# IM K–5 MATH™ v.I

Illustrative Mathematics®
LEARN MATH FOR LIFE

## IM K–5 Math™ v.I certified by Illustrative Mathematics®

*The final piece to a fully aligned K–12 curriculum*

@IllustrateMath

Illustrative Mathematics

#LearnWithIM

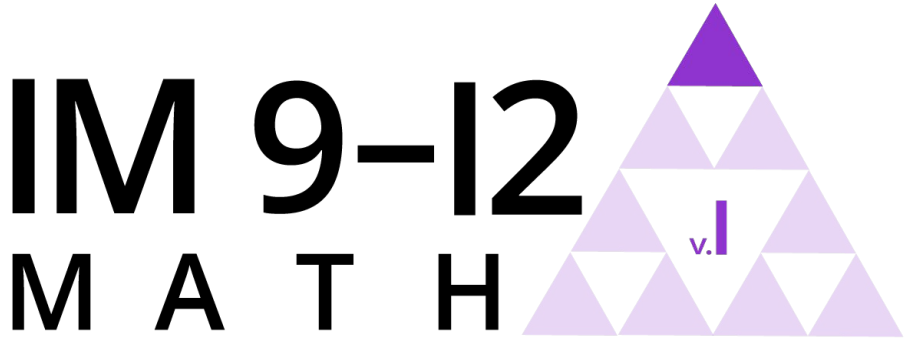
# Today's Presenters

**Sarah Caban**
**Grade 5 Lead Writer K-5 Math**
**Illustrative Mathematics**

Sarah believes that all students learn math by doing math together, and sees great value in listening to students' ideas.
She started her teaching career on an island off the coast of Maine, teaching in a K–8 one-room schoolhouse, then she spent four years teaching grade 4. Since 2001 she has been a K–12 math coach and coordinator, and she also presents professional development at local, state, and national conferences.
At IM, she facilitated professional learning for the Illustrative Mathematics Middle School Curriculum, and her work as a Grade 5 Lead puts her passion for listening and collaboration at the heart of every lesson she designs, teaches, and places in a teacher's hand.

# A seamless, coherent, and aligned mathematical experience





# Creating a world where all learners know, use, and enjoy mathematics

# IM Mission

In the chat, respond to the following question:

**What does it mean for a curriculum to support *all learners?***

# Equitable Practice and Structures

# K–5 Beta Experience

*"The tools and adaptations to meet the needs of diverse learners are written right into the curriculum.* ***All students are welcomed into the math*** *with accessible routines and materials."*

*"Students are talking about math like they never have before and* ***students who would typically not speak up in math class now join the conversation****."*

# Discussion Topics

- Problem-based Lesson Structure
- The Student Experience
- The Teacher Experience
- K-5 Curriculum Assessment
- Supporting Student Learning
- Learning Through the Work of Teaching

# Problem-based Curriculum

# Why problem-based curriculum?

"Students learn mathematics as a result of solving problems. Mathematical ideas are the *outcomes* of the problem-solving experience rather than the elements that must be taught before problem solving."

Hiebert, J., et. al. (1996).
Problem solving as a basis for reform in curriculum and instruction

# Problem-based curriculum

## Students...

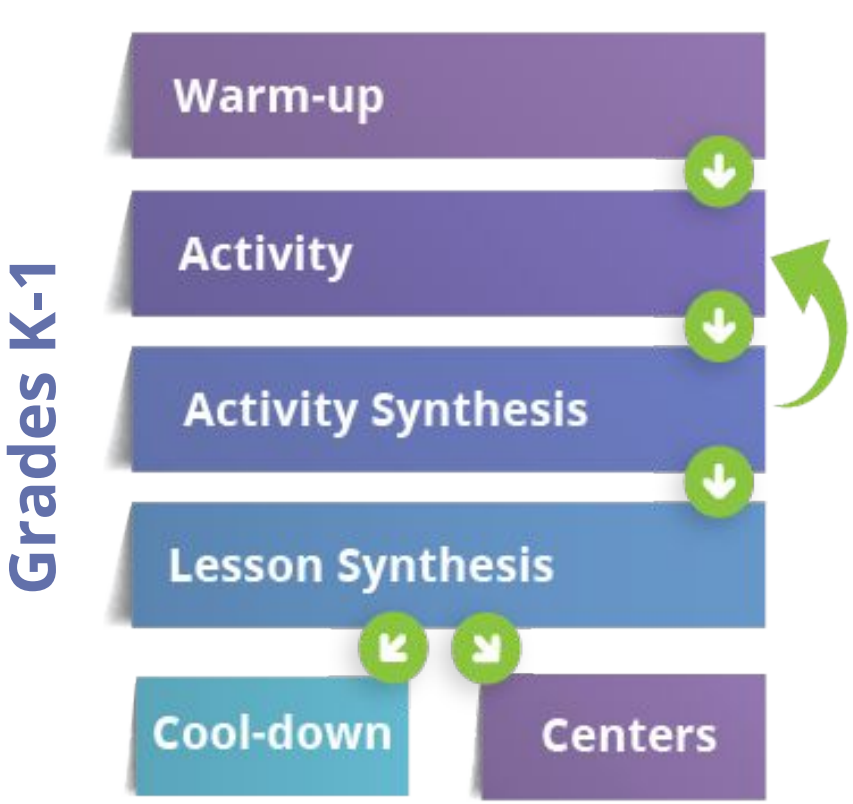
- have invitational and accessible entry points
- work on activities individually and in groups
- take various approaches to problems

## Teachers...

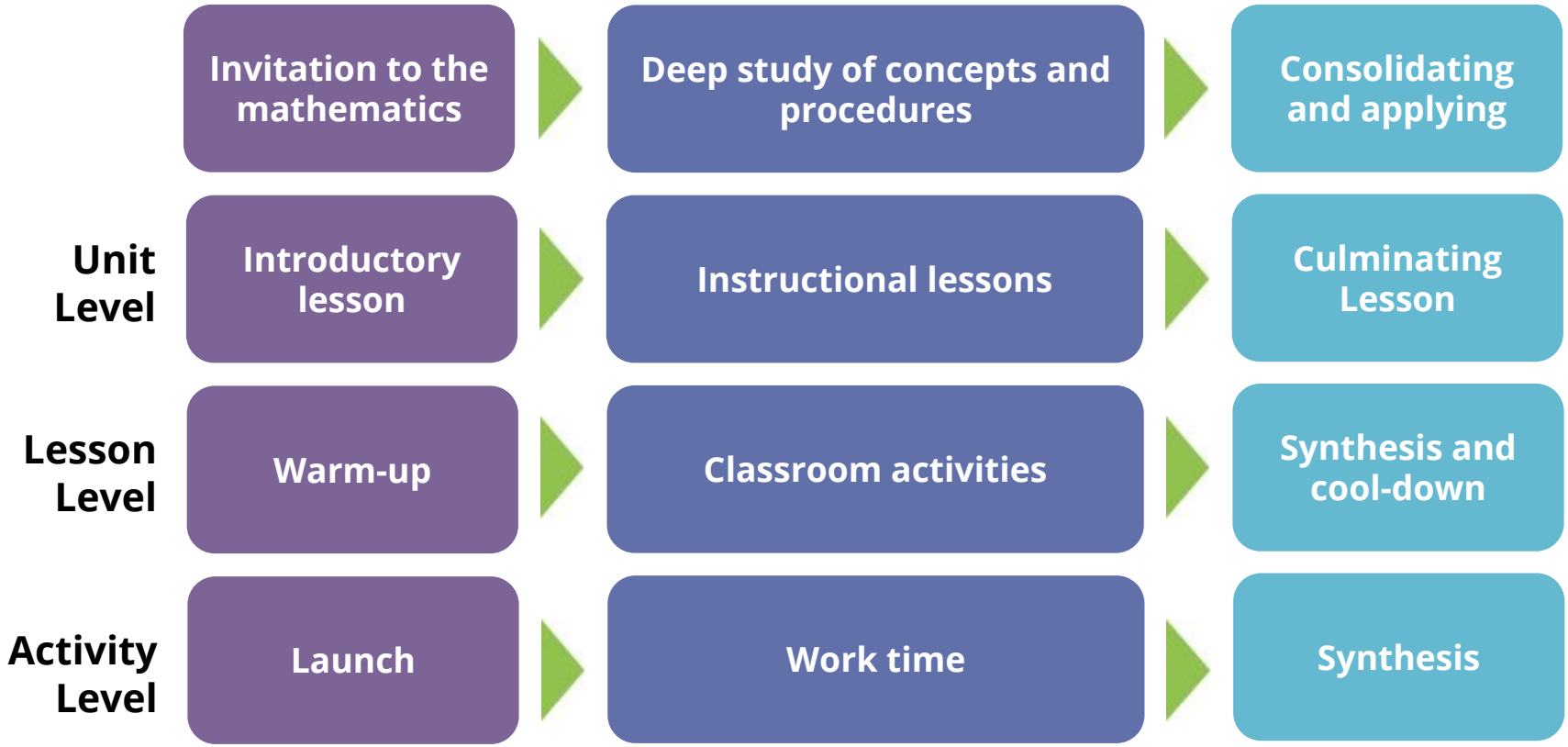
- ensure students understand the problem from the start
- listen and learn from student work and discussions
- synthesize learning at the end of each activity and lesson

