Illustrative Mathematics®
LEARN MATH FOR LIFE

GRADE 4

Teacher Guide

UNIT 2

K5_vII

20231010

Unit 2: Fraction Equivalence and Comparison

Goals

- Students generate and reason about equivalent fractions and compare and order fractions with the following denominators: 2, 3, 4, 5, 6, 8, 10, 12, and 100.

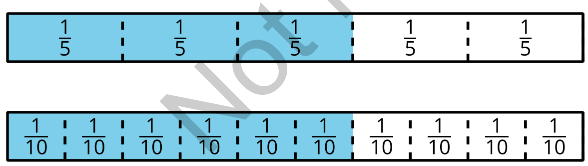
Narrative

In this unit, students extend their prior understanding of equivalent fractions and comparison of fractions.

In grade 3, students partitioned shapes into parts with equal area and expressed the area of each part as a unit fraction. They learned that any unit fraction $\frac{1}{b}$ results from a 1 partitioned into b equal parts. Students used unit fractions to build non-unit fractions, including fractions greater than 1, and represented them on fraction strips and tape diagrams. The denominators of these fractions were limited to 2, 3, 4, 6, and 8. Students also worked with fractions on a number line, establishing the idea of fractions as numbers and equivalent fractions as the same point on the number line.

Here, students follow a similar progression of representations. They use fraction strips, tape diagrams, and number lines to make sense of the size of fractions, generate equivalent fractions, and compare and order fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Students generalize that a fraction $\frac{a}{b}$ is equivalent to fraction $\frac{(n \times a)}{(n \times b)}$ because each unit fraction is being broken into n times as many equal parts, making the size of the part n times as small $\frac{1}{(n \times b)}$ and the number of parts in the whole n times as many $(n \times a)$. For example, we can see $\frac{3}{5}$ is equivalent to $\frac{6}{10}$ because when each fifth is partitioned into 2 parts, there are 2×3 or 6 shaded parts, twice as many as before, and the size of each part is half as small, $\frac{1}{(2 \times 5)}$ or $\frac{1}{10}$.



As the unit progresses, students use equivalent fractions and benchmarks, such as $\frac{1}{2}$ and 1, to reason about the relative location of fractions on a number line and to compare and order fractions.

Throughout The Unit

Students continue to develop strategies for multiplying numbers mentally—building on their fluency from grade 3 and applying the properties of multiplication. The *Number Talks* in this unit support this goal, focusing on factors 2, 4, 5, 6, 8, 10, and 12. Students engage in strategies, such as doubling and halving, and relate these strategies to folding of fraction strips and partitioning of tape diagrams into smaller unit fractions.

Here is a sampling of the *Number Talk* warm-ups in the unit.

lesson 5	lesson 9	lesson 16
2×12 4×12 8×12 16×12	10×6 10×12 10×24 5×24	5×6 5×12 6×12 11×12

These factors are intentionally chosen to build flexibility with the unit fractions in this unit. As students see the relationship between these factors and their products in *Number Talks*, they can become more efficient in determining equivalency and comparing fractions with these denominators.

Materials Needed

Lesson	Materials to Gather	Materials to Copy
Lesson 1	• Straightedges: Activity 1, Activity 2	• Fraction Strips Template (1 copy for every 2 students): Activity 1
Lesson 2	• Straightedges: Activity 1, Activity 2 • Materials from a previous lesson: Activity 2	
Lesson 3		
Lesson 4	• Straightedges: Activity 1	
Lesson 5	• Straightedges: Activity 1	
Lesson 6		• Card Sort Where Do They Belong? Cards (1 copy for every 2 students): Activity 2
Lesson 7	• Tools for creating a display: Activity 2	
Lesson 8	• Tape (painter's or masking): Activity 1	

Lesson 9	• Rulers or straightedges: Activity 1 • Sticky notes: Activity 2	• How Do You Know Cards (1 copy for every 15 students): Activity 2
Lesson 10		
Lesson 11		• Card Sort Fractions Galore Cards (1 copy for every 3 students): Activity 3
Lesson 12	• Colored pencils: Activity 2	
Lesson 13		
Lesson 14	• Tools for creating a display: Lesson	
Lesson 15		
Lesson 16		• Fraction Cards with Denominators 2, 3, 4, and 6 (1 copy for every 2 students): Activity 1 • Fraction Cards with Denominators 5, 8, 10, 12, and 100 (1 copy for every 2 students): Activity 1
Lesson 17	• Markers: Activity 1, Activity 2 • Paper: Activity 1, Activity 2 • Paper clips: Activity 1, Activity 2 • Tape (painter's or masking): Activity 1, Activity 2	

Section A: Size and Location of Fractions

Standards

Building On **3.NF.A.1, 3.NF.A.2, 3.NF.A.2.a, 3.NF.A.3, 3.NF.A.3.b, 3.NF.A.3.d, 3.OA.B.5**

Addressing **4.NF.A.1, 4.NF.A.2**

Building Towards **4.NBT.B.4, 4.NF.A, 4.NF.A.1, 4.NF.A.2**

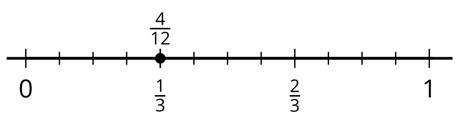
Goals

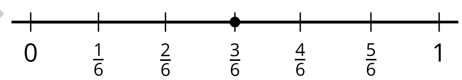
- Make sense of fractions with denominators 2, 3, 4, 5, 6, 8, 10, and 12 through physical representations and diagrams.
- Reason about the location of fractions on the number line.

Narrative

In this section, students revisit ideas and representations of fractions from grade 3, working with denominators that now include 5, 10, and 12. Students use physical fraction strips, diagrams of fraction strips, tape diagrams, and number lines to make sense of the size of fractions and fractional relationships.

Students reason about the relationship between fractions where one denominator is a multiple of the other denominator (such as $\frac{1}{5}$ and $\frac{1}{10}$, or $\frac{1}{6}$ and $\frac{1}{12}$). They consider different ways to represent these relationships. Students also compare fractions to benchmarks, such as $\frac{1}{2}$ and 1.





The work in this section prepares students to reason about equivalence and comparison of fractions in the subsequent lessons.

Suggested Centers

Lesson 1
- Get Your Numbers in Order (1–5), Stage 3: Denominators 2, 3, 4, 6 (Addressing)
- Mystery Number (1–5), Stage 3: Fractions with Denominators 2, 3, 4, 6 (Supporting)

Lesson 2
- Get Your Numbers in Order (1–5), Stage 3: Denominators 2, 3, 4, 6 (Addressing)
- Mystery Number (1–5), Stage 3: Fractions with Denominators 2, 3, 4, 6 (Supporting)

Lesson 3
- Get Your Numbers in Order (1–5), Stage 3: Denominators 2, 3, 4, 6 (Addressing)
- Mystery Number (1–5), Stage 3: Fractions with Denominators 2, 3, 4, 6 (Supporting)

Lesson 4
- Get Your Numbers in Order (1–5), Stage 3: Denominators 2, 3, 4, 6 (Addressing)
- Number Line Scoot (2–4), Stage 3: Halves, Thirds, Fourths, Sixths, and Eighths (Supporting)

Lesson 5

- Get Your Numbers in Order (1–5), Stage 3: Denominators 2, 3, 4, 6 (Addressing)
- Number Line Scoot (2–4), Stage 3: Halves, Thirds, Fourths, Sixths, and Eighths (Supporting)

Lesson 6

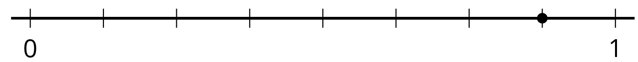
- Get Your Numbers in Order (1–5), Stage 3: Denominators 2, 3, 4, 6 (Addressing)
- Number Line Scoot (2–4), Stage 3: Halves, Thirds, Fourths, Sixths, and Eighths (Supporting)

Section A Checkpoint

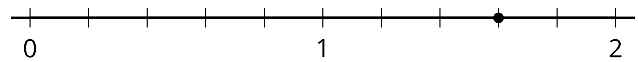
1 Student Task Statement

Label the point on each number line with a fraction it represents.

a.



b.



Solution

a. $\frac{7}{8}$. There are 8 equal parts in 1, and the point is on the 7th tick mark from 0 to show 7 parts.

b. $\frac{8}{5}$. There are 5 equal parts in each whole, and the point is on the 8th tick mark from 0 to show 8 parts.

Responding To Student Thinking

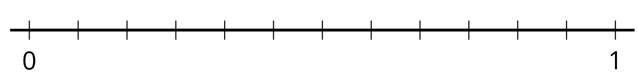
Students may not yet accurately interpret the partitions on a number line as unit fractions or the tick marks as distances away from 0. For example, on the first number line students may count the tick marks between 0 and 1 and label the point as $\frac{7}{7}$.

Students represent fractions less than 1 but may not yet understand the numerator can be greater than the denominator for fractions greater than 1. For example, students label $\frac{8}{5}$ as $\frac{5}{8}$.

During the next section, when students represent equivalent fractions with number lines, monitor for and invite selected students to share the ways they determine what the tick marks represent. Ask how number lines with fractions are the same as and different from number lines with whole numbers. Emphasize the meaning of the numerator and denominator in each fraction and how those meanings are connected to interpreting the partitions and points on a number line.

2 Student Task Statement

Is $\frac{7}{12}$ greater than or less than $\frac{1}{2}$? Explain your reasoning. Use the number line if it is helpful.



Solution

$\frac{7}{12}$ is greater than $\frac{1}{2}$. Sample responses: $\frac{7}{12}$ is the 7th tick mark from 0 on this number line, and it's more than halfway to 1. Half of 12 is 6 and $\frac{1}{2}$ is $\frac{6}{12}$, so $\frac{7}{12}$ is greater.

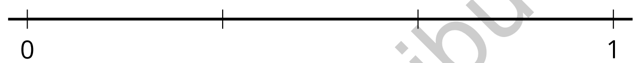
Responding To Student Thinking

Students recognize the greater fraction but do not yet show they are reasoning about benchmarks. For example, they think $\frac{7}{12}$ is greater because 7 and 12 are both greater than 1 and 2.

Encourage students to play *Compare*, Stage 6 in Section C. Monitor for the ways students reason about their comparisons using what they know about $\frac{1}{2}$ and 1 or by creating common denominators. Invite students to share their reasoning with others and to illustrate their reasoning with a diagram.

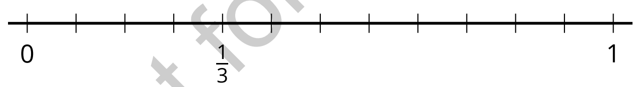
3 Student Task Statement

Explain why $\frac{4}{12}$ is equivalent to $\frac{1}{3}$. Use the number line if it is helpful.



Solution

Sample response: If I divide each third into 4 equal pieces, each of those pieces is 1 twelfth, and $\frac{1}{3}$ is on the 4th tick mark from 0, showing it is equivalent to 4 twelfths.



Responding To Student Thinking

Students show they are not yet reasoning about the size of fractions or multiples of numerators and denominators.

Encourage students to play *Compare*, Stage 6 in Section C. Monitor for the ways students reason about equivalent fractions when the denominators are multiples. Invite students to share their reasoning with others and to illustrate their reasoning with a diagram. Connect strategies that include drawing more partitions on the number lines with thinking about common denominators.

Practice Problems

12 Problems

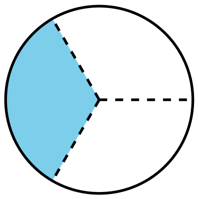
1 Pre-unit

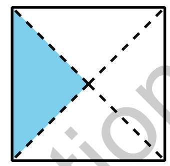
Standards

Practicing **3.NF.A.1**

Student Task Statement

What fraction of each figure is shaded?





