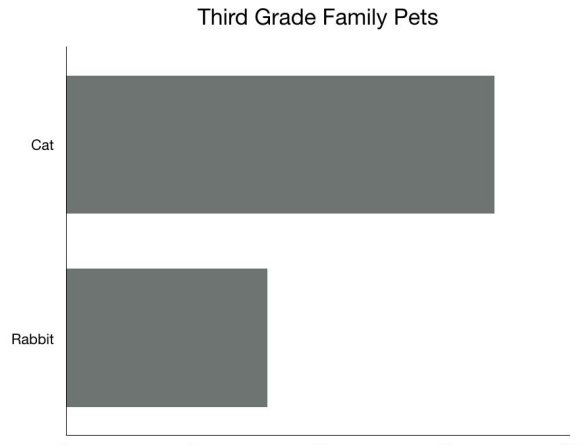
The mathematical purpose of this lesson is to invite students to use the structure of a bar graph to solve compare problems. The bar graph allows them to see the difference between the two quantities and physically use the grid lines to count on and count back. This leads to work in the next lesson in which students relate these strategies to operations and represent them with equations.

Students begin by solving a compare problem in two different ways. The synthesis for Activity 1 highlights the counting on and counting back strategies. In Activity 2, students solve a series of compare problems and are introduced to different ways to work compare problems.

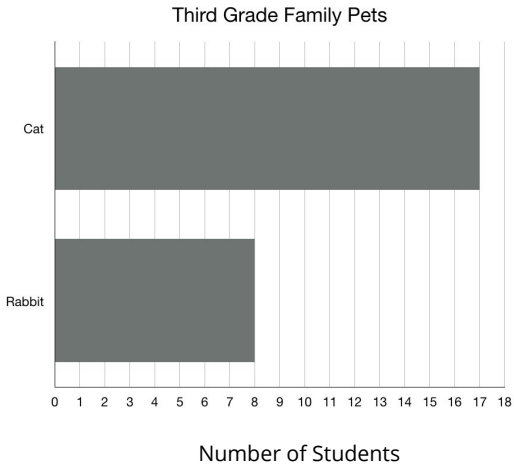
Student-facing Learning Goal: Let's solve compare problems about data.

Warm-up Narrative: Number Talk: Make 10 with 3 Addends		**Building on CCSS:** 2.OA.B.2
10 min	The purpose of this Number Talk is to activate students' previous experiences with the strategy of making a ten. The ability to make 10 will help students develop fluency within 20 and will be helpful later in this lesson and in upcoming lessons when students add and subtract within 20.	
Task Statement Find the value of each sum mentally. 3 + 7 3 + 7 + 2 5 + 7 2 + 4 + 8		**Launch/Activity** • Display one problem. • "Give me a signal when you have an answer and can explain how you got it."

Student Responses • I know that 3 + 7 = 10. • I know 3 + 7 = 10 and 2 more is 12. • I still made 10 with 3 and 7 first. Then 2 more is 12. • I put the 2 and 8 together to get 10 and then there were 4 left, so the answer is 14.	• 1 minute: quiet think time • record answers and strategies • keep problems and work displayed • repeat with each problem **Synthesis** • How does making 10 make these problems easier to solve? Consider asking: • "Who can restate _____'s reasoning in a different way?" • "Did anyone have the same strategy but would explain it differently?" • "Did anyone approach the problem in a different way?" • "Does anyone want to add on to _____"s strategy?"
Activity 1 Narrative: Comparing Strategies	**Addressing CCSS:** 2.MD.10
15 min 	The purpose of this activity is to highlight the counting up and counting back strategies to solve compare problems. This builds on grade 1 understandings of the relationship between addition and subtraction. The structure of the bar graph is useful for students to show that counting up and counting back both work to find the difference between two of the bars. If students do not generate representations of both strategies during the activity, create and display work so that each strategy can be discussed during the synthesis.
Task Statement Launch	**Launch** • Groups of 2 • Display image (graph with no scale).



Activity



How many more students have cats than rabbits?
Show two ways to find the answer.

Student Responses

9. Possible responses: I counted back from 17 to 8. I counted up from 8 to 17 (17 - 8 = 9).

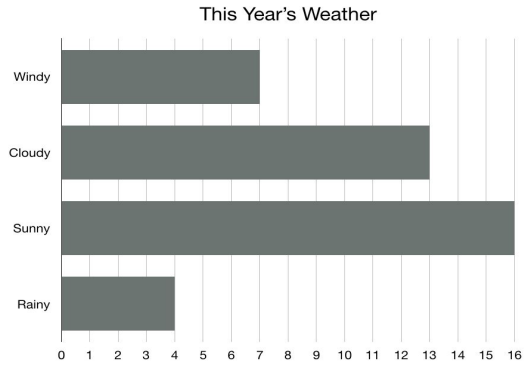
- "A group of third-grade students were asked, 'What pets do you have?' Their responses are shown in the bar graph. What do you notice about the data in the graph?"
- 1 minute: partner discussion
- Monitor for students to say: "More students have cats than rabbits," or wonder, "How many more students have cats than rabbits?"
- Share the comparisons with the class.