# Overarching Design Structure

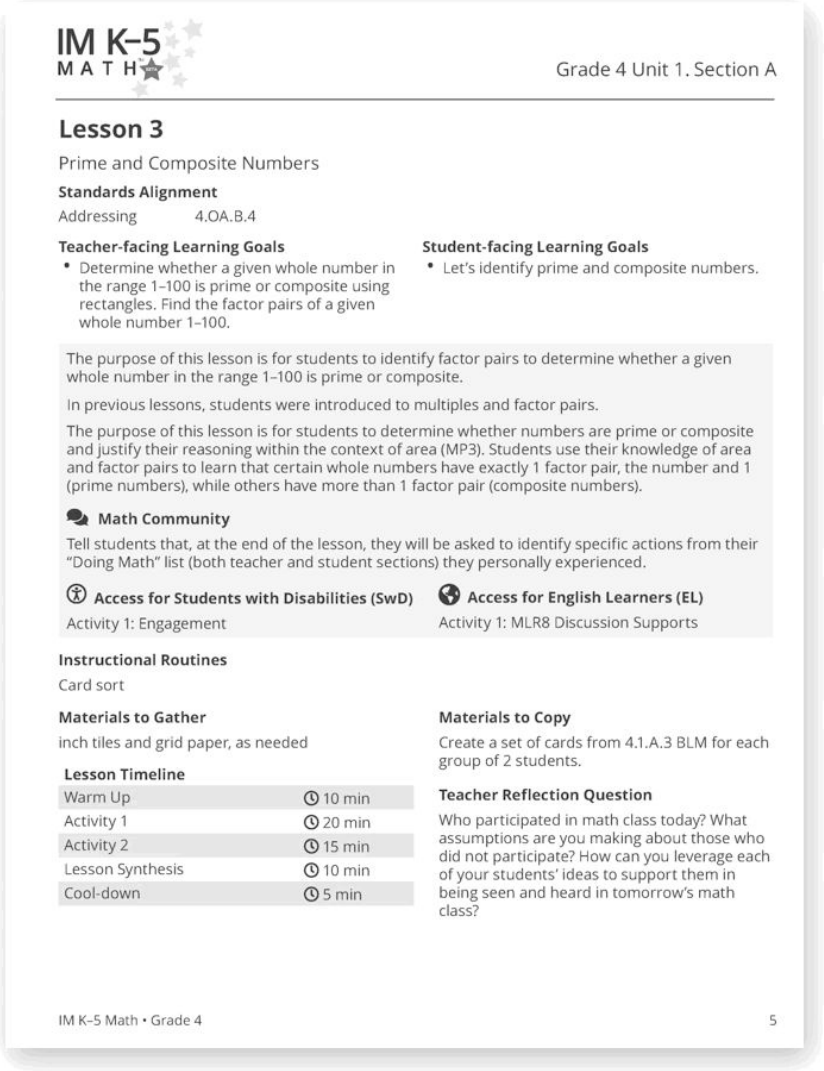
# Structure of a 60-minute Lesson




# Overarching Design Structure



# Structure of Materials



IM K–5 MATH



## Lesson 3

Prime and Composite Numbers

**Standards Alignment**

Addressing 4.OA.B.4

**Teacher-facing Learning Goals**

- Determine whether a given whole number in the range 1–100 is prime or composite using rectangles. Find the factor pairs of a given whole number 1–100.

**Student-facing Learning Goals**

- Let's identify prime and composite numbers.

The purpose of this lesson is for students to identify factor pairs to determine whether a given whole number in the range 1–100 is prime or composite.

In previous lessons, students were introduced to multiples and factor pairs.

The purpose of this lesson is for students to determine whether numbers are prime or composite and justify their reasoning within the context of area (MP3). Students use their knowledge of area and factor pairs to learn that certain whole numbers have exactly 1 factor pair, the number and 1 (prime numbers), while others have more than 1 factor pair (composite numbers).

**Math Community**

Tell students that, at the end of the lesson, they will be asked to identify specific actions from their "Doing Math" list (both teacher and student sections) they personally experienced.

**Access for Students with Disabilities (SwD)**

Activity 1: Engagement

**Access for English Learners (EL)**

Activity 1: MLR8 Discussion Supports

**Instructional Routines**

Card sort

**Materials to Gather**

inch tiles and grid paper, as needed

**Materials to Copy**

Create a set of cards from 4.1.A.3 BLM for each group of 2 students.

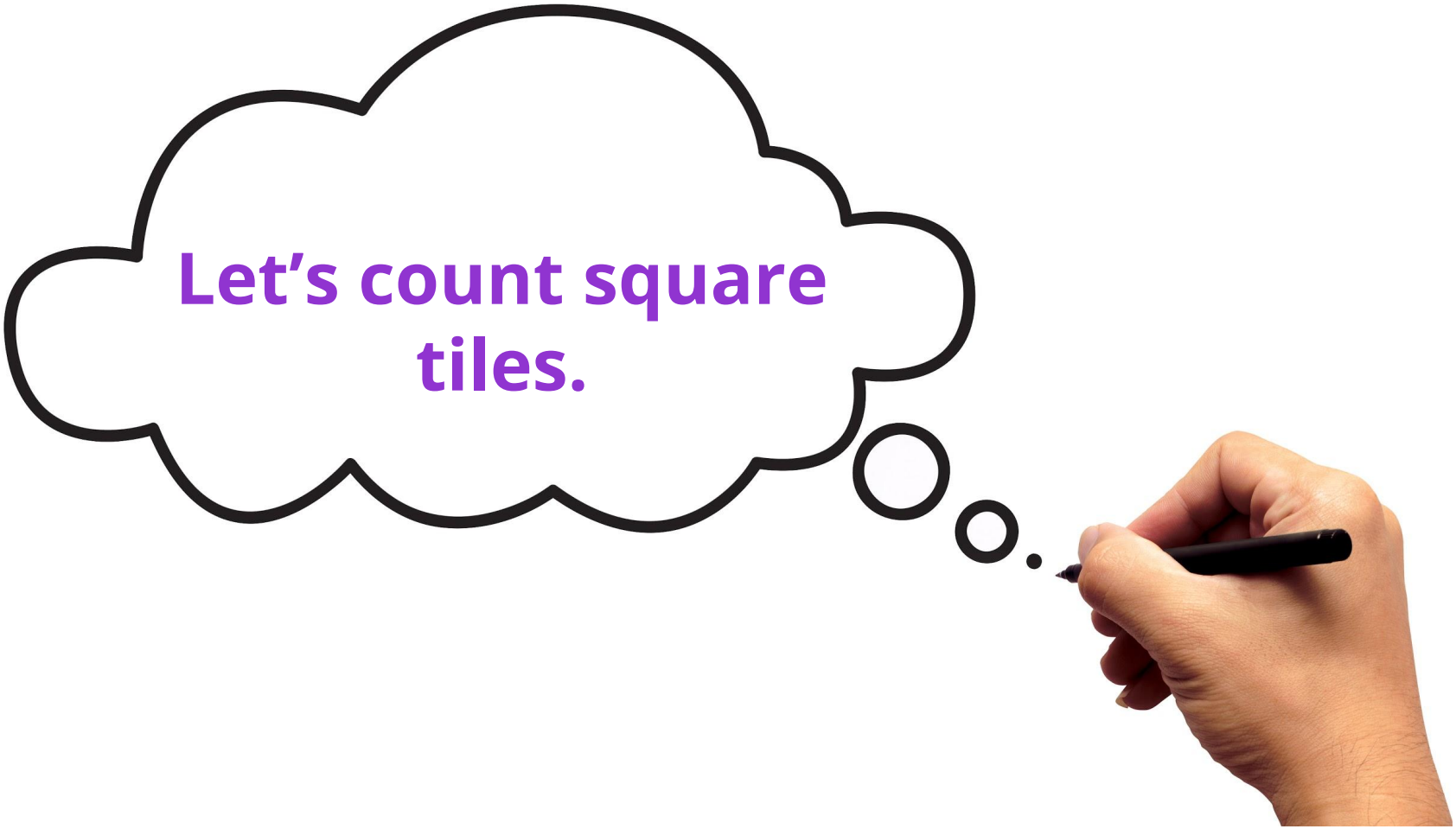
**Lesson Timeline**

| | |
|---|---|
| Warm Up | 10 min |
| Activity 1 | 20 min |
| Activity 2 | 15 min |
| Lesson Synthesis | 10 min |
| Cool-down | 5 min |

**Teacher Reflection Question**

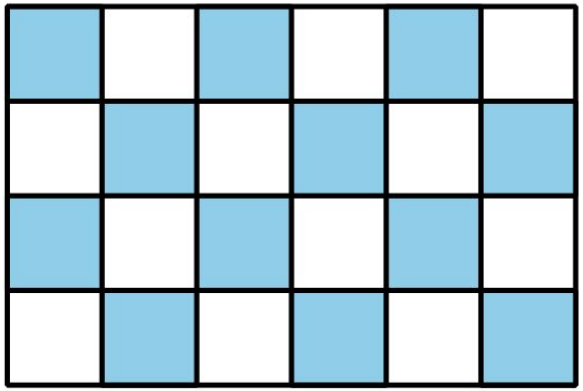
Who participated in math class today? What assumptions are you making about those who did not participate? How can you leverage each of your students' ideas to support them in being seen and heard in tomorrow's math class?

# The Student Experience

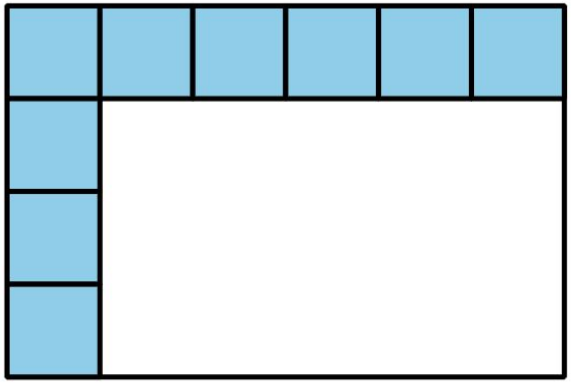

Let’s count square tiles.