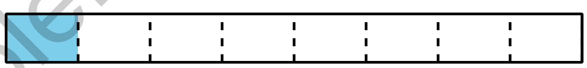
Solution

$\frac{1}{3}$ of the circle and $\frac{1}{4}$ of the square

2

Student Task Statement

Explain why the shaded part represents $\frac{1}{8}$ of the whole rectangle.



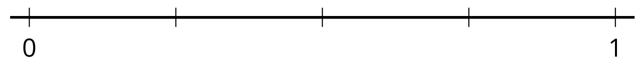
Solution

There are 8 equal parts in the rectangle, and 1 part is shaded.

3

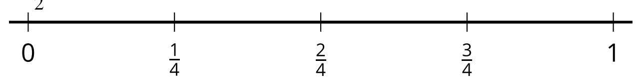
Student Task Statement

Label each tick mark with the number it represents. Explain your reasoning.



Solution

Some students may label $\frac{2}{4}$ as $\frac{1}{2}$.



There are 4 equal parts, so each part is $\frac{1}{4}$.

4

Pre-unit

Standards

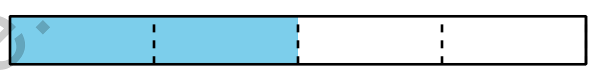
Practicing **3.NF.A.3.b**

Student Task Statement

Explain or show why $\frac{1}{2}$ and $\frac{2}{4}$ are equivalent fractions.

Solution

Sample response:



The shaded portion represents $\frac{2}{4}$ because 2 of 4 equal parts of the diagram are shaded. It also represents $\frac{1}{2}$ because the shaded portion is 1 of 2 equal parts of the diagram.

5 from Unit 2, Lesson 1

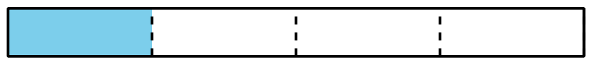
Student Task Statement

a. The whole diagram represents 1. Shade the diagram to represent $\frac{1}{4}$.

b. To represent $\frac{1}{6}$ on the diagram, would you shade more or less than what you shaded for $\frac{1}{4}$? Explain your reasoning.

Solution

a. Sample response:

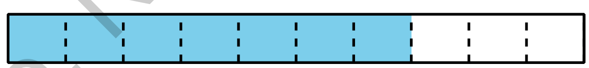


b. Less. Sample response: When there are 6 equal parts in 1 whole, each part is smaller than when there are 4 parts in the same size whole.

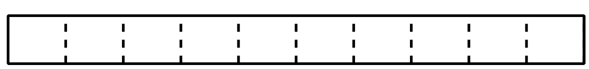
6 from Unit 2, Lesson 2

Student Task Statement

a. The whole diagram represents 1. What fraction does the shaded part represent? Explain your reasoning.



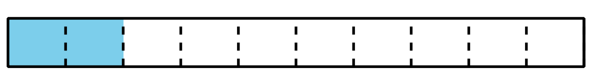
b. Shade this diagram to represent $\frac{2}{10}$.



Solution

a. $\frac{7}{10}$. Sample response: The rectangle is broken into 10 equal parts, and 7 of them are shaded.

b. Sample response:



7 from Unit 2, Lesson 3

Student Task Statement

Circle the greater fraction in each pair. Explain or show your reasoning.

a. $\frac{1}{8}$ or $\frac{1}{10}$

b. $\frac{4}{10}$ or $\frac{7}{10}$

c. $\frac{4}{5}$ or $\frac{5}{4}$

Solution

Sample reasoning: (Students may create a diagram to show their reasoning for each question.)

a. $\frac{1}{8}$. There are fewer eighths in a whole than tenths in a whole.

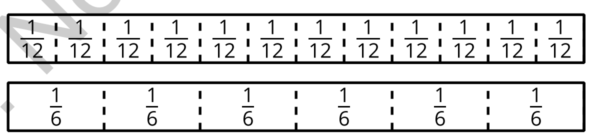
b. $\frac{7}{10}$. I know that 7 tenths is more than 4 tenths.

c. $\frac{5}{4}$. Since $\frac{4}{4}$ is 1 whole, 5 fourths is greater than 1. Since $\frac{5}{5}$ is 1 whole, and 4 fifths is less than 1.

8 from Unit 2, Lesson 4

Student Task Statement

Use the fraction strips to name 3 pairs of equivalent fractions. Explain how you know the fractions are equivalent.



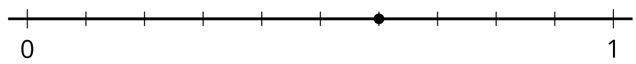
Solution

Sample responses: $\frac{2}{12}$ and $\frac{1}{6}$, $\frac{4}{12}$ and $\frac{2}{6}$, $\frac{6}{12}$ and $\frac{3}{6}$. The fractions in each pair represent the same size, but they are formed by d with different sized parts.

9 from Unit 2, Lesson 5

Student Task Statement

a. Explain or show why the point on the number line represents both $\frac{3}{5}$ and $\frac{6}{10}$.



b. Explain why $\frac{6}{10}$ and $\frac{3}{5}$ are equivalent fractions.

Solution

a. Sample response: There are ticks marking 10 equal-size spaces between 0 and 1 and the point is on the 6th tick mark from 0, so that's $\frac{6}{10}$. The 10 spaces can be seen as 5 groups of 2 spaces, or fifths, and the point is on the third tick that marks fifths, so that's $\frac{3}{5}$.

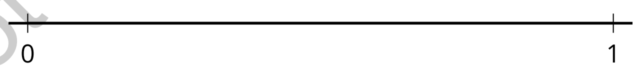
b. $\frac{6}{10}$ and $\frac{3}{5}$ are equivalent fractions because they represent the same point on the number line or the same distance from 0.

10 from Unit 2, Lesson 6

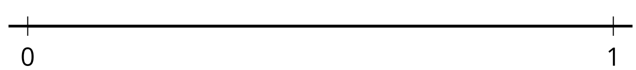
Student Task Statement

For each question, explain your reasoning. Use a number line if you find it helpful.

a. Is $\frac{4}{5}$ greater than or less than $\frac{1}{2}$?



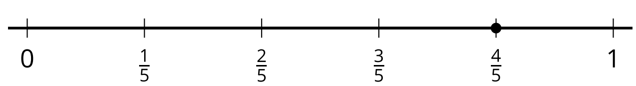
b. Is $\frac{4}{5}$ greater than or less than 1?



Solution

a. $\frac{4}{5}$ is greater than $\frac{1}{2}$ because it is more than half way from 0 to 1.

b. $\frac{4}{5}$ is less than 1 because one whole is $\frac{5}{5}$ and $\frac{4}{5}$ is less than $\frac{5}{5}$ (or because the point for $\frac{4}{5}$ is to the left of 1 on the number line).



11 Student Task Statement

Fold a strip of paper to represent each fraction. How did you fold the paper to make sure the parts are the correct size for each fraction? Use the blank diagrams to show how you folded.

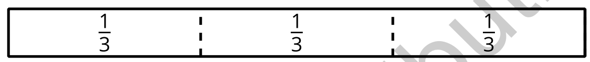
a. $\frac{1}{3}$s

b. $\frac{1}{5}$s

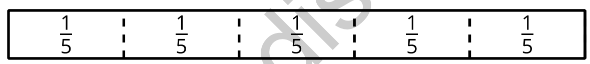
c. $\frac{1}{10}$s

Solution

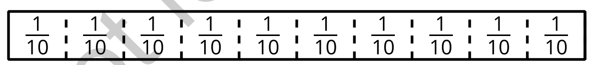
a. Sample response: For thirds, I folded the two ends so that they met in the middle.



b. Sample response: For fifths, I tried to fold the paper 4 times, to the left, right, left and right. Then I tried to get the folds to be close to equal.

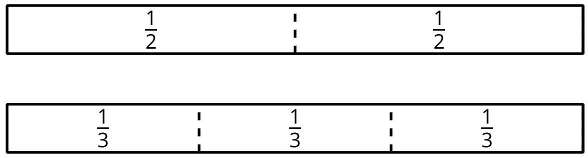


c. Sample response: For tenths, I just folded my fifths back up and then folded that in half since there are two tenths in each fifth.

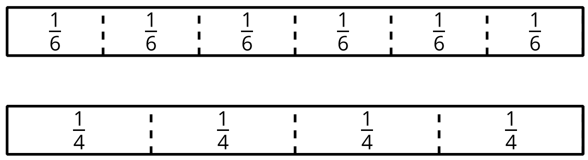


12 Student Task Statement

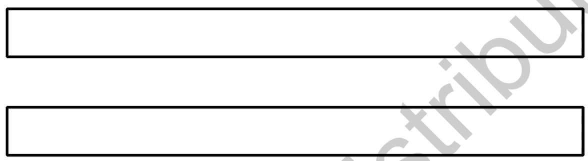
a. Andre looks at these fraction strips and says, “Each $\frac{1}{2}$ is the same size as $\frac{1}{3}$ and another half of $\frac{1}{3}$.” Do you agree with Andre? Explain your reasoning.



b. What relationship do you see between $\frac{1}{6}$ and $\frac{1}{4}$? Explain your reasoning.



c. Can you find a relationship between $\frac{1}{6}$ and $\frac{1}{8}$ using fraction strips? Explain your reasoning.



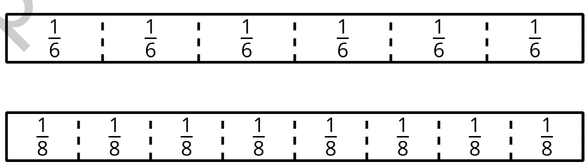
Solution

Sample responses:

a. Andre is correct. The fraction strips show that there are three $\frac{1}{3}$s in 1 and two $\frac{1}{2}$s in 1, so that means there is one $\frac{1}{3}$ and another half of $\frac{1}{3}$ in each $\frac{1}{2}$.

b. $\frac{3}{6}$ is the same as $\frac{2}{4}$, and that means that there is one $\frac{1}{6}$ and another half of $\frac{1}{6}$ in each $\frac{1}{4}$.

c. It looks like $\frac{1}{6}$ is $\frac{1}{8}$ and maybe another third of $\frac{1}{8}$. I can see from the fraction strips that $\frac{3}{6}$ is $\frac{3}{8}$ and another $\frac{1}{8}$. This means that $\frac{1}{6}$ is equal to $\frac{1}{8}$ and another third of $\frac{1}{8}$.



Representations of Fractions (Part 1)

Standards

Building On 3.NF.A.1

Building Towards 4.NF.A.1

Instructional Routines

- MLR1 Stronger and Clearer Each Time
- What Do You Know about _____?

Goals

- Describe (orally and in writing) the relationship between the denominator and the size of the unit fraction.
- Represent and identify unit fractions that have denominators 2, 3, 4, 5, 6, 8, and 12 on physical and visual representations.

Student Facing Learning Goals

Let's name some fractions and represent them visually.

Lesson Purpose

The purpose of this lesson is for students to make sense of unit fractions with denominators 2, 3, 4, 5, 6, 8, 10, and 12, using physical and visual representations.

Narrative

In grade 3, students were introduced to fractions as numbers. They learned to name and represent fractions, to recognize simple equivalent fractions, and to compare fractions with like numerators and denominators (limited to 2, 3, 4, 6, and 8). Students used fraction strips, area diagrams, tape diagrams, and number lines to support their reasoning with fractions.

This lesson activates students' prior knowledge of unit fractions and includes fractions with new denominators 5, 10, and 12. Students revisit the meanings of "numerator" and "denominator," name unit fractions, create representations for them, and recall some strategies and tools for reasoning about fractions.

The idea of equivalence may naturally come up (and will help to prepare students for upcoming work), but it is not the focus of this lesson.