Activity

- "You noticed that more students have cats than rabbits. Your job now is to figure out how many more students have cats than rabbits. Think about two different ways you can find the answer and record them."
- 3–4 minutes: independent work time
- 2–3 minutes: partner discussion
- Monitor for students who used a counting up strategy and counting back strategy.

	Synthesis • Display work from both students. • "How are their strategies similar? How are the strategies different?" • "Can someone show us on the bar graph how you could count up or count back to solve this problem?" • "Why does counting up and counting back work to compare these two bars?" (Because we are finding the distance between them and we can start with the larger one and count back, or start from the smaller one and count on.) • "Where do you see addition or subtraction in the strategies?" (We are adding when we are counting up, and we are subtracting when we are counting back.)
Activity 2 Narrative: Compare Problems with Bar Graphs	**Addressing CCSS:** 2.MD.10
20 min	The purpose of this activity is to give students practice solving compare problems with difference unknown using data represented with a bar graph. Monitor for students who use different operations and represent how they solved the problem with equations. This will be discussed in the synthesis.

Task Statement



Based on the graph, answer the problems and show your reasoning.

1. How many days are represented on the graph?
2. How many more windy days were there than cloudy days?
3. How many more sunny days were there than rainy days?
4. How many fewer cloudy days were there than sunny days?

Are you ready for more?

How many more sunny and rainy days were there than windy and cloudy?

Student Response:

1. 40 days
2. 6 more windy days
3. 12 more sunny days
4. 4 fewer cloudy days

Launch/Activity

- "We just saw that there are different ways to compare numbers on a graph. Now you are going to solve some problems about data on your own. Be careful—they aren't all the same type of problem."
- 10 minutes: independent work time
- Monitor for students who used different strategies to solve question 4.

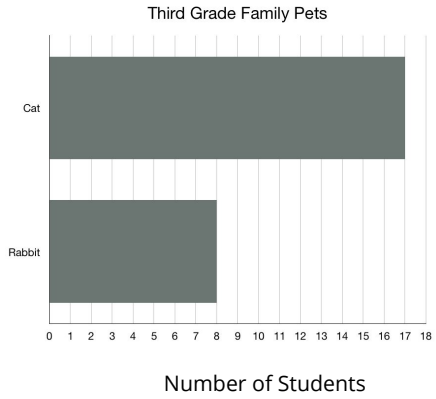
Synthesis

- What was different about question 4?
- What strategy did you use? Is it the same strategy you used for questions 2 and 3?
- "How can we represent our strategies as addition or subtraction?" Give time for a turn and talk and follow with a whole-class discussion.

Lesson Synthesis

"Today we solved problems about data represented with bar graphs. We learned that we can add or subtract to compare two categories represented by two different bars. For example, in the following graph, if we were comparing the difference between the number of students with cats and rabbits, we could add or subtract."

Display the pet graph from Activity 1.



“Turn and talk to your neighbor. One partner describes how you would add, and the other describes how you would subtract.” Ask one student to explain how they added and another student how they subtracted. Record their responses for all to see.

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SAMPLE LESSON

Grade 3 Unit 2

Lesson 11: Disappearing Squares

Unit 2: Area and Multiplication	**Lesson 11**: Disappearing Squares

Teacher-facing Learning Goals

- Solve real-world and mathematical problems involving area.
- Multiply side lengths to find areas of rectangles with whole-number side lengths.

Addressing CCSS: 3.MD.C.7b

Lesson Purpose

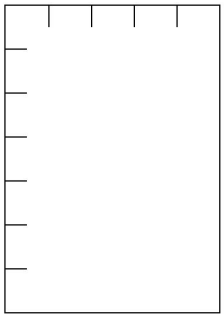
The mathematical purpose of this lesson is for students to solve real-world and mathematical problems involving area. Problems in the lesson progress from the use of partially tiled rectangles to labeled side lengths to encourage students to multiply side lengths when determining the area.

Materials Needed

- display image for lesson synthesis

Cool-down: Where are the Squares?

The rectangle is marked off in unit lengths. What is the area of the rectangle?



Student Responses

35 square units

Teacher Reflection Question

How did students change their method for finding area when faced with only partially tiled rectangle or marks along the side lengths in rectangles?

Lesson Narrative
In previous lessons, students learned how area is related to multiplication. Also, students have solved real-world problems involving area. The purpose of this lesson is for students to solve problems involving area. Students will work with rectangles where the unit squares are less and less visible, encouraging students to multiply the side lengths to find the area. In this lesson, this takes the form of partially tiled rectangles and rectangles with side lengths marked off in linear units. Students will consider strategies they use and be encouraged to multiply side lengths to solve problems involving area.