# Warm Up: Which One Doesn't Belong

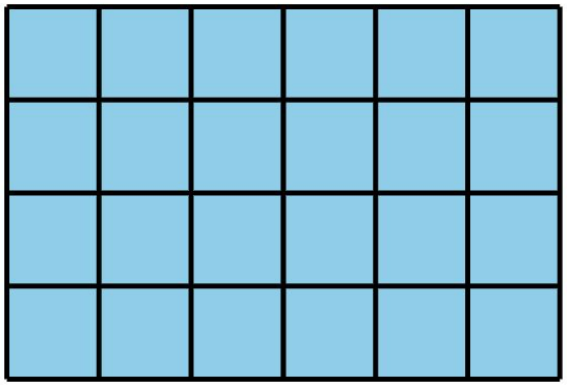
A



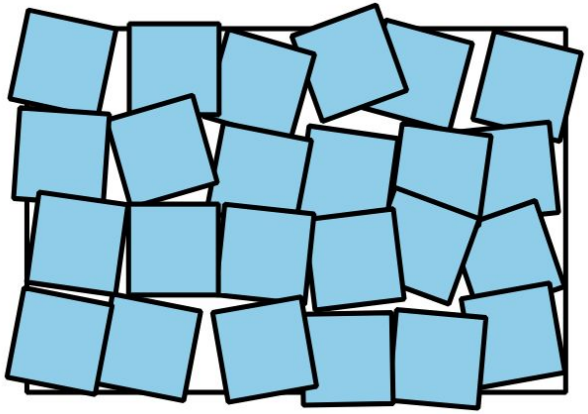
C



B

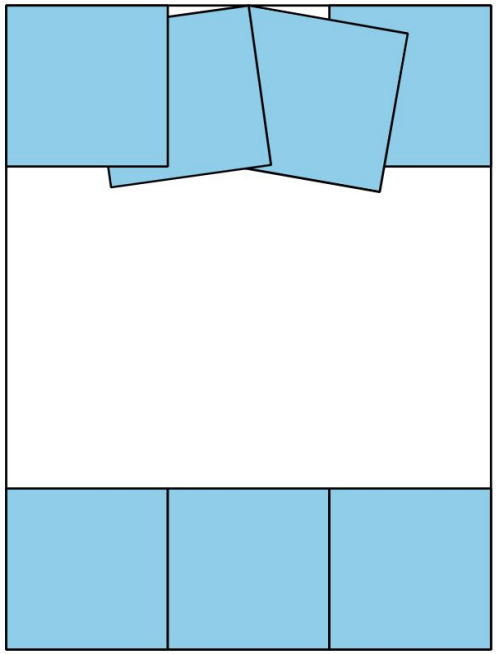


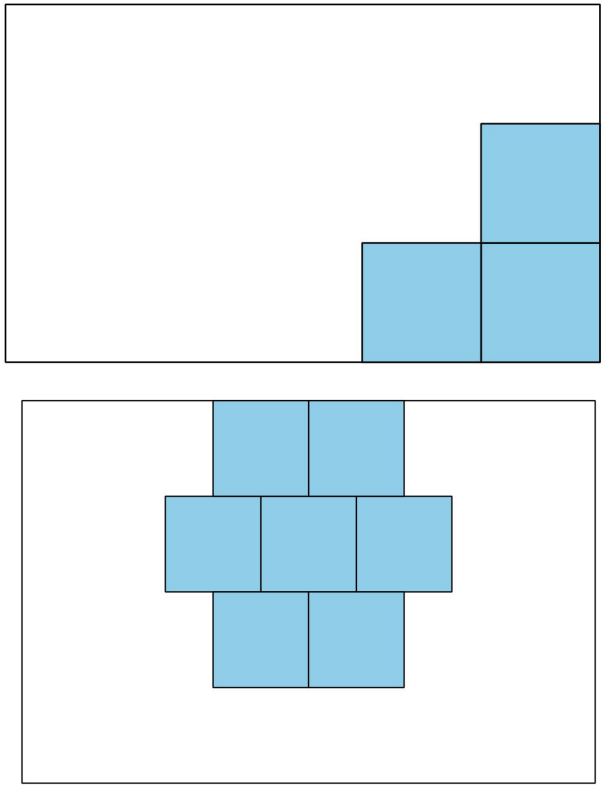
D



# Activity 1: Time to Tile

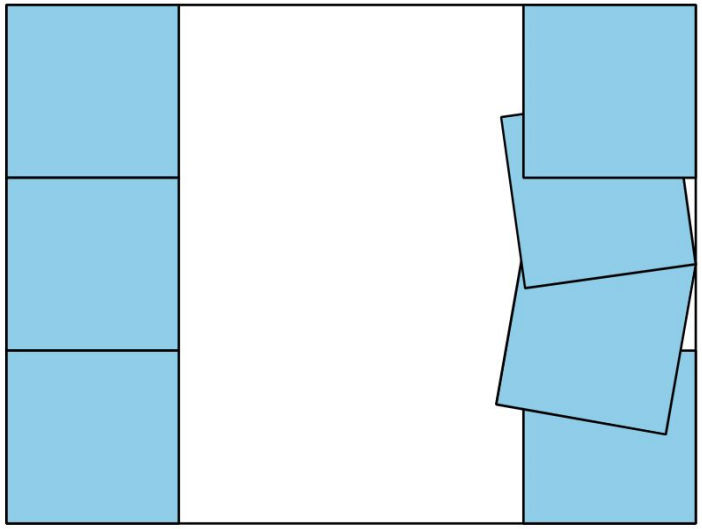
Describe or show how to use the square tiles to measure the area of each rectangle. You can place square tiles on the handout where squares are already shown. You can also move the tiles, if needed.

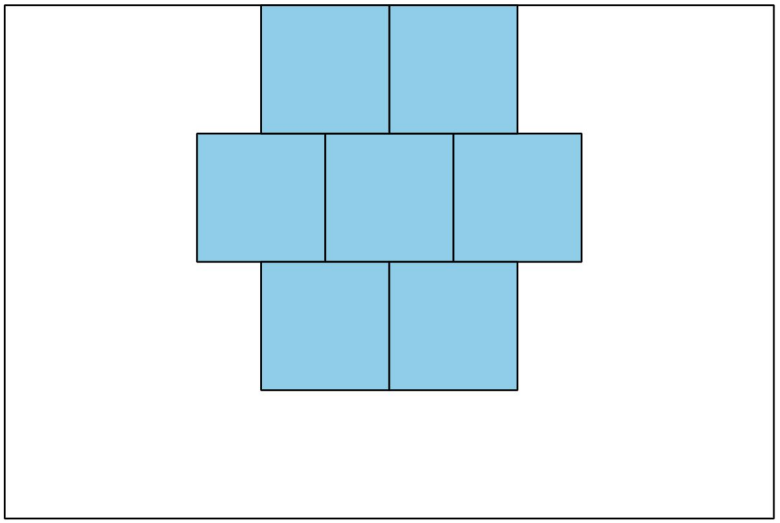




# Activity 1: Synthesis

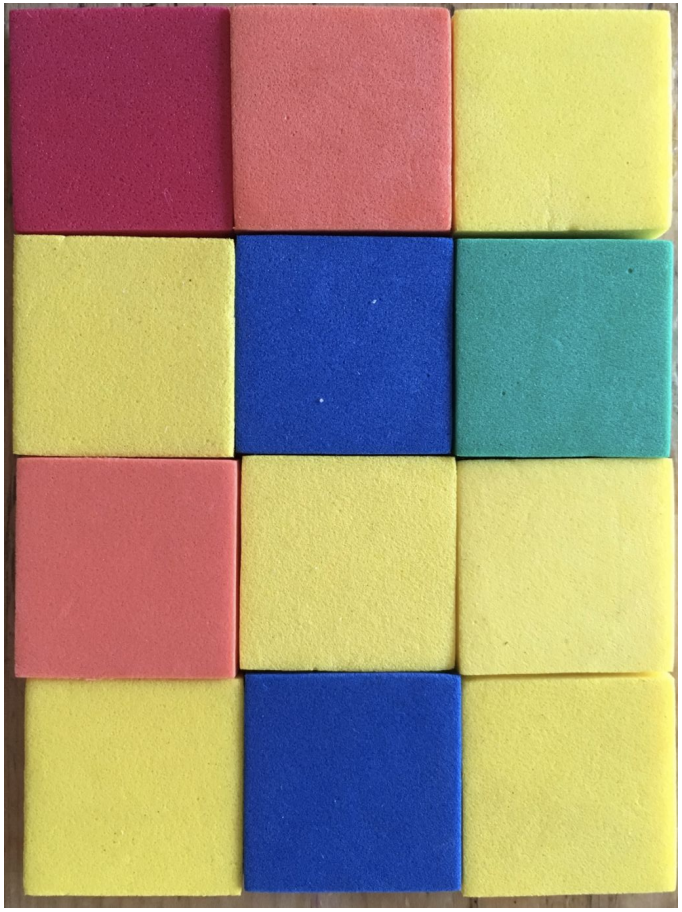
"Why did the square tiles in some of the rectangles need to be adjusted before we could finish finding the area of the rectangle?"

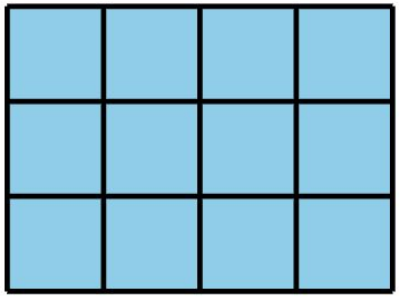


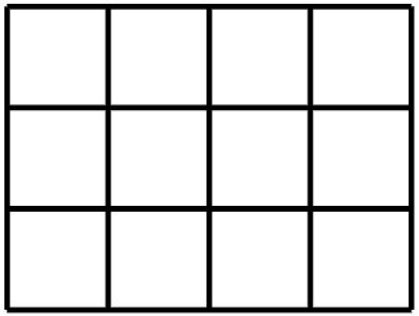


# Activity 2: Card Sort: Rectangles Launch

What do you notice?  What do you wonder?



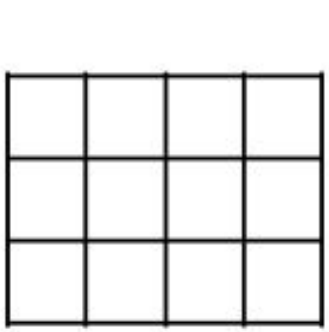


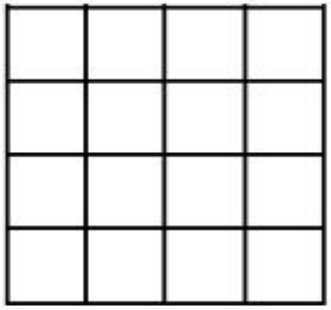


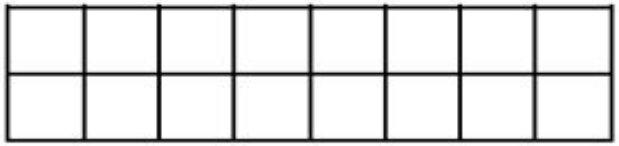
# Activity 2: Card Sort: Rectangles

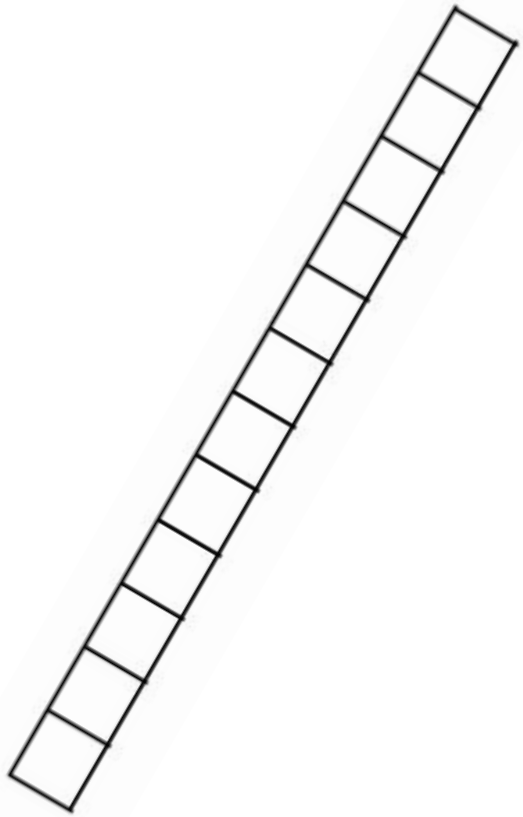
Your teacher will give you a set of cards that show rectangles. Sort the cards into categories of your choosing. Be prepared to explain your categories.