Access For Students with Disabilities

- Engagement

Required Materials

Materials To Gather

- Straightedges: Activity 1, Activity 2

Materials To Copy

- Fraction Strips Template (1 copy for every 2 students): Activity 1

Lesson Timeline

Warm-up	10 mins
Activity 1	20 mins
Activity 2	15 mins
Synthesis Estimate	10 mins
Cool-down	5 mins

Teacher Reflection Questions

What did you learn about each student and their foundational understanding of fractions based on their work today?

Warm-up

10 mins

What Do You Know about $\frac{1}{2}$?

Standards

Building On **3.NF.A.1**

Building Towards **4.NF.A.1**

Instructional Routines

- What Do You Know about ____?

The purpose of this *Warm-up* is to invite students to share what they know about the number $\frac{1}{2}$ and elicit ways in which it can be represented. It gives the teacher the opportunity to hear students' understandings about and experiences with fractions, $\frac{1}{2}$ in particular. The fraction $\frac{1}{2}$ is familiar to students and will be central in the next activity.

This is the first time students experience the *What Do You Know About* ____ routine in grade 4. Students should be familiar with this routine from a previous IM grade. However, they may benefit from a brief review of the steps involved.

Student Task Statement

What do you know about $\frac{1}{2}$?

Student Response

Sample responses:

- It is a fraction.
- I shared half of my sandwich with my friend.
- It is what we get when we split something into two equal parts.
- We can "halve" something.
- Dividing by 2.
- It is halfway between 0 and 1 on a number line.
- It is less than 1.

Launch

- Groups of 2
- Display the number $\frac{1}{2}$.
- "What do you know about this number?"
- 1 minute: quiet think time

Activity

- "Discuss your thinking with your partner."
- 2 minutes: partner discussion
- Share and record responses.

Activity Synthesis

- "What different ways can we represent $\frac{1}{2}$?" (Cut an object, a rectangle, or another shape into two equal parts, mark the middle point between 0 and 1 on a number line.)

Activity 1

20 mins

Fraction Strips

Standards

Building On	**3.NF.A.1**
Building Towards	**4.NF.A.1**

The purpose of this activity is for students to use fraction strips to represent halves, fourths, and eighths. The denominators in this activity are familiar from grade 3. The goal is to remind students of the relationships between fractional parts in which one denominator is a multiple of another. Students should notice that each time the unit fractions on a strip are folded in half, there are twice as many equal-size parts on the strip and that each part is half as large.

In the discussion, use the phrases "number of parts" and "size of the parts" to reinforce the meaning of a fraction.

Access for Students with Disabilities

Engagement: Provide Access by Recruiting Interest. Provide choice and autonomy. Provide access to different colored strips of paper students can use to differentiate each fraction.
Supports accessibility for: Organization, Visual-Spatial Processing

Required Materials

Materials To Gather

- Straightedges: Activity 1

Materials To Copy

- Fraction Strips Template (1 copy for every 2 students): Activity 1

Required Preparation

- Each group of 2 needs 4 strips of equal-size paper (cut lengthwise from letter-size or larger paper or use the provided blackline master).

Student Task Statement

Your teacher will give you strips of paper. Each strip represents 1.

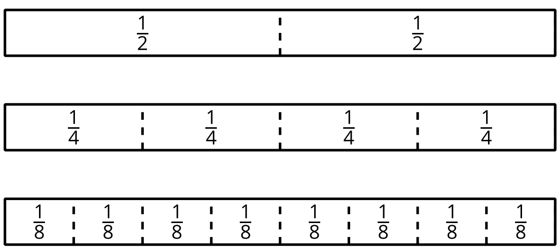
1. Use the strips to represent halves, fourths, and eighths.

   Use one strip for each fraction and label the parts.

2. What do you notice about the number of parts or the size of the parts? Make at least 2 observations.

Student Response

1.



2. Sample responses:
   - Each time we folded, there were more parts.
   - Each time we folded, the parts got smaller.
   - The $\frac{1}{8}$ parts are each half the size of the $\frac{1}{4}$ parts.
   - The $\frac{1}{4}$ parts are each half of the $\frac{1}{2}$ parts.

Launch

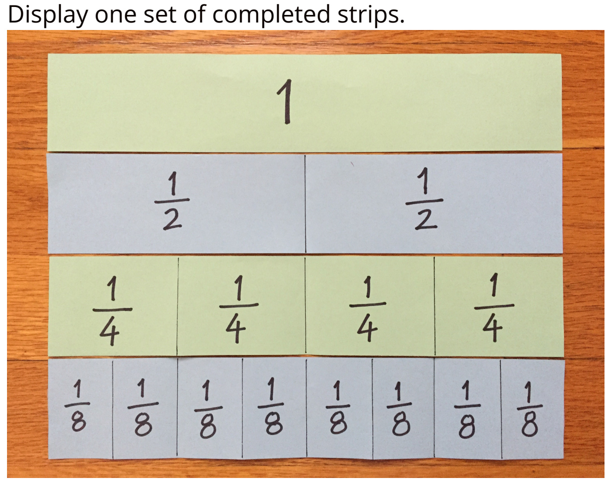
- Groups of 2
- Give each group 4 paper strips and a straightedge.
- Hold up one strip for all to see.
- "Each strip represents 1."
- Label that strip with "1" and tell students to do the same on one of their strips.
- "Take a new strip. How would you fold it to show halves?"
- 30 seconds: partner think time
- "Think about how to show fourths on the next strip and eighths on the last strip."

Activity

- "Work with your partner on the task."
- 10 minutes: partner work time
- Monitor for students who notice that each denominator is twice the next smaller denominator.

Activity Synthesis

- Select a group to share their paper strips and how they found the fractional parts. Ask if others also found them the same way.
- Display one set of completed strips.



- Invite students to share what they noticed about the number and size of the parts on the strips. Highlight the ideas noted in *Student Responses*.
- If no students mentions the relationships between the fractions on different strips, encourage them to work with a partner to look for some.
- If the terms "numerator" and "denominator" did not

arise during discussion, ask students about them.

- Remind students that the **denominator** (the number at the bottom of a fraction) tells us the number of equal-size parts in 1 whole, and the **numerator** (the number at the top of a fraction) refers to how many of those parts are being described. Consider displaying these terms and their meanings for students to reference.
- Ask students to save the fraction strips for a future lesson.

Advancing Student Thinking

If students create a fraction strip that has different sized parts, consider asking:

- "How could we make sure that each part on a strip is equal?"
- "What tools might we use to help make precise folds?"

Activity 2

15 mins

Fractions, Represented

Standards

Building On	**3.NF.A.1**
Building Towards	**4.NF.A.1**

Instructional Routines

- MLR1 Stronger and Clearer Each Time

The purpose of this activity is for students to revisit the meaning of unit fractions with familiar and unfamiliar denominators (3, 5, 6, 10, and 12) and recall how to name and represent them.

While drawing tape diagrams to represent these fractions, students have opportunities to look for structure and to make use of the relationships between the denominators of the fractions (MP7). For example, to make a diagram with twelfths, students can cut each of 6 sixths in half.

To support students in drawing straight lines on the tape diagrams, provide access to a straightedge or ruler. Students should not, however, use rulers to measure the location of a fraction on any diagram.

This activity uses MLR1 Stronger and Clearer Each Time. Advances: Reading, Writing.

Required Materials

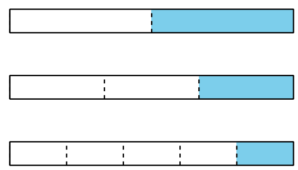
Materials To Gather

- Straightedges: Activity 2

Student Task Statement

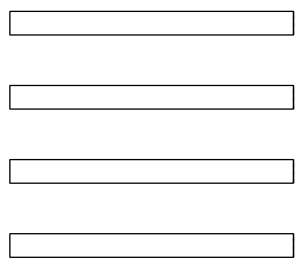
1. Each whole diagram represents 1. What fraction does the shaded part of each diagram represent?

   a.
   b.
   c.



2. Here are four blank diagrams. Each diagram represents 1. Partition each diagram and shade 1 part so that the shaded part represents the given fraction.

   a. $\frac{1}{6}$
   b. $\frac{1}{8}$
   c. $\frac{1}{10}$
   d. $\frac{1}{12}$

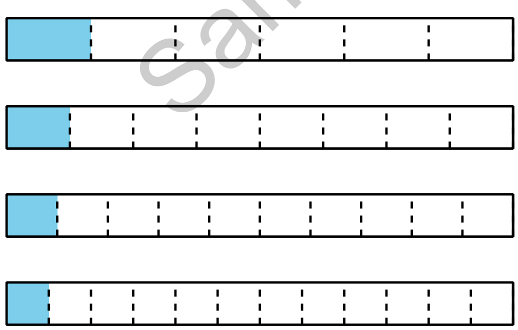


3. Suppose you use the same blank diagram to represent $\frac{1}{20}$. Would the shaded part be larger or smaller than the shaded part in the diagram of $\frac{1}{10}$? Explain how you know.

Student Response

1. a. $\frac{1}{2}$
   b. $\frac{1}{3}$
   c. $\frac{1}{5}$
2. Sample response:

   a.
   b.
   c.
   d.



3. It will be smaller, because the whole will be split into 20 equal parts instead of 10. So, each part will be smaller (half the size) than the parts representing

Launch

- Groups of 2
- Give each student a straightedge.
- "Let's look at some other fractions and draw diagrams to represent them. Consider using a straightedge when you draw."

Activity

- 7–8 minutes: independent work time
- "Discuss your responses with your partner. Be sure to talk about how you created diagrams for $\frac{1}{6}$, $\frac{1}{10}$, and $\frac{1}{12}$."
- 2–3 minutes: partner discussion
- Monitor for students who:
  - Notice the relationship of thirds, sixths, and twelfths, and of fifths and tenths.
  - Use the given diagrams to help partition the other diagrams.

Activity Synthesis

- "How did you know how to partition the diagrams in the second question?"
- Select students who use the given diagrams or the relationships between denominators to display their diagrams and share their reasoning.
- "What relationships do you see between the fractions in this activity?" (Sample responses:
  - As the denominator gets larger, each fractional part gets smaller.
  - A fifth is twice the size of a tenth, or a tenth is half as big as a fifth.
  - Thirds, sixths, and twelfths are related in that a third is 2 sixths and a sixth is 2 twelfths. Fifths and tenths are related in the same way.)

MLR1 Stronger and Clearer Each Time

- "Share your response to the last question with your partner. Take turns being the speaker and the listener. If you are the speaker, share your response. If you are the listener, ask questions and give feedback to help your partner improve their work."
- 3–5 minutes: structured partner discussion.
- Repeat with 2–3 different partners.

tenths.

- “Revise your initial response based on the feedback from your partners.”
- 2–3 minutes: independent work time

Lesson Synthesis

“Today we refreshed our memory about fractions. We used fraction strips and diagrams to represent some familiar and some new fractions.”

Based on students’ work during the lesson, choose the questions that need more discussion:

- “In general, what does the denominator in a fraction represent?” (the number of equal parts in 1 whole)
- “What does the fraction $\frac{1}{5}$ tell us?” (the size of one part if 1 whole is split into 5 equal parts)
- “What did you notice about the size of a fraction as the denominator gets larger?” (The size of the fraction gets smaller.) “Why might that be?” (There are more equal parts in 1 whole, so each part gets smaller.)
- “What relationships did we see between the fractions that we studied today?” (The denominators of some fractions are multiples of other fractions. A representation of one fraction can be split into two or three parts to represent another fraction.)

Suggested Centers

- Get Your Numbers in Order (1–5), Stage 3: Denominators 2, 3, 4, 6 (Addressing)
- Mystery Number (1–5), Stage 3: Fractions with Denominators 2, 3, 4, 6 (Supporting)

Cool-down

5 mins

What Do the Diagrams Show?