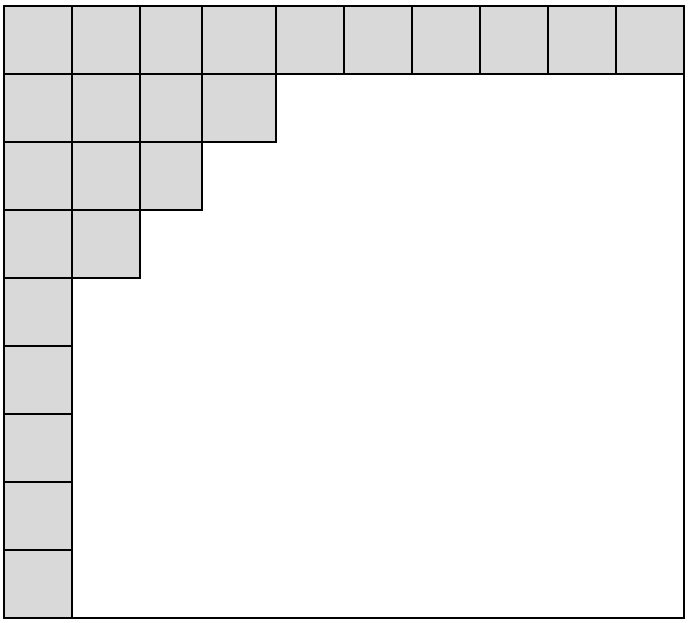
Student-facing Learning Goal: Let's solve problems involving area.	
Warm-up Narrative: Number Talk: One More Group	**Addressing CCSS:** 3.OA.B.5
10 minutes	The purpose of this Number Talk is to elicit strategies and understandings students have for multiplying by 6. Multiplying by 6 can be challenging for students so this number talk is meant to give students a way of using what they know about products with factors of 5 to find products with factors of 6. These understandings help students develop fluency and will be helpful later in this lesson when students will be encouraged to multiply to find the area of a rectangle. In this activity, students have an opportunity to notice and make use of structure (MP7) because they may use strategies based on the properties of multiplication to find unknown facts. Students may commute the factors to create a fact they know. Students may think about "one more group" as they move from the first problem to the second problem (or the third to the fourth). Also, students may say that they "just know" the product. All of these responses are acceptable because students will be in different stages as they progress towards fluency.
Task Statement Find the value of each product mentally. 5 x 2 6 x 2 5 x 6 6 x 6	**Launch/Activity** • Display one problem. • "Give me a signal when you have an answer and can explain how you got it." • 1 minute: quiet think time • Record answers and strategy.

Student Responses • 10, because I counted by 2. I just knew 5 x 2 = 10. • 12, because I know that 5 groups of 2 is 10, so one more group of 2 would be 12. I just knew 2 x 6 = 12, so I knew that 6 x 2 is 12 because the factors were just in a different order. I just knew it. • 80, because I counted by 10. I just knew 8 x 10 = 80. • 90, because I know that 8 groups of 10 is 80, so one more group of 10 would make 90.	• Keep problems and work displayed. • Repeat with each problem. **Synthesis** • Focus question: • "What happens when we increase one of the factors by 1?" (The product goes up by 1 of the other factor.) • "What makes this happen?" (The amount increases by 1 group. Each group gets 1 more.) • Consider asking: • "Who can restate _____'s reasoning in a different way?" • "Did anyone have the same strategy but would explain it differently?" • "Did anyone approach the problem in a different way?" • "Does anyone want to add on to _____'s strategy?"
Activity 1 Narrative: Partially Tiled	**Addressing CCSS:** 3.MD.C.7b
15 min	The purpose of this activity is to begin withdrawing the full grid from problems involving area. This encourages students to multiply to solve problems involving area, but still provides some visual support to see the arrangement of the rows and columns. This problem includes a product of 10, with which students should be increasingly comfortable. The number of square units is large in order to discourage one-by-one counting.

Task Statement

This rectangle represents a room that is partially tiled. Each gray tile has a side length of 1 foot.

1. How many gray tiles will it take to tile the whole room?
2. What do you need to know to find the area of *any* rectangle?



Student Responses

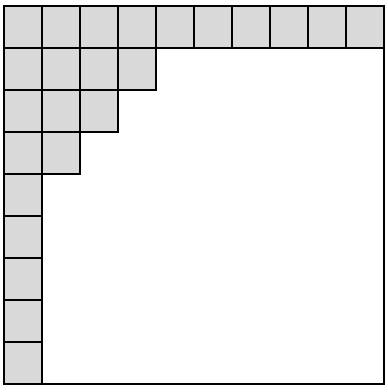
1. 90 gray tiles
2. the side lengths

Launch/Activity

- Groups of 2
- "This problem involves finding the area of a room that is partially tiled with square tiles. Think about how you might solve this problem."
- 1 minute: quiet think time
- 3–5 minutes: partner work
- Monitor for students who use multiplication to find the area of the rectangles.
- Consider asking:
  - "How did you find the area of the rectangle?"
  - "Is there a way that's faster than counting every square one-by-one?"