*Examples:*







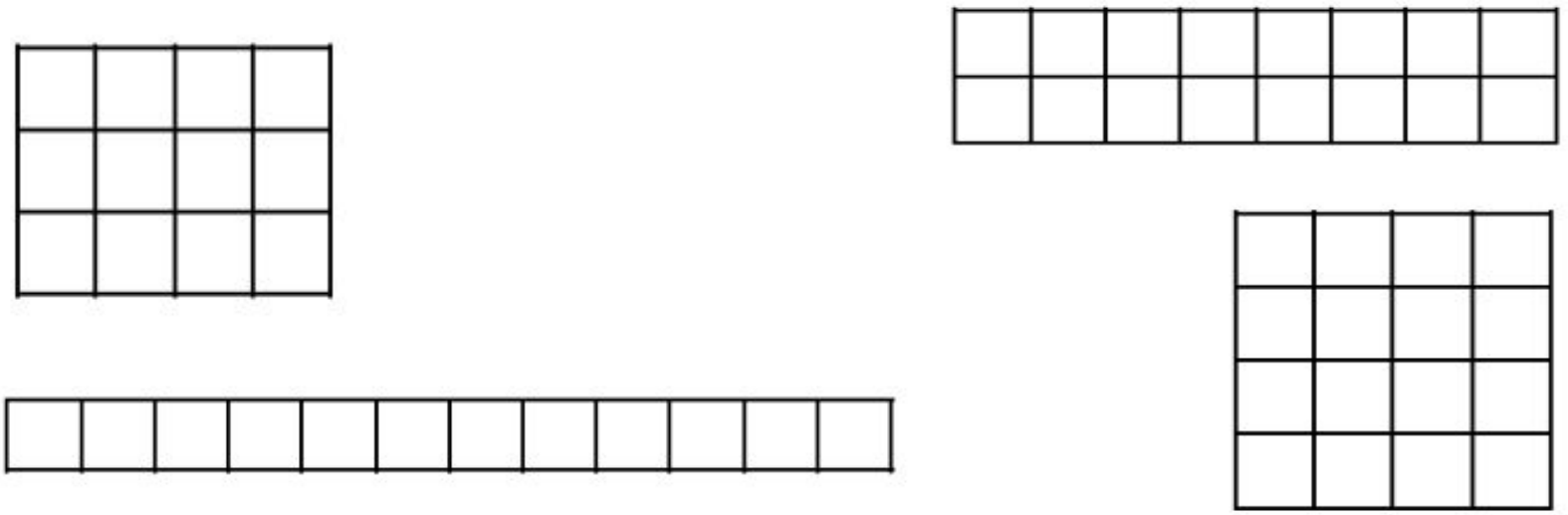


# Activity 2: Synthesis

“How did you sort the rectangles?”

"How did you decide that a rectangle belonged a category?"

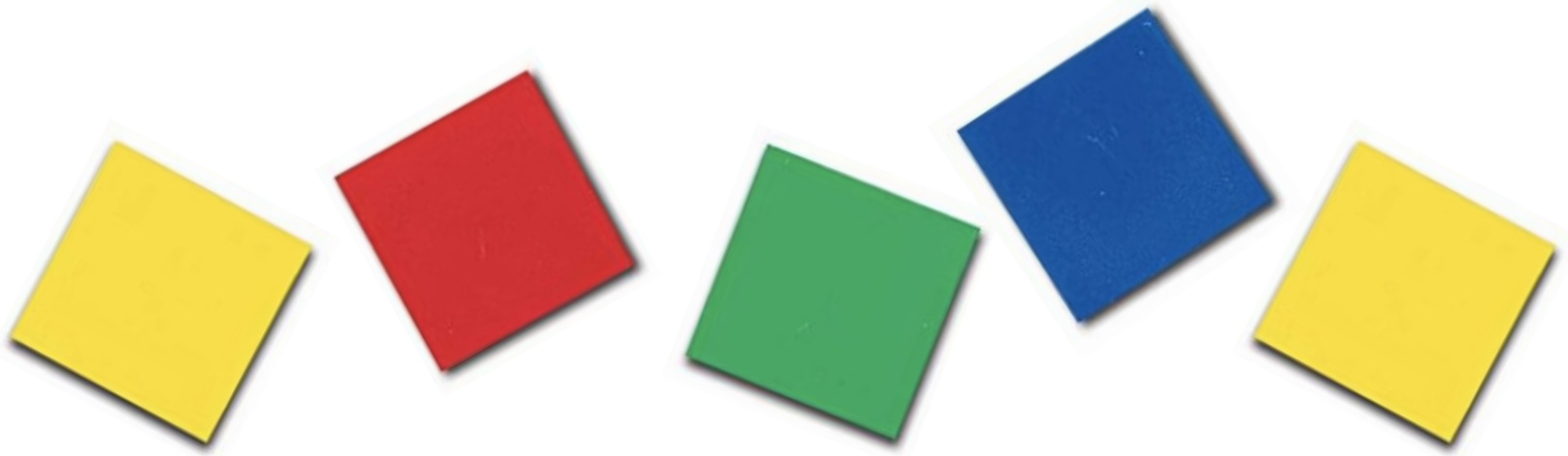
"How did you know that the rectangle you created had the same area as the other rectangles in that category?"



# Lesson Synthesis

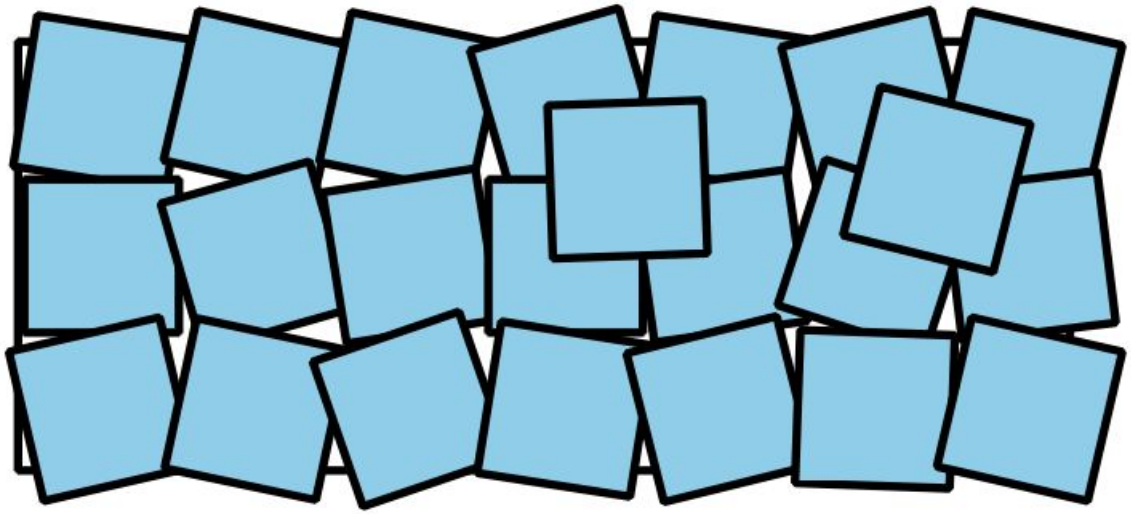
"Today we learned we can draw squares in rectangles to represent tiling. We can count the squares to find the area of a rectangle just like we would count tiles."

"What helpful features do rectangles have that help us find their area?"



# Cool-down

Priya says this rectangle has an area of 23 square units because she covered it with 23 square tiles.

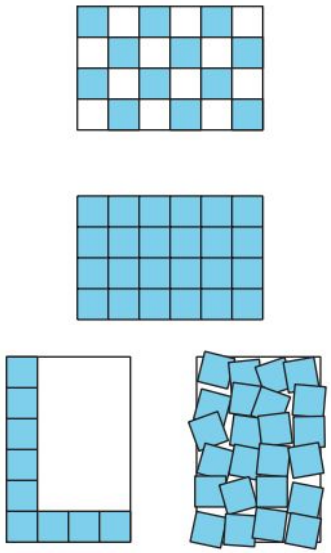


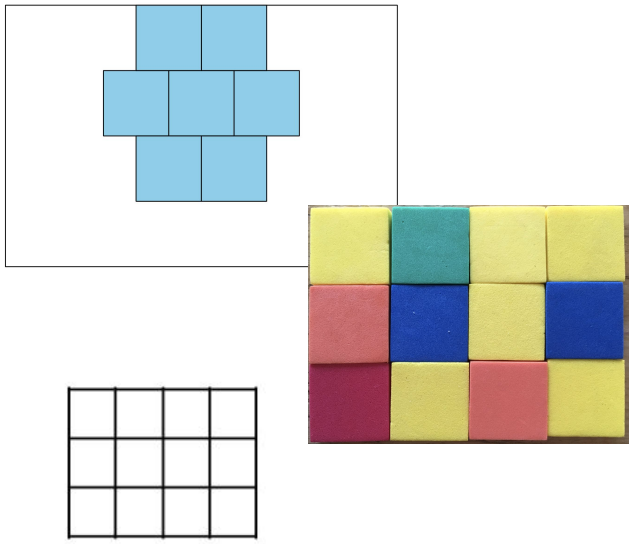
Do you agree with Priya? Explain or show your reasoning.