Standards

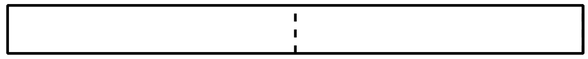
Building Towards **4.NF.A.1**

Student Task Statement

Each whole diagram represents 1.

1. What fraction does each shaded part represent?

2. Explain or show how you could use this diagram to represent sixths.



Student Response

1. $\frac{1}{6}$, $\frac{1}{10}$, and $\frac{1}{5}$.
2. Sample response: Split each half into 3 equal parts so there will be a total of 6 parts. Each part is a sixth.

Responding To Student Thinking

Students describe what the second tape diagram shows but not how it could be used to show sixths.	Next Day Supports At the start of the next lesson, invite students to share how they would show sixths on a blank diagram and on a diagram already partitioned into halves.
The work in this lesson builds from concepts of fractions addressed in a prior unit.	Prior Unit Support Grade 3, Unit 5, Section A Introduction to Fractions

Unit 2, Lesson 2

Representations of Fractions (Part 2)

Standards

Building On **3.NF.A.1**

Building Towards **4.NF.A, 4.NF.A.1**

Instructional Routines

- Which Three Go Together?

Goals

- Explain (orally) how to determine what non-unit fraction is represented by a tape diagram.
- Represent a non-unit fraction on a tape diagram.

Student Facing Learning Goals

Let's name some other fractions and represent them with diagrams.

Lesson Purpose

The purpose of this lesson is for students to make sense of non-unit fractions (including those greater than 1) that have denominators 2, 3, 4, 5, 6, 8, 10, and 12.

Narrative

In the previous lesson, students made sense of the meanings of the numerator and denominator in unit fractions. They identified fractions represented by diagrams, and partitioned diagrams to represent given fractions. In this lesson, students reason in similar ways—using numerical and visual representations—about non-unit fractions and fractions that are greater than 1.

Students are reminded of what they learned in grade 3: that a non-unit fraction $\frac{a}{b}$ can be understood as a parts of a unit fraction $\frac{1}{b}$, and that fractions with different numerators and denominators can be equivalent. Unlike in grade 3, the denominators students see now include 5, 10, and 12.

As in the previous lesson, rulers can be provided to help students draw, extend, or align partition lines, but they should not be used to measure the location of a fraction on any diagram.

Access For Students with Disabilities

- Representation

Access For English Learners

- MLR2

Required Materials

Materials To Gather

- Straightedges: Activity 1, Activity 2
- Materials from a previous lesson: Activity 2

Lesson Timeline

Warm-up	10 mins
Activity 1	20 mins
Activity 2	15 mins
Synthesis Estimate	10 mins
Cool-down	5 mins

Teacher Reflection Questions

Who participated in math class today? What assumptions are you making about those who did not participate? How can you leverage each of your students' ideas to support them in being seen and heard in tomorrow's math class?

Warm-up

10 mins

Which Three Go Together: All Cut Up

Standards

Building On **3.NF.A.1**

Building Towards **4.NF.A.1**

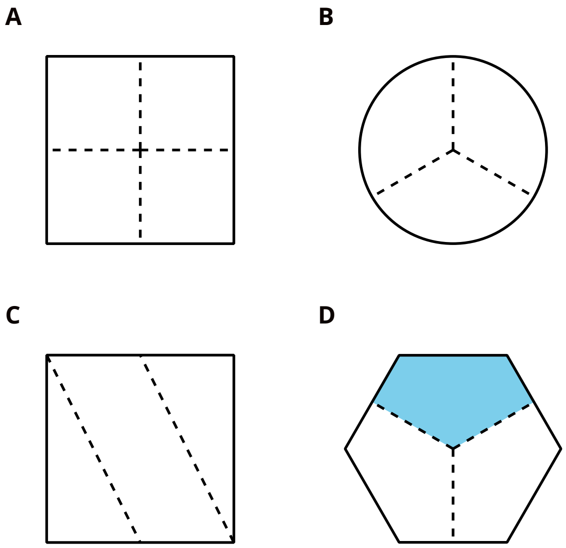
Instructional Routines

- Which Three Go Together?

This *Warm-up* prompts students to carefully analyze and compare the features of four partitioned shapes. It allows the teacher to hear the terminologies students use to talk about fractions and fractional parts. In making comparisons, students have a reason to use language precisely (MP6).

Student Task Statement

Which 3 go together?



Student Response

Sample response:

- A, B, and C go together because the parts are clear, or unshaded.
- A, B, and D go together because they are partitioned into equal parts.
- A, C, and D go together because they have straight sides.
- B, C, and D go together because they are partitioned into 3 parts.

Launch

- Groups of 2
- Display the image.
- "Pick 3 that go together. Be ready to share why they go together."
- 1 minute: quiet think time

Activity

- "Discuss your thinking with your partner."
- 2–3 minutes: partner discussion
- Share and record responses.

Activity Synthesis

- "What does the shaded part in D represent?" ($\frac{1}{3}$ or one-third of the shape)
- Shade one part of B and C.
- "Is each shaded part one-third of the shape as well?" (Yes for B, No for C)
- "Why is the shaded part not one-third of the square in C?" (The parts aren't equal in size.)
- Shade one part of A. "Is it a third of the square?" (No, it is $\frac{1}{4}$ or one-fourth.)

Activity 1

A Diagram for Each Fraction

20 mins

Standards

Building On	**3.NF.A.1**
Building Towards	**4.NF.A.1**

The purpose of this activity is to activate what students know about the meaning and size of non-unit fractions. Students match a set of fractions with diagrams that represent them. There are 3 sets of equivalent fractions to prompt students to share what they know about equivalent fractions.

To add movement to the activity, students can check their matches with other groups in the room before the *Activity Synthesis*.

Access for Students with Disabilities

Representation: Internalize Comprehension. Use visual details, such as color or arrows, to illustrate connections between representations. For example, use the same color for the numerator and the shaded portion of the corresponding diagram.
Supports accessibility for: Visual-Spatial Processing

Required Materials

Materials To Gather

- Straightedges: Activity 1

Student Task Statement

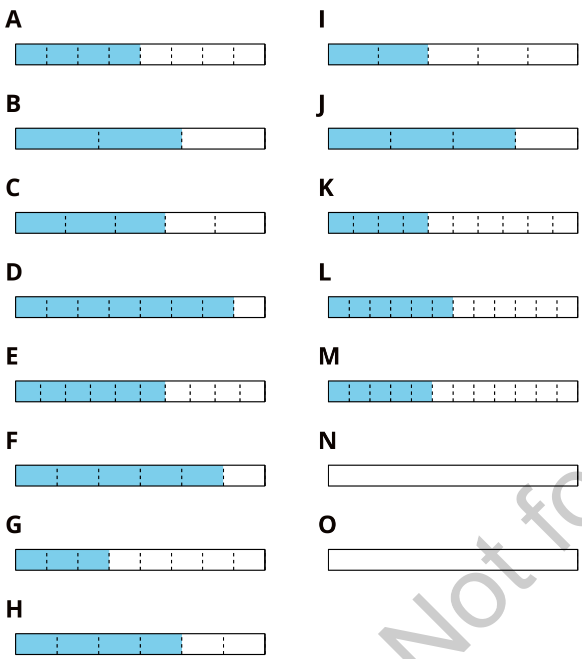
Each whole diagram represents 1. Match each fraction to a diagram whose shaded part represents that fraction.

Two of the fractions are *not* represented. Use the blank diagrams to represent each of them.

$\frac{2}{3}$: ________ $\frac{3}{8}$: ________ $\frac{4}{10}$: ________ $\frac{4}{6}$: ________ $\frac{6}{6}$: ________

$\frac{3}{5}$: ________ $\frac{4}{8}$: ________ $\frac{6}{12}$: ________ $\frac{6}{10}$: ________ $\frac{3}{4}$: ________

$\frac{5}{6}$: ________ $\frac{2}{5}$: ________ $\frac{5}{12}$: ________ $\frac{7}{10}$: ________ $\frac{7}{8}$: ________



Student Response

$\frac{2}{3}$: B or H

$\frac{3}{8}$: G

$\frac{4}{10}$: K

$\frac{4}{6}$: H or B

$\frac{6}{6}$: N or O

$\frac{3}{5}$: C or E

$\frac{4}{8}$: A

$\frac{6}{12}$: L

$\frac{6}{10}$: E or C

$\frac{3}{4}$: J

$\frac{5}{6}$: F

$\frac{2}{5}$: I

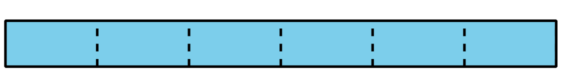
$\frac{5}{12}$: M

$\frac{7}{10}$: O or N

$\frac{7}{8}$: D

The missing diagrams for $\frac{6}{6}$ and $\frac{7}{10}$:

- N



Launch

- Groups of 2
- Give each student a straightedge
- Record and display the fraction $\frac{1}{4}$.
- "Describe to your partner what the diagram would look like for this fraction."
- 30 seconds: partner discussion
- Record and display the fraction $\frac{2}{4}$.
- "Describe what the diagram would look like for this fraction."
- 30 seconds: partner discussion
- Share responses.
- "In an earlier lesson, we looked at fractions with 1 for the numerator. Now let's look at fractions with other numbers for the numerator."
- As a class, read aloud the word name of each fraction in the task.

Activity

- "Take a minute to think quietly about how you might match each fraction with a diagram that represents it."
- 1 minute: quiet think time
- "Work with a partner to match each fraction with a diagram. Two of the fractions have no matching diagrams. Use the blank diagrams to create representations for them."
- 10 minutes: group work time

Activity Synthesis

- Invite students to share how they went about making the matches.
- Highlight explanations that emphasize the meanings of the numerator and denominator in a fraction.
- Ask students if they noticed that some diagrams have the same amount shaded, but the fractions they represent have different numbers. "Which diagrams show this?" (A and L, B and H, C and E, I and K)
- "What does it mean that the diagrams representing those fractions are the same?" (The fractions are the same size or have the same value. The term

- O

"equivalent" may or may not come up at this point.)

Activity 2

15 mins

Diagrams for Some Other Fractions

Standards

Building On	**3.NF.A.1**
Building Towards	**4.NF.A.1**

This activity extends students' reasoning about the meaning of a fraction's numerator and denominator and the size of non-unit fractions to include fractions greater than 1. Students see the size of 1 whole marked in a couple of diagrams and learn that the same size applies to all diagrams. Students are prompted to both interpret diagrams and create them: they write a fraction to represent the shaded part of a diagram and partition a diagram to represent a given fraction.

Some students may benefit from having physical manipulatives to help them conceptualize fractions that are greater than 1. Consider using fraction strips to support those students. For instance, ask them to fold as many strips as needed to represent 4 halves or 5 fourths.

Access for English Language Learners

MLR2 Collect and Display. Collect the language students use to reason about fractions greater than one. Display words and phrases, such as "fraction," "numerator," "denominator," "1 whole," "greater than," and "equal-size parts." During the activity, invite students to suggest ways to update the display: "What are some other words or phrases we should include?" Invite students to borrow language from the display as needed.
Advances: Conversing, Reading

Required Materials

Materials To Gather

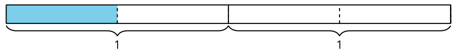
- Materials from a previous lesson: Activity 2
- Straightedges: Activity 2

Required Preparation

- Each student needs access to their fraction strips from a previous lesson.

Student Task Statement

1. What fraction does the shaded part of each diagram represent?

   a. 

   b. 

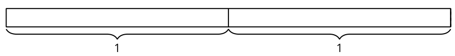
   c. 

   d. 

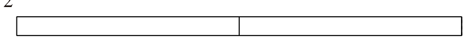
   e. 