Synthesis

Display:



Ask:

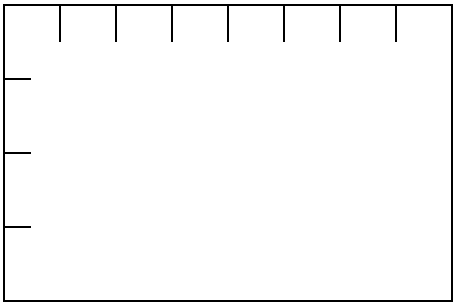
- "How were you able to see equal groups in this problem?" (There was a complete row and if you tiled the rest of the rows,

	each row would be an equal group. There was a complete column and if you tiled the rest of the columns, each column would be an equal group.) • "How did you figure out how many equal groups there were?" (Since I knew how many were in each row, I counted how many squares were in the column and this told me that there were 9 rows of 10. Since I knew how many were in each column, I counted how many squares were in the row and this told me that there were 10 columns of 9.) • "What would we multiply to find the total number of tiles needed to cover the room?" (The number in each row times the number in a column. The side lengths. 9 x 10.)
Activity 2 Narrative: Going, Going, Gone	**Addressing CCSS:** 3.MD.C.7b
20 min	The purpose of this activity is to gradually withdraw the grid from problems involving area, but still leave enough support that students can visualize the unit squares in the rectangle. Throughout the lesson, students will be encouraged to visualize rows and columns. This structure is important for students to visualize, even in the absence of the grid, because it helps confirm that multiplying the side lengths of the rectangle results in the total number of unit squares.

Task Statement

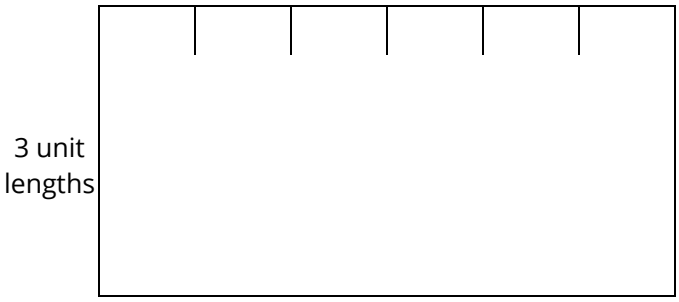
Part 1

The rectangle is marked off in unit lengths. What is the area of the rectangle in square units?



Part 2

This rectangle is marked off in unit lengths along the top and labeled with the length in unit lengths on the left side. What is the area of this rectangle in square units?



Student Responses

Part 1

32 square units

Part 2

18 square inches

Launch/Activity

Part 1

- Groups of 2
- "This problem involves finding the area of a rectangle that is marked off in unit lengths. Think about how you might solve this problem."
- 1 minute: quiet think time
- 3–5 minutes: partner work
- Monitor for students who draw the missing grid lines before they can multiply to find the area of the rectangle.
- Consider asking:
  - "How would you describe the rows and columns if you pictured the squares in the rectangle?"
  - "How could you find the total number of square units?"

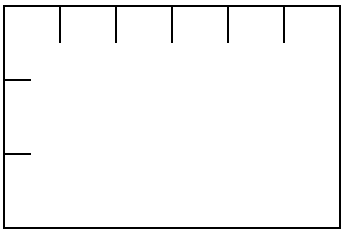
Part 2

- "This rectangle is marked off in unit lengths along the top and is labeled with the length in units on the side. Think about how you might find the area of this rectangle."
- 1 minute: quiet think time
- 3–5 minutes: partner work
- Monitor for students who create the missing grid lines before they can multiply to find the area of the rectangle.

	• Consider asking:
	  • "How would you describe the rows and columns if you pictured the squares in the rectangle?"
	  • "How could you find the total number of square units?"
	
	Synthesis
	• Display: samples of student work where student created the missing grid lines.
	• Ask: "How can creating the grid from the tick marks help us see the missing groups?" (I can see all the squares in a row. I can see all the squares in a column. I can see how many rows there are. I can see how many columns there are.)

Lesson Synthesis

Display:



Ask: "How would you describe the equal groups of unit squares to someone who was having trouble picturing them?" (The marks across the top show that there would be 6 squares in each row. The marks across the top show that there would be 6 columns. The marks along the side show that there would be 3 rows. The marks along the side show that there would be 3 squares in each column.)