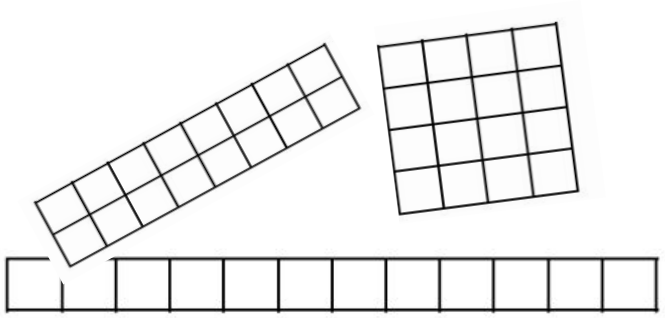
# The Structure of a Lesson

**Invitation to the mathematics** → **Deep study of concepts and procedures** → **Consolidating and applying**

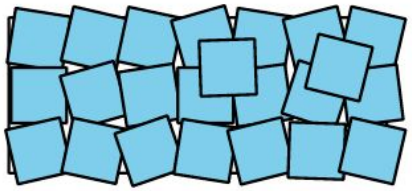
Which one doesn't belong?







Priya says this rectangle has an area of 23 square units because she covered it with 23 square tiles.



Do you agree with Andre? Explain your reasoning.

# K–5 Beta Experience

*"This curriculum has taken all of the **fantastic instructional routines** teachers were pulling from multiple sources and ties them up with a great big bow into a rigorous, coherent curriculum resource!"*

*"The **clearly written 'math story'** helps me know what has come before and how the material connects to what comes next."*

# The Teacher Experience

# Unit Breakdown

Grade 3 Unit 2: *Area and Multiplication*

| Section A | Section B | Section C |
| --- | --- | --- |
| *Concepts of Area Measurement* | *Relate Area to Multiplication* | *Area of Figures Composed of Rectangles* |

# Unit Narrative

## Unit 2: Area and Multiplication

### Unit Learning Goals

- Students learn about area concepts and relate area to multiplication and to addition.

In this unit, students are introduced to the concept of area as how much space a two-dimensional shape covers. Initially, students compare shapes directly to decide which covers more space. Students then work more precisely by tiling shapes using different units such as pattern blocks and square tiles. They learn the importance of tiling figures with no gaps or overlaps and count squares to determine the area of a rectangle.

Students relate the measurement of area to the operation of multiplication as they view rectangles as arrays of squares and see the rows and columns as equal groups. Then they explicitly connect the area of rectangles to multiplication expressions and understand that multiplying the side lengths of rectangles gives the same number as counting squares that tile a rectangle. Students are introduced to some standard area units: square inches, square centimeters, square feet, and square meters. They measure and multiply side lengths of rectangles to find area.

In the second section in this unit, representations progress from rectangles that are tiled or shown with full grids to partial grids, or from sides with tick marks to rectangles with only side length measurements. This facilitates the transition from counting squares to multiplying side lengths to find area. By the end of the section, students multiply side lengths to solve real-world and mathematical problems related to area.

$$6 \times 3 = 18$$

# Section Narrative

## Section A: Concepts of Area Measurement

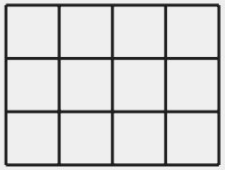
### Standards Alignments

| | |
|---|---|
| Addressing | 3.MD.C.5, 3.MD.C.5.a, 3.MD.C.5.b, 3.MD.C.6, 3.OA.A.1 |
| Building Towards | 3.MD.C.5 |

### Section Learning Goals

- Describe area as the number of unit squares that cover a plane figure without gaps and overlaps.
- Measure the area of rectangles by counting unit squares.

In this section, students spend time reasoning about area as an attribute of two-dimensional shapes. They develop a sense of area as the amount of space covered by a shape. Then they tile shapes with squares. They learn that the area of a shape is the number of squares that are used to tile the shape with no gaps or overlaps.



# Lesson Overview Page



## Lesson 3: Let's Tile Rectangles!

### Standards Alignments

Addressing 3.MD.C.5.b, 3.MD.C.6

### Teacher-facing Learning Goals

- Explain that rectangles that can be covered by the same number of unit squares without gaps or overlaps have the same area.
- Find the area of rectangles (within 24 square units) by counting unit squares.

### Student-facing Learning Goals

- Let's count square tiles.

### Lesson Purpose

The purpose of this lesson is for students to measure area by counting square tiles and create rectangles with a given area.

In previous lessons, students learned that counting square tiles that cover a shape gives the area of the shape in square units. In this lesson, students further explore tiling and find the area of partially tiled rectangles. Students also create rectangles of a specific area to demonstrate they understand that rectangles covered by the same number of square tiles without gaps or overlaps have the same area.

Students should have access to square tiles throughout the lesson and also be encouraged to draw the partitioned rectangles they create with the tiles.

### Students with Disabilities (SwD)

- Representation (Activity 2)

### English Learners (EL)

- MLR8 (Activity 2)

### Instructional Routines

Card Sort (Activity 2), MLR1 Stronger and Clearer Each Time (Activity 1), Which One Doesn't Belong? (Warm-up)

### Materials to Gather

- Inch tiles: Activity 1, Activity 2

### Materials to Copy

- Time to Tile (groups of 1): Activity 1
- Rectangular Area Cards V1 (groups of 2): Activity 2





### Lesson Timeline

| | |
|---|---|
| Warm-up | 10 min |
| Activity 1 | 15 min |
| Activity 2 | 20 min |
| Lesson Synthesis | 10 min |
| Cool-down | 5 min |

### Teacher Reflection Question

Reflect on students' counting strategies. What strategies are they comfortable using as they find area? What strategies could use more practice?

## Cool-down (to be completed at the end of the lesson) 5 min

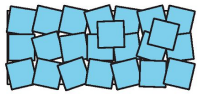
Tile a Rectangle

### Standards Alignments

Addressing 3.MD.C.5.b

### Student-facing Task Statement

Andre says this rectangle has an area of 23 square units because he covered it with 23 square tiles.



Do you agree with Andre? Explain your reasoning.

### Student Responses

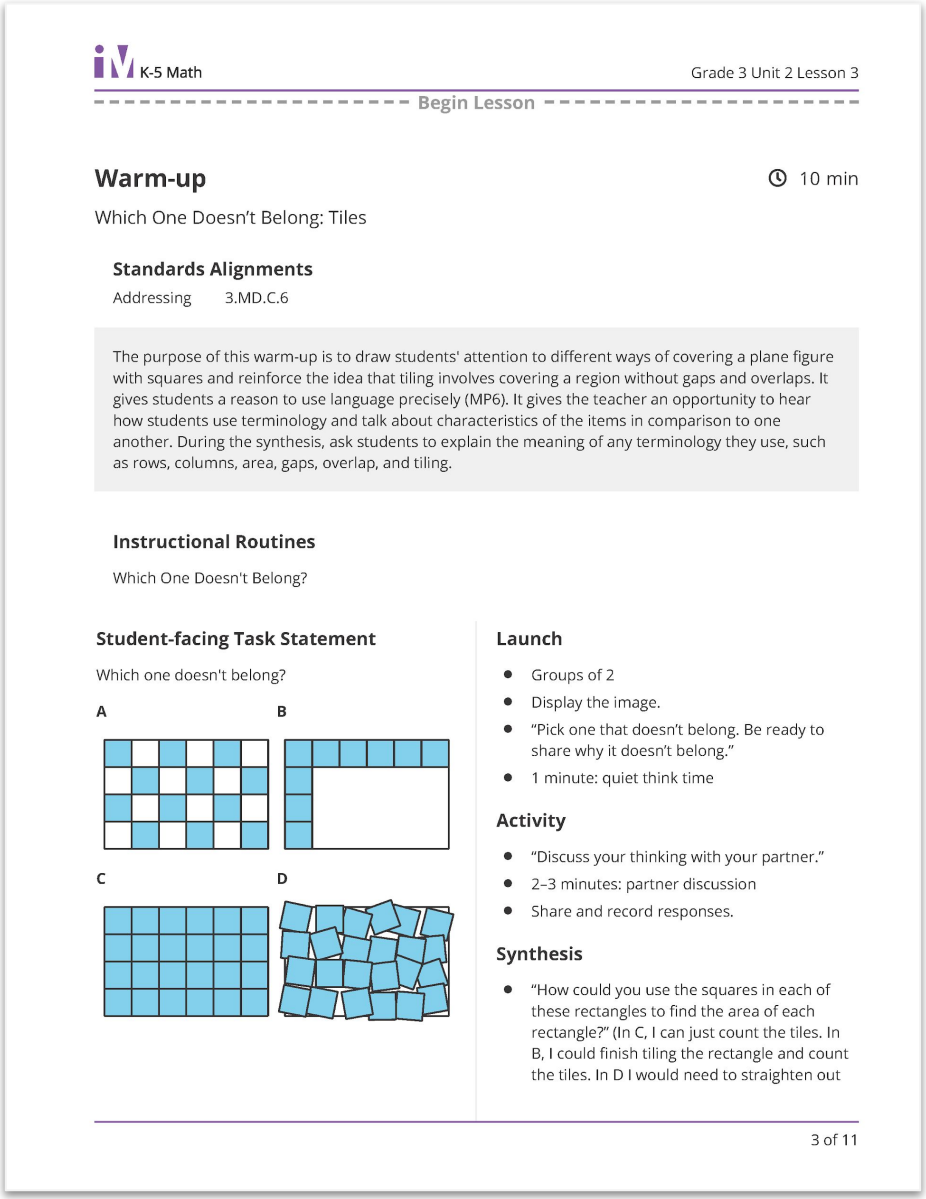
Sample responses:

- No, even though it is 23 tiles, we don't know that completely fills the rectangle because you can see some of the rectangle and some of the squares are overlapping.
- No, even though it is 23 tiles, you can see that some of the space in the rectangle isn't covered.

# Lesson Plan

2-column format

- Student view
- Teacher view



K-5 Math

Grade 3 Unit 2 Lesson 3

Begin Lesson

## Warm-up

10 min

Which One Doesn't Belong: Tiles

### Standards Alignments

Addressing 3.MD.C.6

The purpose of this warm-up is to draw students' attention to different ways of covering a plane figure with squares and reinforce the idea that tiling involves covering a region without gaps and overlaps. It gives students a reason to use language precisely (MP6). It gives the teacher an opportunity to hear how students use terminology and talk about characteristics of the items in comparison to one another. During the synthesis, ask students to explain the meaning of any terminology they use, such as rows, columns, area, gaps, overlap, and tiling.