2. Here are 4 fractions and 4 blank diagrams. Partition each diagram and shade the parts to represent the fraction.

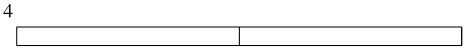
   a. $\frac{2}{2}$

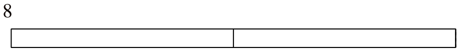
   b. $\frac{4}{2}$

   c. $\frac{5}{4}$

   d. $\frac{10}{8}$

Student Response

1. Accept all equivalent forms of each response.
   a. $\frac{1}{2}$
   b. $\frac{3}{2}$
   c. $\frac{4}{3}$
   d. $\frac{4}{4}$
   e. $\frac{6}{4}$
2. Sample responses:

   a. 

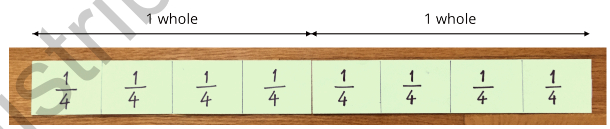
   b. 

   c. 

   d. 

Launch

- Groups of 2
- Give each student a straightedge and access to their fraction strips from a previous lesson.
- "How can you show $\frac{3}{4}$ with fraction strips?" (Find the strip showing fourths and highlight 3 parts, or fourths.)
- "How can you show $\frac{8}{4}$?"
- 1 minute: partner discussion
- If students say that they don't have enough strips to show 8 fourths, ask them to combine their strips with another group's.
- Invite groups to share their representations of $\frac{8}{4}$. Students may use different fractional parts (fourths and halves, or fourths and eighths).
- Display two strips that show fourths side by side, as shown.



- Count the fourths: "$\frac{1}{4}$, $\frac{2}{4}$, . . . , $\frac{8}{4}$."
- "How many fourths did we count?" (8) "How many wholes was that?" (2)

Activity

- "Each bracket in the first diagram shows 1 whole. The size of 1 whole is the same in all the diagrams."
- "Take a moment to work on the first question. Then, discuss your responses with your partner."
- "Be prepared to explain how you know what fractions the diagrams represent."
- 2–3 minutes: independent work time on the first question
- 2 minutes: partner discussion
- Pause for a whole-class discussion.
- "How did you determine what fractions the diagrams represent?" (Count the number of parts in 1, and then count the number of shaded parts.)
- Display students' work, or display and annotate the tape diagrams as they explain.
- Consider labeling each part with the unit fraction and counting each shaded part aloud ("1 half, 2 halves, 3

halves," or "1 third, 2 thirds, 3 thirds, 4 thirds") before writing the represented fractions ($\frac{3}{2}$ or $\frac{4}{3}$).

- "Work with your partner on the second question. You may use a straightedge to help you draw your diagrams."
- 5–7 minutes: partner work time

Activity Synthesis

- Select students to share their completed diagrams.
- "How did you know how many equal parts to partition each diagram into and how many parts to shade?" (Divide each 1 whole portion into as many equal parts as the number in the denominator. Shade as many parts as the number in the numerator.)
- "How did you partition a diagram into fourths?" (Split each 1 whole into 2 halves, and then split each half into 2 equal parts again.)
- "How did you partition a diagram into eighths?" (Split each fourth into 2 equal parts.)

Advancing Student Thinking

If students only partition some diagrams correctly, consider asking:

- "What does each part in your diagram represent? How do you know?"
- "Where is 1 whole in your diagram?"

Lesson Synthesis

"Today we made sense of and created diagrams that represent fractions, including fractions greater than 1."

"Did you notice anything interesting about the last two diagrams you created and the fractions they represent?" (They are both greater than 1. The shaded parts are the same size. They have the same amount of shading. The numerator and denominator in one fraction are twice the numerator and denominator in the other. The fractions are equivalent.)

Suggested Centers

- Get Your Numbers in Order (1–5), Stage 3: Denominators 2, 3, 4, 6 (Addressing)
- Mystery Number (1–5), Stage 3: Fractions with Denominators 2, 3, 4, 6 (Supporting)

Cool-down

5 mins

What Do the Diagrams Show?

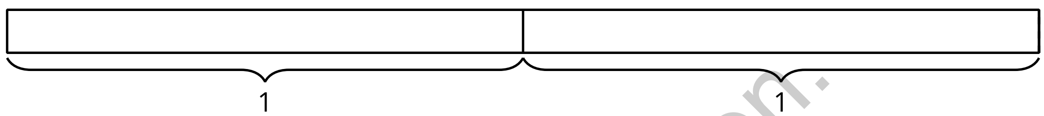
Standards

Building Towards **4.NF.A**

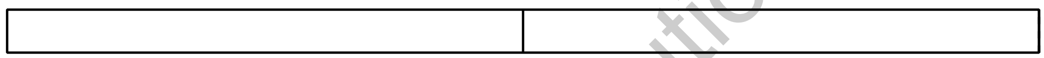
Student Task Statement

Use a blank diagram to create a representation for each fraction. Both blank diagrams represent the same quantity.

1. $\frac{5}{8}$



2. $\frac{9}{8}$



Student Response

Sample response:

1. $\frac{5}{8}$

2. $\frac{9}{8}$

Responding To Student Thinking

Students do not attend to the size of 1 whole in representing the fractions, or do not recognize $\frac{9}{8}$ as greater than 1.

Next Day Supports
Add this *Cool-down* to Activity 1 to review how to represent fractions greater than 1.

The work in this lesson builds from concepts of fractions greater than 1 addressed in a prior unit.

Prior Unit Support
Grade 3, Unit 5, Section B Fractions on the Number Line

Unit 2, Lesson 3

Same Denominator or Numerator

Standards

Building On	**3.NF.A.3.d**
Addressing	**4.NF.A.2**
Building Towards	**4.NBT.B.4, 4.NF.A.2**

Instructional Routines

- MLR1 Stronger and Clearer Each Time
- Number Talk

Goals

- Describe (orally) characteristics of the numerators or denominators when comparing fractions with the same numerator or same denominator.
- Explain (orally) strategies for comparing fractions with the same numerator or same denominator, including using physical or visual representations.

Student Facing Learning Goals

Let's compare fractions with the same numerator or the same denominator.

Lesson Purpose

The purpose of this lesson is for students to use the meaning of numerator and denominator and to compare fractions with the same numerator or the same denominator.

Narrative

In this lesson, students reason about the relative size of fractions based on the meaning of their numerators and denominators, and use fraction strips to support their reasoning.

Students first compare pairs of fractions with the same denominator but different numerators. They recall that fractions with the same denominator are composed of the same unit fractions or have parts that are the same size. So, the numerators (number of parts) can tell us how the fractions compare: the greater the numerator, the greater the fraction.

Next, students compare fractions with the same numerator but different denominators. They recognize that these fractions have the same number of parts, but the size of the parts are different. A greater denominator means more parts in 1 whole, which means the size of each part is smaller. So, the greater the denominator, the lesser the fraction.

Access For Students with Disabilities

- Engagement

Lesson Timeline

Warm-up	10 mins
Activity 1	15 mins
Activity 2	20 mins
Synthesis Estimate	10 mins
Cool-down	5 mins

Teacher Reflection Questions

Most students may find it more intuitive to compare fractions with a common denominator than those with a common numerator. Did you see students who grasp both comparisons equally well? How did they conceptualize the latter?

Warm-up

10 mins

Number Talk: Hundreds More

Standards

Building Towards **4.NBT.B.4**

Instructional Routines

- Number Talk

The purpose of this *Number Talk* is to elicit strategies and understandings students have for adding and subtracting multi-digit numbers. These understandings help students develop fluency and will be helpful in later units as students add and subtract multi-digit numbers fluently using the standard algorithm.

When students decompose addends to support mental addition, they are looking for and making use of the base-ten structure of numbers (MP7).

Student Task Statement

Find the value of each expression mentally.

- $136 + 100$
- $136 + 300$
- $136 + 370$
- $136 + 378$

Student Response

- $236: 136 + 100 = 236$
- $436: 136 + 300 = 436$
- 506: This is 70 more than 436. I know $430 + 70 = 500$, so $436 + 70 = 506$.
- 514: $300 + 100 = 400$, $30 + 70 = 100$, and $6 + 8 = 14$, and $400 + 100 + 14 = 514$.

Launch

- Display the first expression.
- "Give me a signal when you have an answer and can explain how you got it."
- 1 minute: quiet think time

Activity

- Record answers and strategies.
- Keep expressions and work displayed.
- Repeat with each expression.

Activity Synthesis

- "How did the first couple of expressions help you reason about last two expressions?"
- Consider asking:
    - "Who can restate ____'s reasoning in a different way?"
    - "Did anyone use the same strategy but would explain it differently?"
    - "Did anyone approach the expression in a different way?"
    - "Does anyone want to add on to ____'s strategy?"

Activity 1

15 mins

Fractions with the Same Denominator

Standards

Building On 3.NF.A.3.d

Building Towards 4.NF.A.2

The purpose of this activity is to prompt students to reason about the relative sizes of two fractions with the same numerator and articulate how they know which one is greater. Students have done similar reasoning work (and used similar tools to support their reasoning) in grade 3, but here the fractions include those with denominators 5 and 10. When students observe that 5 equal parts are greater than 3 of the same equal part, regardless of the size of those parts, they see regularity in repeated reasoning (MP8).

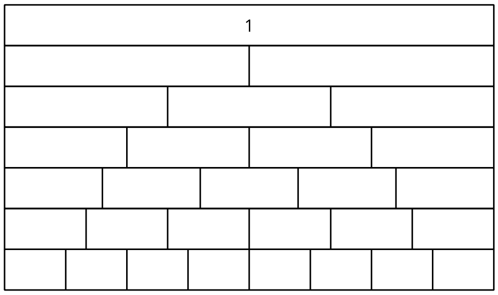
To add movement to this activity and if time permits, assign each group a pair of fractions in the second question and ask them to create a visual display showing their reasoning. Then allow a few minutes for a Gallery Walk. Ask students to identify any patterns they notice on the displays.

Access for Students with Disabilities

Engagement: Provide Access by Recruiting Interest. Provide choice and autonomy. Provide access to colored pencils that students can use to label each rectangle.
Supports accessibility for: Attention, Organization

Student Task Statement

1. This diagram shows a set of fraction strips. Label each part of each strip with the fraction it represents.



2. Circle the greater fraction in each pair. If helpful, use the diagram of fraction strips.

   a. $\frac{3}{4}$ or $\frac{5}{4}$

   b. $\frac{3}{5}$ or $\frac{5}{5}$

   c. $\frac{3}{6}$ or $\frac{5}{6}$

   d. $\frac{3}{8}$ or $\frac{5}{8}$

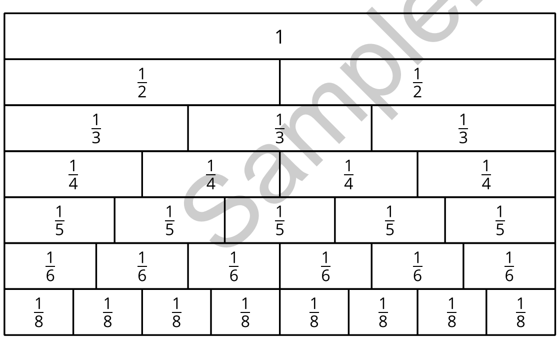
   e. $\frac{3}{10}$ or $\frac{5}{10}$

3. What pattern do you notice about the circled fractions? How can you explain the pattern?

4. Which fraction is greater: $\frac{7}{3}$ or $\frac{10}{3}$? Explain your reasoning.

Student Response

1.