### Instructional Routines

Which One Doesn't Belong?

### Student-facing Task Statement

Which one doesn't belong?

A



B



C



D



### Launch

- Groups of 2
- Display the image.
- "Pick one that doesn't belong. Be ready to share why it doesn't belong."
- 1 minute: quiet think time

### Activity

- "Discuss your thinking with your partner."
- 2–3 minutes: partner discussion
- Share and record responses.

### Synthesis

- "How could you use the squares in each of these rectangles to find the area of each rectangle?" (In C, I can just count the tiles. In B, I could finish tiling the rectangle and count the tiles. In D I would need to straighten out

3 of 11

# K–5 Beta Experience

*"...it is an educative curriculum.* ***Teachers are learning so much right along with their students****."*

***"My favorite part of math this year is everything."***
*- 4th Grade Student*

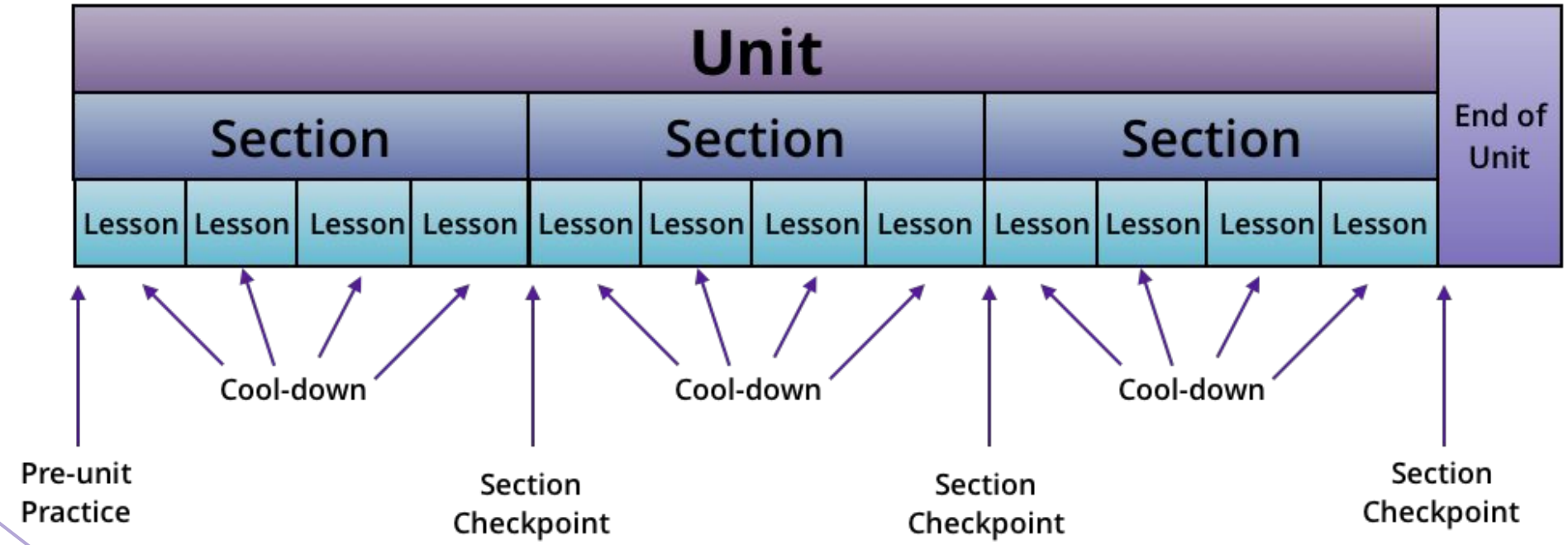
*"It is the most* ***inviting, rigorous and exciting*** *curriculum I've ever taught..."*

# K-5 Curriculum Assessment

# Opportunities for Assessment

1. Pre-unit Practice Problems
2. Cool-downs
3. Section Checkpoints
4. End-of-Unit Assessment

# Opportunities for Assessment



# Pre-unit Practice Problems

## Section A Practice Problems

1. **Pre-unit**

a. Partition the rectangle into 4 equal rows and 5 equal columns.

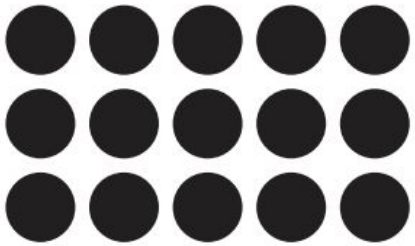
b. How many small squares are there in the rectangle?

# Pre-unit Practice Problems

**Pre-unit**

How many dots are in each array? Explain your reasoning.

a.

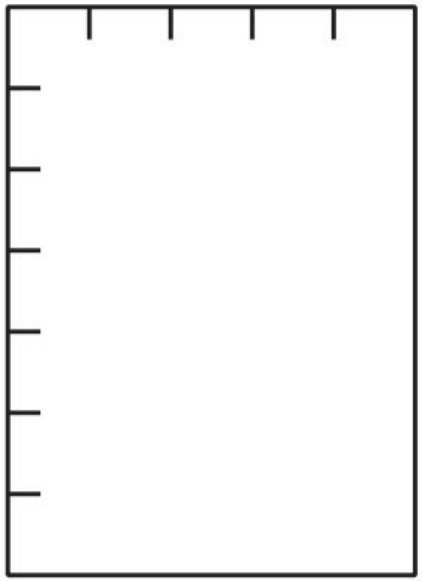


b.

# Cool-down: Response to Student Thinking

**Cool-down:** Where are the Squares?

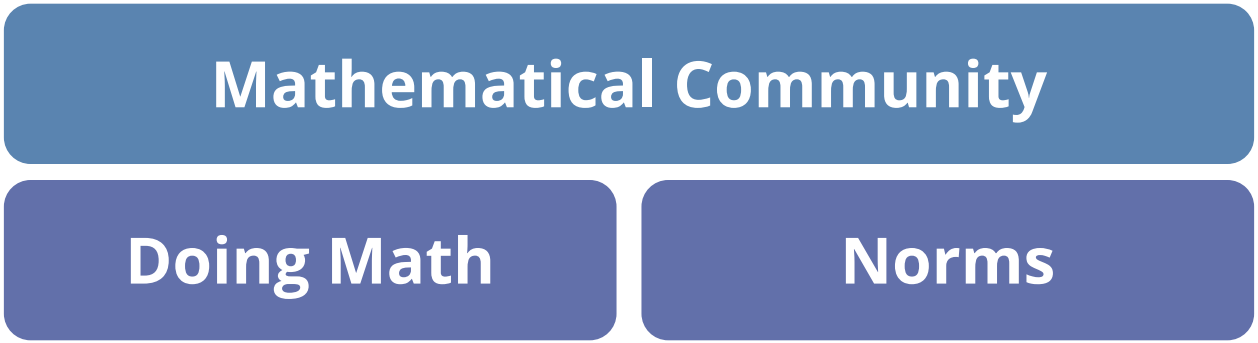
The tick marks on the sides of the rectangle are 1 foot apart. What is the area of the rectangle? Explain or show your reasoning.



| Next day support: | Prior-unit support: |
| --- | --- |
| • During the launch of the next day's lesson, have students brainstorm together ways to find the total number of square units in a rectangle where no square units are visible. | • Grade 2, Unit 3, Section A: Metric Measurement |

# Supporting Student Learning

# Building Mathematical Community




What does it look and sound like to do math together as a mathematical community?

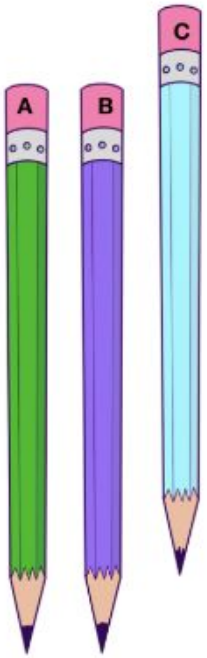
What norms, or expectations, should we be mindful of as we do math together in our mathematical community?

# Equitable Practice and Structures

# Instructional Routines

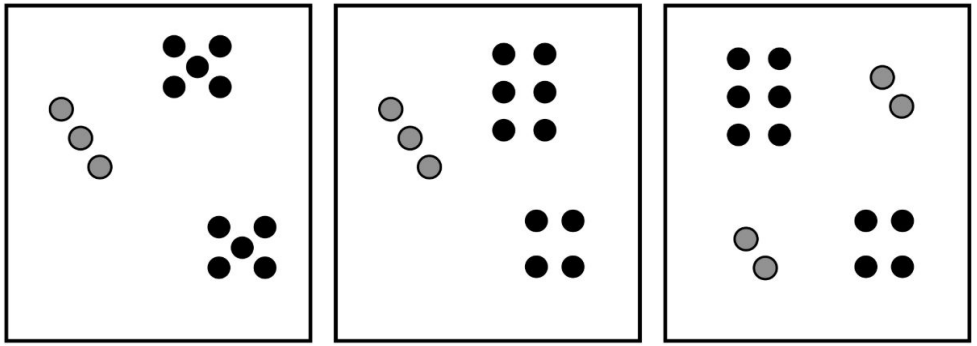
What do you know about _____?

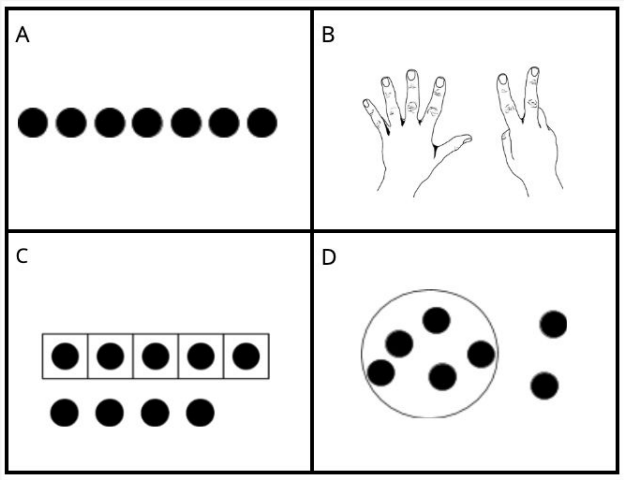
Notice and Wonder



# Instructional Routines

How Many Do You See?






Which One Doesn't Belong?