2. The greater fractions are $\frac{5}{4}$, $\frac{5}{5}$, $\frac{5}{6}$, $\frac{5}{8}$, and $\frac{5}{10}$.

3. Sample responses:
   - All of the circled fractions have a numerator of 5.
   - In each pair, the fraction with 5 for the

Launch

- Groups of 2

Activity

- "Take a few quiet minutes to answer the first three questions. Discuss your work with your partner before moving on to the last question."
- 5–7 minutes: independent work time on the first 3 questions
- 3–4 minutes: partner discussion
- 2–3 minutes: independent work time on the last question

Activity Synthesis

- "What do you notice about each pair of fractions in the second question?" (They all have 3 and 5 for the numerators, and each pair has the same denominator.)
- "What does it mean when two fractions, say $\frac{3}{8}$ and $\frac{5}{8}$, have the same denominator?" (They are made up of the same fractional part—eighths in this case.)
- "How can we tell which fraction is greater when they have the same denominator?" (Because the fractional parts are the same size, we can compare the numerators. The fraction with the greater numerator is greater because it has more parts.)

numerator is greater than the one with 3 for the numerator. The denominator in each pair is the same, say, 4, so 5 fourths will be greater than 3 fourths.

- When the fractional parts are the same size, 5 of those parts will always be greater than 3.

4. Sample response: $\frac{10}{3}$ is greater. Ten $\frac{1}{3}$s are greater than seven $\frac{1}{3}$s. The parts are the same size, so 10 parts is greater than 7 parts.

Activity 2

20 mins

Fractions with the Same Numerator

Standards

Building On	**3.NF.A.3.d**
Building Towards	**4.NF.A.2**

Instructional Routines

- MLR1 Stronger and Clearer Each Time

The purpose of this activity is for students to reason about the relative sizes of two fractions with the same numerator. As before, a diagram of fraction strips can be used to help students visualize the sizes of various fractional parts. When students discuss and improve their explanation for why $\frac{70}{20}$ is greater than $\frac{70}{100}$ they develop their mathematical communication skills (MP3).

This activity uses *MLR1 Stronger and Clearer Each Time.* Advances: *Reading, Writing.*

Student Task Statement

1. Circle the greater fraction in each pair. If helpful, use the diagram of fraction strips from Activity 1.
   a. $\frac{1}{3}$ or $\frac{1}{5}$
   b. $\frac{2}{3}$ or $\frac{2}{5}$
   c. $\frac{3}{3}$ or $\frac{3}{5}$
   d. $\frac{4}{3}$ or $\frac{4}{5}$
   e. $\frac{9}{3}$ or $\frac{9}{5}$
2. What pattern do you notice about the circled fractions? How can you explain the pattern?
3. Which fraction is greater: $\frac{7}{12}$ or $\frac{7}{8}$? Explain your reasoning.
4. Tyler is comparing $\frac{4}{10}$ and $\frac{4}{6}$. He says, "I know 10 is greater than 6. So, $\frac{4}{10}$ is greater than $\frac{4}{6}$." Explain or show why Tyler's conclusion is incorrect.

Student Response

1.
   a. $\frac{1}{3}$
   b. $\frac{2}{3}$
   c. $\frac{3}{3}$
   d. $\frac{4}{3}$
   e. $\frac{9}{3}$
2. Sample responses:
   a. All of the circled fractions have a denominator of 3.
   b. In each pair, the fraction with 3 for the denominator is greater than the fraction with 5 for the denominator. A third is greater than a fifth, so 2 thirds is greater than 2 fifths, 3 thirds is greater than 3 fifths, and so on.
   c. When the number of fractional parts are the same, the fraction with the larger parts is greater.
3. $\frac{7}{8}$ is greater than $\frac{7}{12}$, because 1 eighth is greater than 1 twelfth, so 7 eighths are greater than 7 twelfths.

Launch

- Groups of 2
- "What do you notice about the fractions in the first question?" (Each pair has the same numerators and has 3 and 5 for the denominators.)

Activity

- "Think quietly for a moment about how you can find out which fraction in each pair is greater. Then share your thinking with your partner."
- 1 minute: quiet think time
- 2 minutes: partner discussion
- Monitor for students who use the size of a unit fraction or one fractional part to help them make comparisons.
- "Take a few quiet minutes to work on the questions."
- 7–8 minutes: independent work time

Activity Synthesis

- Select students to share their responses to the first two questions.
- "In the set of fractions you saw, why are the fractions with 3 for the denominator always greater than fractions with 5 for the denominator?" (A third is always greater than a fifth, so some number of thirds will always be greater than the same number of fifths.)

MLR1 Stronger and Clearer Each Time

- "Share your response to the third question with your partner. Take turns being the speaker and the listener. If you are the speaker, share your explanation. If you are the listener, ask questions and give feedback to help your partner improve their explanation."
- 3–5 minutes: structured partner discussion
- Repeat with 2–3 different partners.
- "Revise your initial explanation based on the feedback from your partners."
- 2–3 minutes: independent work time

4. Sample response: The fact that 10 is greater than 6 means the whole is divided into more parts, so each part is smaller. Each tenth is smaller than each sixth, so 4 tenths is less than 4 sixths.

Lesson Synthesis

"Today we looked at fractions with the same denominator and those with the same numerator."

Select students to share their explanations on the last question in the second activity. "What might have Tyler misunderstood? What would you say to help clear it up for him?"

"Based on your work today, how would you complete these sentence starters?"

Display and read aloud:

- "If two fractions have the same denominator, I can tell which one is greater by . . . ."
  (looking at which one has a greater numerator, because it means more of the same fractional parts)
- "If two fractions have the same numerator, I can tell which one is greater by . . . ."
  (looking at which denominator is smaller, because the smaller denominator means a larger fractional part)

Suggested Centers

- Get Your Numbers in Order (1–5), Stage 3: Denominators 2, 3, 4, 6 (Addressing)
- Mystery Number (1–5), Stage 3: Fractions with Denominators 2, 3, 4, 6 (Supporting)

Cool-down

5 mins

Sizing Up Fractions

Standards

Addressing **4.NF.A.2**

Student Task Statement

In each pair of fractions, which one is greater? Explain or show your reasoning.

1. $\frac{7}{8}$ or $\frac{10}{8}$
2. $\frac{4}{10}$ or $\frac{4}{5}$

Student Response

1. $\frac{10}{8}$ is greater, because the two fractions have the same fractional parts (eighths). There are more eighths in $\frac{10}{8}$ than in $\frac{7}{8}$.

2. $\frac{4}{5}$ is greater because 1 fifth is greater than 1 tenth, so 4 fifths are greater than 4 tenths.

Responding To Student Thinking

Students who used tape diagrams to reason about $\frac{4}{10}$ and $\frac{4}{5}$ may reason correctly about each fraction but draw different lengths to represent 1 whole (for instance, a longer tape to show tenths and a much shorter one for fifths), leading to the wrong conclusion about which fraction is greater.

Next Day Supports
Before the *Warm-up*, display and discuss tape diagrams for $\frac{4}{10}$ and $\frac{4}{5}$. Ask students to notice and wonder. Explain that to compare two fractions, the 1 whole must be the same size.

The work in this lesson builds from comparing fractions with the same numerator or denominator addressed in a prior unit.

Prior Unit Support
Grade 3, Unit 5, Section D Fraction Comparisons

Unit 2, Lesson 4

Same Size, Related Sizes

Standards

Building On	**3.NF.A.2, 3.NF.A.2.a, 3.NF.A.3.b**
Addressing	**4.NF.A.1**
Building Towards	**4.NF.A.1**

Instructional Routines

- MLR5 Co-Craft Questions
- Notice and Wonder

Goals

- Describe (orally) the relationship between fractions in which one denominator is the multiple of another (for example, $\frac{1}{2}$ and $\frac{1}{4}$).
- Explain (orally) strategies for locating fractions on the number line.
- Justify (orally) that two fractions are equivalent if they have the same size, using visual representations.

Student Facing Learning Goals

Let's find some fractions that are the same size.

Lesson Purpose

The purpose of this lesson is for students to use visual representations to reason about the fractions that have the same size and to locate them on the number line.

Narrative

In grade 3, students reasoned about equivalent fractions, using fraction strips, tape diagrams, and number lines. Here, they begin to revisit the idea of equivalence. Students examine fractions that have the same size but are expressed with different numerators and denominators. Students use diagrams of fraction strips, now expanded to include fractions with denominator 10 and 12, and then transition to using number lines to support their reasoning.

The relationships between fractions, such as $\frac{1}{4}$ and $\frac{1}{8}$, $\frac{1}{5}$ and $\frac{1}{10}$, and $\frac{1}{6}$ and $\frac{1}{12}$, in which one denominator is a multiple of the other, continue to be highlighted and offer many opportunities for students to look for and make use of structure (MP7).

Later in the unit, students will take a closer look at equivalence and investigate new ways to reason about equivalence.

As in earlier activities, rulers can be provided to help students draw, extend, or align partition lines, but they should not be used to measure the location of a fraction.

Access For Students with Disabilities

- Engagement

Required Materials

Materials To Gather

- Straightedges: Activity 1

Lesson Timeline

Warm-up	10 mins
Activity 1	20 mins
Activity 2	15 mins
Synthesis Estimate	10 mins
Cool-down	5 mins

Teacher Reflection Questions

This lesson is students' first experience with the number line in IM Grade 4. What understandings or misunderstandings about the number line did you observe today as students worked? Did you see students relating the idea of partitioning a tape diagram to partitioning a number line?

Warm-up

10 mins

Notice and Wonder: A Fraction Strip and a Number Line

Standards

Building On	**3.NF.A.2**
Building Towards	**4.NF.A.1**

Instructional Routines

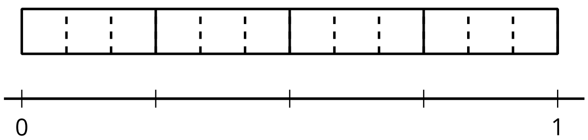
- Notice and Wonder

The purpose of this *Warm-up* is to revisit the idea from IM Grade 3 that tape diagrams and number lines are related, which will be useful later in the lesson, when students transition from using fraction strips to using the number line to represent fractions and reason about their size.

While students may notice and wonder many things about these representations, the connections between the tape diagram and number line (the number and size of the parts in relation to 1) are important to note.

Student Task Statement

What do you notice? What do you wonder?



Student Response

Student may notice:

- The number line is partitioned into fourths.
- There are 12 parts (or 12 twelfths) in the entire fraction strip.
- The entire length of the fraction strip is 1.
- There are 3 parts of $\frac{1}{12}$ for each tick mark on the number line.

Student may wonder:

- Why is the number line only partitioned into fourths?
- Why is the fraction strip partitioned into twelfths but the number line is not?
- Where is 0 on the fraction strip?
- Can we add tick marks to the number line to show twelfths?

Launch

- Groups of 2
- Display the image.
- "What do you notice? What do you wonder?"
- 1 minute: quiet think time

Activity

- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Share and record responses.

Activity Synthesis

- "How are these representations alike? How are they different?"
- "Some tick marks on the number line are not labeled. What labels do you think would be appropriate for them?" ($\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, or $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, or $\frac{3}{12}$, $\frac{6}{12}$, $\frac{9}{12}$)

Activity 1

20 mins

Same Size, Different Numbers

Standards

Building On	**3.NF.A.3.b**
Addressing	**4.NF.A.1**

This activity serves two main goals: to revisit the idea of equivalence from grade 3, and to represent non-unit fractions with denominators 10 and 12. Students use diagrams of fraction strips, which allow them to see and reason about fractions that are the same size. In the next activity, students will apply a similar process of partitioning to represent these fractional parts on number lines.

Access for Students with Disabilities

Engagement: Provide Access by Recruiting Interest. Provide choice and autonomy. Provide access to colored pencils students can use to label each rectangle.

Supports accessibility for: Attention, Organization

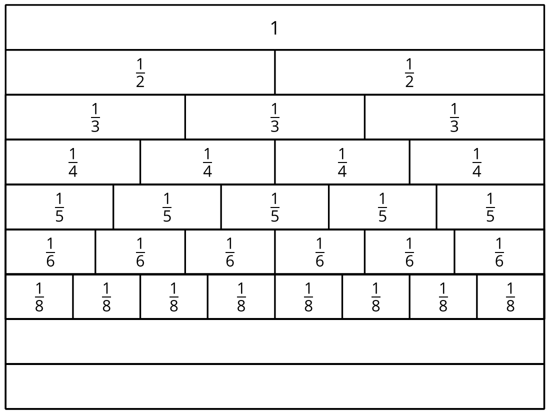
Required Materials

Materials To Gather

- Straightedges: Activity 1

Student Task Statement

Here's a diagram of fraction strips, with two blank strips added.



1. Use one blank strip to show tenths. Label the parts. How did you partition the strip?
2. Use the other blank strip to show twelfths. Label the parts. How did you partition the strip?
3. Jada says, "I notice that 1 of the $\frac{1}{2}$ parts is the same size as 2 of the $\frac{1}{4}$ parts and 3 of the $\frac{1}{6}$ parts. So, $\frac{1}{2}$, $\frac{2}{4}$, and $\frac{3}{6}$ must be equivalent fractions." Jada's reasoning is correct.

   Find a fraction in the diagram that is equivalent to each of the following fractions. Be prepared to explain your reasoning.

   a. $\frac{1}{6}$

   b. $\frac{2}{10}$

   c. $\frac{3}{3}$

Student Response

1. Sample response: We can split each fifth into 2 equal parts so that there are 10 equal parts in the entire row. Each part is 1 tenth.
2. Sample response:
   - Each sixth can be split into 2 equal parts, for a total of 12 equal parts. Each part is 1 twelfth.
   - Each fourth can be split into 3 equal parts, for a total of 12 parts.

Launch

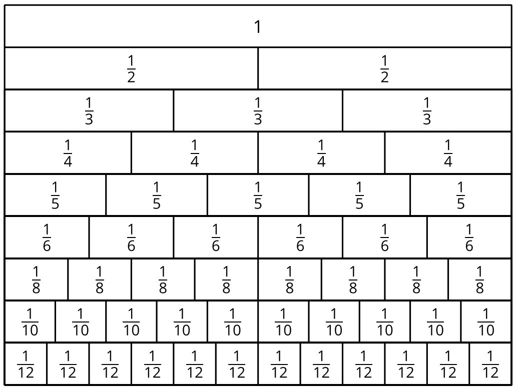
- Groups of 2
- Give each student a straightedge.
- "Here is a diagram of fraction strips you saw before, with two new rows added."
- "How can we show tenths and twelfths in the two rows? Think quietly for a minute."
- 1 minute: quiet think time

Activity

- "Work on the first two questions on your own. Afterward, discuss your responses with your partner."
- "Use a straightedge when drawing your diagram."
- 5–6 minutes: independent work time
- 2–3 minutes: partner discussion
- Monitor for students who found the size of tenths and twelfths as noted in *Student Responses*.
- Pause for a brief discussion. Select students who used different strategies to find tenths and twelfths to share.
- After each student shares, ask if others in the class did it the same way or if they have anything to add to the explanation.
- "Look at your completed diagram. What can you say about the relationship between $\frac{1}{5}$ and $\frac{1}{10}$?" (There are two $\frac{1}{10}$s in every $\frac{1}{5}$. One fifth is twice one tenth. One fifth is the same size as 2 tenths.)
- "What can you say about the relationship between $\frac{1}{6}$ and $\frac{1}{12}$?" (There are two $\frac{1}{12}$s in every $\frac{1}{6}$. One sixth is twice one twelfth. One sixth is the same size as 2 twelfths.)
- "Take 2 minutes to answer the last question."
- 2 minutes: independent or group work time

Activity Synthesis

- Invite students to share their response to the last question and explain how they found **equivalent fractions.**
- Highlight the idea that two fractions that are the same size are equivalent, even if they have different numbers for the numerators and denominators.



3. Sample response:
   a. $\frac{2}{12}$
   b. $\frac{1}{5}$
   c. $\frac{2}{2}$, $\frac{4}{4}$, $\frac{5}{5}$, $\frac{6}{6}$, $\frac{8}{8}$, $\frac{10}{10}$, and $\frac{12}{12}$

- If needed, ask: "How many fractions that are equivalent to $\frac{3}{3}$ do you see on the diagram?" (Every strip on the diagram shows a fraction equivalent to $\frac{3}{3}$.)

Activity 2

15 mins

Fractions on Number Lines

Standards

Building On **3.NF.A.2.a**

Building Towards **4.NF.A.1**

Instructional Routines

- MLR5 Co-Craft Questions

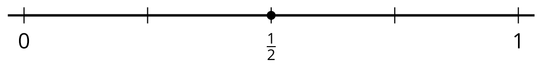
The purpose of this activity is to remind students of their work in grade 3 using number lines as a way to reason about fractions. Students see that they can partition number lines in a similar way as they partitioned fraction strips and diagrams.

This activity gives students another opportunity to notice the relationship between two fractions where one denominator is a multiple or a factor of the other, and then use this relationship to locate fractions on a number line. In doing so, students practice looking for and making use of structure (MP7).

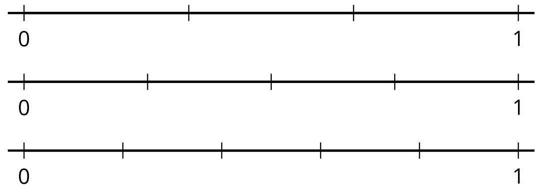
The work prepares students to use number lines to think about equivalent fractions in the next lesson.

Student Task Statement

1. The point on this number line shows the fraction $\frac{1}{2}$.



Label the tick marks on each number line.



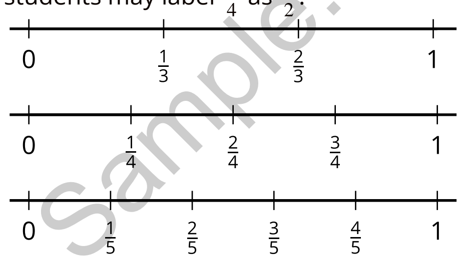
2. You will locate $\frac{1}{6}$, $\frac{1}{8}$, and $\frac{1}{10}$ on one of the number lines.
   a. Which number line would you use for each fraction? Be prepared to explain your reasoning.
   b. Locate and label each fraction ($\frac{1}{6}$, $\frac{1}{8}$, and $\frac{1}{10}$) on a different number line.
3. Locate and label each fraction on one of the number lines.

$\frac{2}{6}$ $\frac{2}{8}$ $\frac{6}{8}$ $\frac{8}{10}$ $\frac{4}{6}$

$\frac{4}{8}$ $\frac{4}{10}$ $\frac{6}{6}$ $\frac{6}{10}$ $\frac{8}{8}$

Student Response

1. Some students may label $\frac{2}{4}$ as $\frac{1}{2}$.



2. a. Sample response:
   - For $\frac{1}{6}$, I'd use the number line that shows thirds, and partition each third into 2 equal parts.
   - For $\frac{1}{8}$, I'd use the number line that shows fourths, and partition each fourth into 2 equal parts.

Launch

- Groups of 2

MLR5 Co-craft Questions

- "Keep your books closed."
- Display only the four number lines without revealing the question(s).
- "Write a list of mathematical questions that could be asked about these number lines."
- 2 minutes: independent work time
- 2–3 minutes: partner discussion
- Invite several students to share one question with the class. Record responses.
- "What do these questions have in common? How are they different?"
- Reveal the task (students open books), and invite additional connections.

Activity

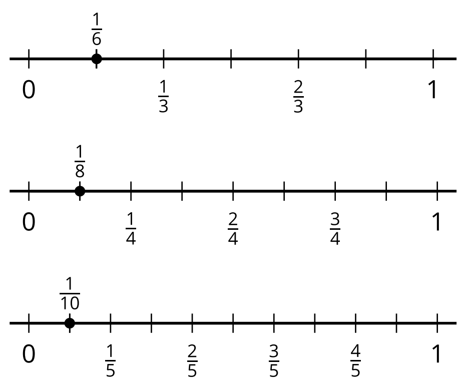
- "Take 5 minutes to complete the first two questions."
- 5 minutes: independent work time
- "Discuss your work with a partner. Make sure you and your partner agree on the labels for the number lines and can explain how you know before moving on to the last question."
- 2 minutes: partner discussion
- Monitor for students who use the tick marks for $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$ on the given number lines to locate $\frac{1}{6}$, $\frac{1}{8}$, and $\frac{1}{10}$.

Activity Synthesis

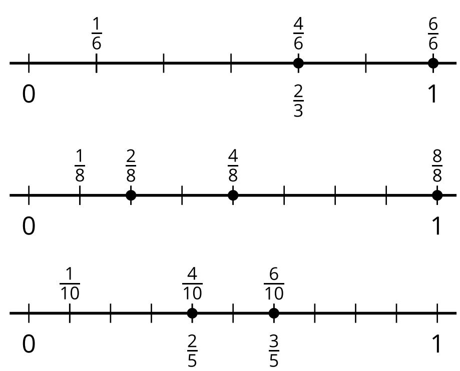
- Select 1–2 students to share their completed number lines for the second problem.
- "How did you know which number line to choose for $\frac{1}{6}$, $\frac{1}{8}$, and $\frac{1}{10}$?" (We could locate $\frac{1}{6}$, for example, on any of the number lines. But since we know that $3 \times 2 = 6$, we can split each part in the number line that shows thirds into 2 equal parts to make 6 equal parts. Having 6 equal parts makes it easiest to locate $\frac{1}{6}$.)

- For $\frac{1}{10}$, I'd use the number line that shows fifths, and partition each fifth into 2 equal parts.

b.



3.



Advancing Student Thinking

If students place the labels between the tick marks on the number line, rather than at or below the tick marks, consider asking:

- "How did you decide to label the tick marks on each line?"
- "How can you use the number line that shows $\frac{1}{2}$ to help you label the tick marks on the other number lines?"

Lesson Synthesis

Select 1–2 students to share their completed number lines from the last problem in the last activity, with points marked on the lines to represent the given fractions.

Consider asking:

- "How did you know where to put a point for a fraction? Use one of the given fractions to explain. (Choose the number line whose number of parts matches the denominator. Starting from 0, count as many tick marks as the number in the numerator. Example: For $\frac{4}{10}$, count 4 tick marks on the number line that show tenths.)

Display a completed diagram of fraction strips from the first activity.

"How is representing a fraction like $\frac{6}{10}$ on a number line like representing it on a fraction strip? How is it different?" (They both involve identifying the right fractional parts—by looking at the denominator—and then counting as many parts as the numerator of the fraction. It's different because one involves the size of parts that are folded and the other involves a specific place on the number line.)

Suggested Centers

- Get Your Numbers in Order (1–5), Stage 3: Denominators 2, 3, 4, 6 (Addressing)
- Number Line Scoot (2–4), Stage 3: Halves, Thirds, Fourths, Sixths, and Eighths (Supporting)

Cool-down

5 mins

Where on the Number Line?

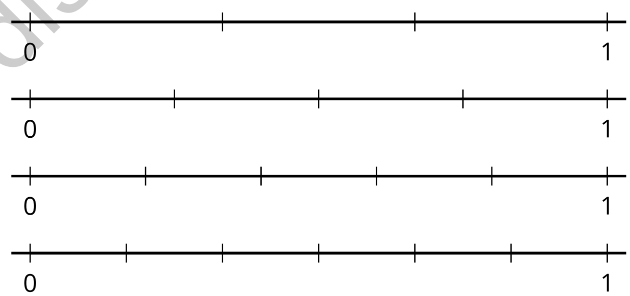
Standards

Addressing **4.NF.A.1**

Student Task Statement

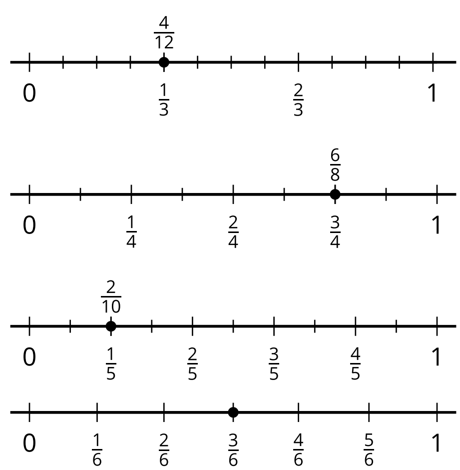
Locate and label each fraction on one of the number lines. Show your reasoning.