# Instructional Routines

8 x 20 = 8 x 2 x 10

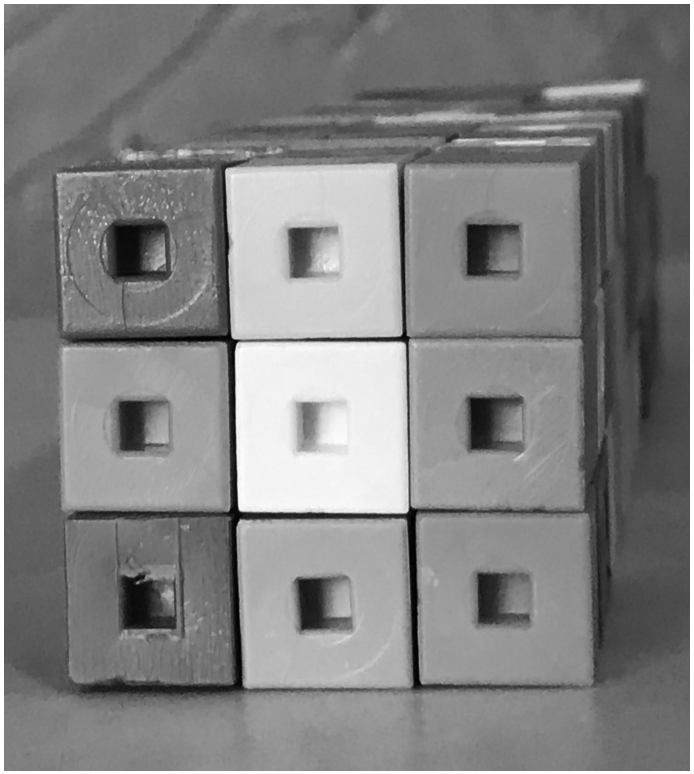
8 x 20 = 2 x 8 x 10

8 x 20 = 16 x 10

6 x 20 = 12 x 10

True or False?

Estimation Exploration



# Instructional Routines

Choral Count

| | | |
|---|---|---|
| $\frac{1}{6}$ | $\frac{2}{6}$ | $\frac{3}{6}$ |
| $\frac{4}{6}$ | $\frac{5}{6}$ | $\frac{6}{6}$ |
| $\frac{7}{6}$ | $\frac{8}{6}$ | $\frac{9}{6}$ |
| $\frac{10}{6}$ | $\frac{11}{6}$ | $\frac{12}{6}$ |
| $\frac{13}{6}$ | $\frac{14}{6}$ | $\frac{15}{6}$ |
| $\frac{16}{6}$ | $\frac{17}{6}$ | $\frac{18}{6}$ |

Number Talk

$5 \times \frac{1}{2}$

$5 \times \frac{3}{2}$

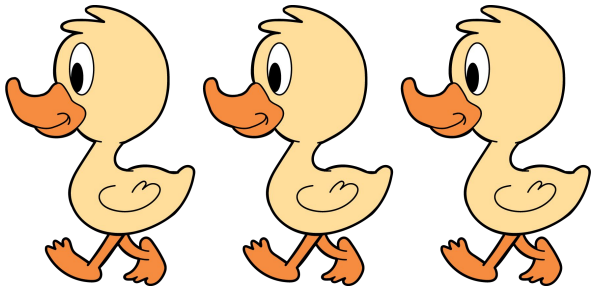
$6 \times \frac{1}{3}$

$6 \times \frac{2}{3}$

# Instructional Routines: Kindergarten

## Act it Out

3 little ducks went out one day,
over the hill and far away.
Mother duck said, “Quack, quack, quack.”
Then 3 little ducks came back.



## Questions About Us

- “How can we figure out how many of us are here?” (We can count each person. We can count each circle in the 5-frame.)
- Count each student.
- “How many of us are here today?”
- Count each circle.
- “How many of us are here today?”

# Practice Problems

## Pre-unit

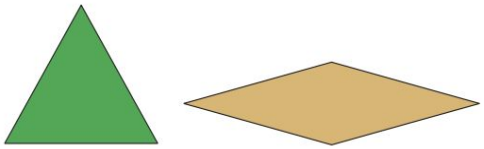
How many dots are in each array? Explain your reasoning.

a.

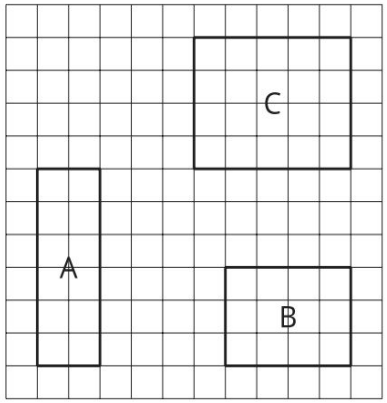
b.

## Exploration

Which shape has greater area, a green triangle pattern block or a tan rhombus pattern block? Explain your reasoning.



## Lesson-aligned




a. Find the area of each rectangle.

b. Can rectangles with different shapes have the same area? Explain your reasoning.

# Purposes of Centers

- Meant for extra practice in school or outside of school
- Build fluency across a year (or any skill that develops over time)

By design, centers:

- are aligned to grade levels and units.
- consist of stages with the same general structure.
- can be repeated with different results each time.
- primarily focus on the major work of grade.

# Two Types of Centers in a Unit

1. ***Addressing*** - Centers that address the work of the grade level unit.
2. ***Supporting*** - Reviews prior stages or supports ongoing fluency.

Prior grade level stages

**Center: Capture Squares**

Students generate a number and connect 2 dots that are adjacent to the number. If that line closes the square, they capture it and shade it in their color. The player to shade in 3 squares first is the winner.

**Stage 1**

**Lessons:**

- Grade3.2.A1 (supporting)
- Grade3.2.A2 (supporting)

**Stage 2**

**Lessons:**

- Grade3.2.A3 (supporting)

**Stage 3**

**Lessons:**

- Grade3.2.B5 (addressing)
- Grade3.2.B6 (addressing)
- Grade3.2.B7 (addressing)

Addresses the current grade level

# Supports for English Learners

- MLRs support language demands: reading, writing, speaking, listening, conversing, representing
- Structured but adaptable formats for amplifying, assessing, and developing students' language.

MLR1: Stronger and Clearer Each Time

MLR2: Collect and Display

MLR3: Clarify, Critique, Correct

MLR4: Information Gap

MLR5: Co-Craft Questions

MLR6: Three Reads

MLR7: Compare and Connect

MLR8: Discussion Supports

# Supports for Students with Disabilities

***Areas of cognitive functioning***

- Conceptual Processing
- Visual-spatial Processing
- Language
- Organization
- Memory
- Attention
- Social-emotional
- Fine-motor Skills

# Units By Grade Level: K–2

| | |
|---|---|
| **K.1** | Math in Our World |
| **K.2** | Numbers 1–10 |
| **K.3** | Flat Shapes All Around Us |
| **K.4** | Understanding Addition and Subtraction |
| **K.5** | Composing and Decomposing Numbers to 10 |
| **K.6** | Numbers 0–20 |
| **K.7** | Solid Shapes All Around Us |
| **K.8** | Putting it All Together |

| | |
|---|---|
| **1.1** | Adding, Subtracting, and Working with Data |
| **1.2** | Addition and Subtraction Story Problems |
| **1.3** | Adding and Subtracting within 20 |
| **1.4** | Numbers to 99 |
| **1.5** | Adding within 100 |
| **1.6** | Length Measurements within 120 Units |
| **1.7** | Geometry and Time |
| **1.8** | Putting it All Together |

| | |
|---|---|
| **2.1** | Adding, Subtracting, and Working with Data |
| **2.2** | Adding and Subtracting within 100 |
| **2.3** | Measuring Length |
| **2.4** | Addition and Subtraction on the Number Line |
| **2.5** | Numbers to 1,000 |
| **2.6** | Geometry, Time and Money |
| **2.7** | Adding and Subtracting within 1,000 |
| **2.8** | Equal Groups |
| **2.9** | Putting it All Together |

# Units By Grade Level: 3–5

| | |
|---|---|
| **3.1** | Introducing Multiplication |
| **3.2** | Area and Multiplication |
| **3.3** | Wrapping Addition and Subtraction within 1,000 |
| **3.4** | Relating Multiplication to Division |
| **3.5** | Fractions as Numbers |
| **3.6** | Measuring Length, Time, Liquid Volume, and Weight |
| **3.7** | Two-dimensional Shapes and Perimeter |
| **3.8** | Putting it All Together |

| | |
|---|---|
| **4.1** | Factors and Multiples |
| **4.2** | Fraction Equivalence and Comparison |
| **4.3** | Extending Operations to Fractions |
| **4.4** | From Hundredths to Hundred-thousandths |
| **4.5** | Multiplicative Comparison and Measurement |
| **4.6** | Multiplying and Dividing Multi-digit Numbers |
| **4.7** | Angles and Angle Measurement |
| **4.8** | Properties of Two-dimensional Shapes |
| **4.9** | Putting it All Together |

| | |
|---|---|
| **5.1** | Finding Volume |
| **5.2** | Fractions and Quotients and Fraction Multiplication |
| **5.3** | Multiplying and Dividing Fractions |
| **5.4** | Wrapping Up Multiplication and Division with Multi-digit Numbers |
| **5.5** | Place Value Patterns and Decimal Operations |
| **5.6** | More Decimals and Fraction Operations |
| **5.7** | Shapes on the Coordinate Plane |
| **5.8** | Putting it All Together |

# Learning Through the Work of Teaching

# Learning Through the Work of Teaching

**Curriculum**

**Professional Learning**

Four categories:

**Pedagogy**

**Student Thinking**

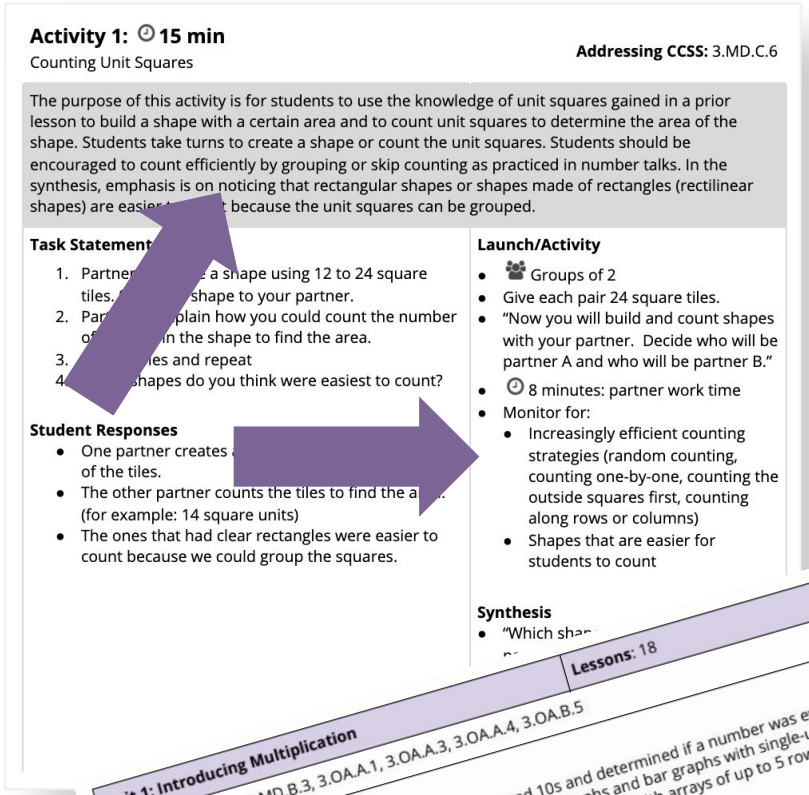
**Math Content**

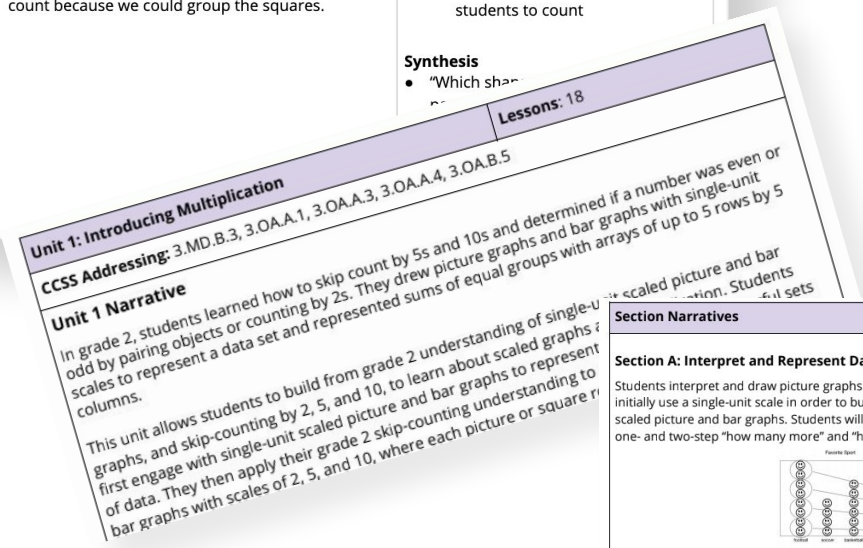
**Beliefs & Positioning**

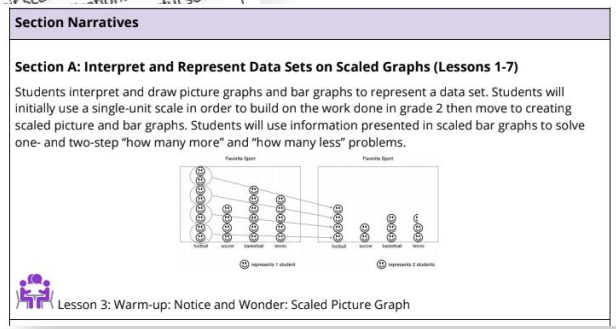
# Curriculum: Educative Components

- Mathematical Narratives
- Teacher Reflection Questions
- Professional Learning Community (PLC) Activities

# Mathematical Narratives




- Mathematical Narratives
  - Unit
  - Section
  - Lesson
  - Activity

Math Content

Student Thinking

Beliefs & Positioning

Pedagogy

# Teacher Reflection Question

| | |
|---|---|
| **Pedagogy** | Which question did you ask today that best supported students' understanding of area? What did students say or do that showed the question was effective? |
| **Student Thinking** | What strategies were students most comfortable using today? |
| **Math Content** | In grade 2, students skip counted by 2s, 5s, and 10s. In tomorrow's lesson, record how students are building toward fluency in multiplication by 2s, 5s, and 10s. |
| **Beliefs & Positioning** | As students worked in small groups, whose ideas were heard, valued, and accepted? How can you adjust the group structure tomorrow to ensure each student's ideas are a part of the collective learning? |

# PLC Activity: Section Narrative

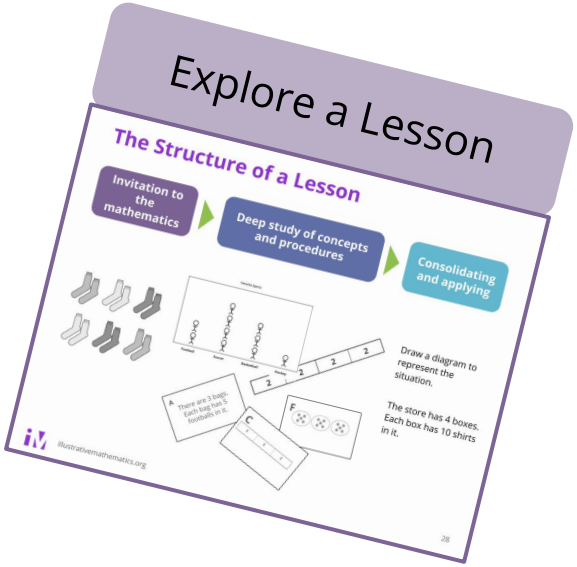
PLC: Lesson 1, Activity 1, Compare Shapes

**Suggested Centers**

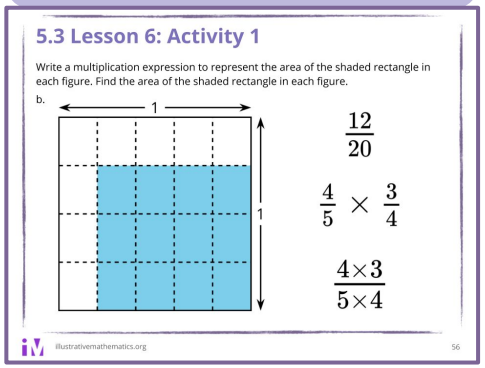
- Can You Build It? (3–5), Stage 1: Rectangles (support)
- Five in a Row: Multiplication (3–5), Stage 1: Factors 1–5 and 10 (reviewing)
- Can You Build It? (3–5), Stage 1: Rectangles (support)
- Five in a Row: Multiplication (3–5), Stage 1: Factors 1–5 and 10 (reviewing)
- Can You Build It? (3–5), Stage 1: Rectangles (support)
- Five in a Row: Multiplication (3–5), Stage 1: Factors 1–5 and 10 (reviewing)

# Professional Learning: Topics

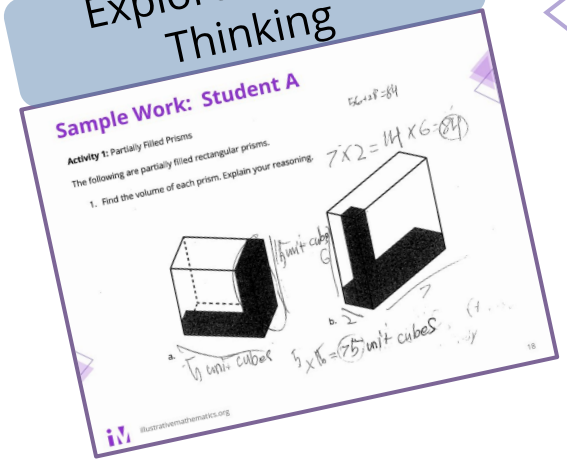
## Explore a Lesson



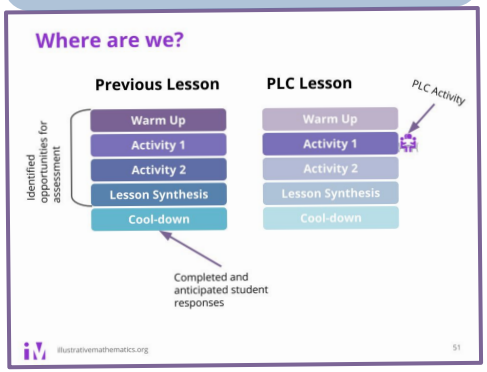

## Deep Dive Into Progressions and Representations



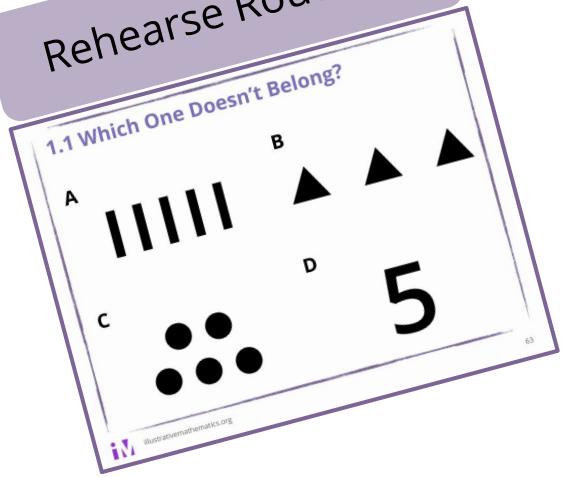

## Explore Student Thinking




## Planning and Learning with Teachers




## Rehearse Routines



# K–12 Math Professional Learning

## Summer:

Two Full Days Onsite:

- over the Summer/ early Fall
- Up to 25 participants per grade span: K-2, 3-5

## Fall:

Two Sessions:

- One session for each grade span: K-2, 3-5
- Content focused

## Spring:

Two Sessions:

- One session for each grade span: K-2, 3-5
- Content focused

# Questions?

At the bottom of the screen, please submit questions using either the:

- Q&A Feature or
- Chat Feature

If we do not get to all of the questions, there is a webinar posted on the IM website.

# THANK YOU

Illustrative Mathematics

## Join IM Communities

@IllustrateMath

Illustrative Mathematics

www.illustrativemathematics.blog

## IM K–5 Math Blog Posts

**First Impressions: The First Units in IM K–5 Math**
*By Kristin Gray*

**Creating an Accessible Mathematical Community with IM K–5: the power of "yet" for students and adults**
*By Maureen D. O'Connell*

**Building a Math Community with IM K–5 Math**
*By Tabitha Eutsler*


70