$\frac{3}{6}$ $\frac{2}{10}$ $\frac{6}{8}$ $\frac{4}{12}$



Student Response

Sample response:



Responding To Student Thinking

Students place the labels between the tick marks on the number line, rather than at or below the tick marks.

Next Day Supports
Consider explaining that, on a fraction strip or a tape diagram, a label like $\frac{1}{2}$ is not placed at the partition line because the number refers to a portion of the tape, rather than to the distance from 0. But on a number line, a label like $\frac{1}{2}$ is placed at the tick mark because it shows that number's distance from 0.

The work in this lesson builds from locating fractions on a number line addressed in a prior unit.

Prior Unit Support
Grade 3, Unit 5, Section B Fractions on the Number Line

Fractions on Number Lines

Standards

Building On **3.OA.B.5**

Addressing **4.NF.A.1**

Instructional Routines

- Number Talk

Goals

- Justify (orally and using other representations) that two fractions are equivalent if they are on the same point on a number line.
- Label equivalent fractions on number line diagrams.

Student Facing Learning Goals

Let's investigate equivalent fractions on a number line.

Lesson Purpose

The purpose of this lesson is for students to recognize that equivalent fractions describe the same point on the number line and to identify such fractions on a number line.

Narrative

Prior to this lesson, students used fraction strips and tape diagrams to visualize and represent fractions that are the same size. Here, they use number lines to do so. Students are reminded that equivalent fractions describe the same point on the number line, or are the same distance from 0.

To determine whether two fractions are equivalent, students rely on their understanding of fractions with related denominators (in which one denominator is a multiple of another). They practice thinking of certain fractions in terms of other fractions (for instance, thinking that they can split 1 third into 2 sixths, or 1 fifth into 2 tenths).

Access For Students with Disabilities

- Representation

Required Materials

Materials To Gather

- Straightedges: Activity 1

Lesson Timeline

Warm-up	10 mins
Activity 1	20 mins
Activity 2	15 mins
Synthesis Estimate	10 mins
Cool-down	5 mins

Teacher Reflection Questions

In the next lesson, students will compare fractions to $\frac{1}{2}$ and 1, applying what they know about equivalence and distance on a number line. How did today's work prepare students for that lesson?

Warm-up

10 mins

Number Talk: A Number Times 12

Standards

Building On **3.OA.B.5**

Instructional Routines

- Number Talk

The purpose of this *Warm-up* is to remind students of doubling as a strategy for multiplication where a factor in one product is twice a factor in another product. The reasoning that students do here with the factors 2, 4, 8, and 16 will support them as they reason about equivalent fractions and find multiples of numerators and denominators.

Student Task Statement

Find the value of each expression mentally.

- 2×12
- 4×12
- 8×12
- 16×12

Student Response

- 24: I just know it.
- 48: 4 is twice 2, so 4×12 is twice 2×12. I doubled 24 to get 48.
- 96: 8 is twice 4, so 8×12 is twice 4×12. I doubled 48 to get 96.
- 192: 16 is twice 8, so 16×12 is twice 8×12. I doubled 96 to get 192.

Launch

- Display the first expression.
- "Give me a signal when you have an answer and can explain how you got it."
- 1 minute: quiet think time

Activity

- Record answers and strategies.
- Keep problems and work displayed.
- Repeat with each expression.

Activity Synthesis

- "How did the first three expressions help you find the value of the last expression?"

Activity 1

20 mins

All Lined Up

Standards

Addressing **4.NF.A.1**

The purpose of this activity is to remind students of a key insight from grade 3—that the same point on the number line can be named with fractions that don't look alike. Students see that those fractions are equivalent, even though their numerators and denominators may be different.

Students have multiple opportunities to look for regularity in repeated reasoning (MP8). For instance, they are likely to notice that:

- Fractions that have the same number for the numerator and denominator all represent 1.
- In fractions that describe the halfway point between 0 and 1, the numerator is always half the denominator, or the denominator twice the numerator.
- In fractions that describe $\frac{1}{4}$, the denominator is 4 times the numerator.

These observations will help students to identify and generate equivalent fractions later in the unit.

Access for Students with Disabilities

Representation: Internalize Comprehension. Synthesis: Invite students to identify which details were most important to solve the problem. Display the sentence frame, "The next time points are in the same place on different number lines, I will . . . ."
Supports accessibility for: Language, Attention

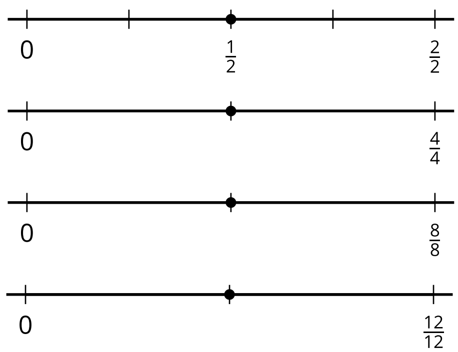
Required Materials

Materials To Gather

- Straightedges: Activity 1

Student Task Statement

1. These number lines have different labels for the tick mark on the far right.



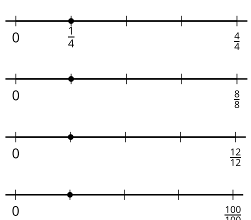
   a. Explain to your partner why the tick marks on the far right can be labeled with different fractions.

   b. Label each point with a fraction it represents (other than $\frac{1}{2}$).

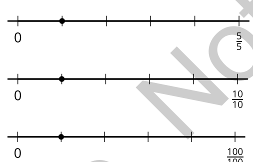
   c. Explain to your partner why the fractions you wrote are equivalent.

2. Label the point on each number line with a fraction it represents. Use a different fraction for each number line. Be prepared to explain your reasoning.

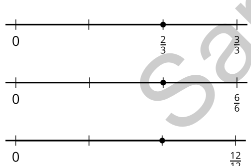
a.



b.



c.



Student Response

1. Sample responses:
   a. No written response is required. Students may discuss: Each fraction has a value of 1, and there are different ways to represent 1 with a fraction.

Launch

- Groups of 2
- Give students access to straightedges. Display the first set of number lines.
- “What do you notice? What do you wonder?” (I notice each number line has different fractions represented. The first number line has a point that is half-way between 0 and 1 labeled $\frac{1}{2}$, but if you label all the tick marks you won’t have 2 in the denominator for all of them. I wonder what fraction goes on each mark? Can you have a number line with both halves and fourths? How many fourths are at the $\frac{1}{2}$ line?)
- 1 minute: quiet think time
- “Share what you noticed and wondered with your partner.”
- 1 minute: partner discussion

Activity

- “Take a moment to work independently on the task. Then discuss your work with your partner.”
- “The labels that you write for the points on different number lines should be different.”
- 7–8 minutes: independent work time
- Monitor for students who:
  - Partition each number line into as many parts as the denominator before naming a fraction for the point on the number line.
  - Use multiplicative relationships between denominators to name a fraction (for instance, $4 \times 3 = 12$, so the line showing twelfths has 3 times as many parts as the one showing fourths).

Activity Synthesis

- Select students to share their responses and reasoning for the first set of questions. Highlight explanations that convey that:
  - Any fraction with the same number for the numerator and denominator has a value of 1.
  - Equivalent fractions share the same location or are the same distance from 0 on the number line.

b. The labels should say $\frac{2}{4}$, $\frac{4}{8}$, and $\frac{6}{12}$.

c. No written response is required. Students may discuss: The fractions are all halfway between 0 and 1 on the number line, so they are the same size. They all represent one-half. They are the same point on the number line.

2. a. $\frac{2}{8}$, $\frac{3}{12}$, and $\frac{25}{100}$

   b. $\frac{1}{5}$, $\frac{2}{10}$, and $\frac{20}{100}$

   c. $\frac{4}{6}$ and $\frac{8}{12}$

- Invite previously selected students to share their responses for the second set of questions.
- Consider asking:
  - “Who can restate _______ 's reasoning in a different way?”
  - “How are these ways for labeling the points the same? How are they different?“

Activity 2

15 mins

How Far to Run?

Standards

Addressing **4.NF.A.1**

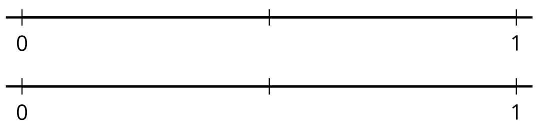
In this activity, students reason about whether two fractions are equivalent in the context of distance. To support their reasoning, students use number lines and their understanding of fractions with related denominators (where one number is a multiple of the other). The given number lines each have only one tick mark between 0 and 1, so students need to partition each line strategically to represent two fractions with different denominators on the diagram.

To help students intuit the distance of 1 mile, consider preparing a neighborhood map that shows the school and some points that are a mile away. Display the map during the *Launch*.

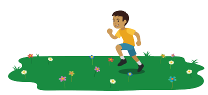
Student Task Statement

1. Han and Kiran plan to go for a run after school.
    - Han says, "Let's run $\frac{3}{4}$ mile. That's how far I run to my soccer practice."
    - Kiran says, "I can only run $\frac{9}{12}$ mile."

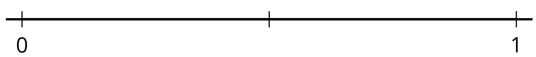
   Which distance should they run? Explain your reasoning. Use one or more number lines to show your reasoning.



2. Mai wants to join Han and Kiran on their run. She says, "How about we run $\frac{7}{8}$ mile?"

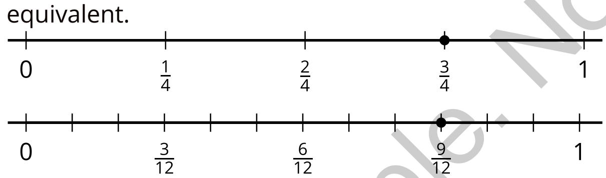


   Is the distance Mai suggests the same as what her friends wanted to run? Explain or show your reasoning.

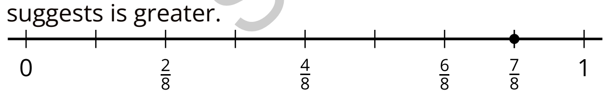


Student Response

1. Sample response: The distances they propose are the same, so either choice works. $\frac{3}{4}$ and $\frac{9}{12}$ are equivalent.



2. Sample response: No, $\frac{7}{8}$ is not the same as what Kiran and Han suggested. $\frac{3}{4}$ is equivalent to $\frac{6}{8}$, not $\frac{7}{8}$. $\frac{7}{8}$ is greater than $\frac{6}{8}$, so the distance Mai suggests is greater.



Launch

- Groups of 2
- "Who has walked a mile? Who has run a mile?"
- "How far is 1 mile? How would you describe it?"
- Consider showing a map of the school and some landmarks or points on the map that are 1 mile away.

Activity

- 6–8 minutes: independent work time
- Monitor for the different ways students reason about the equivalence of $\frac{9}{12}$ and $\frac{3}{4}$. For instance, they may:
    - Know that 1 fourth is equivalent to 3 twelfths and reason that 3 fourths must be 9 twelfths.
    - Note that $\frac{3}{4}$ and $\frac{9}{12}$ are both halfway between $\frac{1}{2}$ and 1 on the number line.
    - Locate $\frac{3}{4}$ and $\frac{9}{12}$ on the same number line (or separate ones) and show that they are in the same location.
- 2–3 minutes: partner discussion

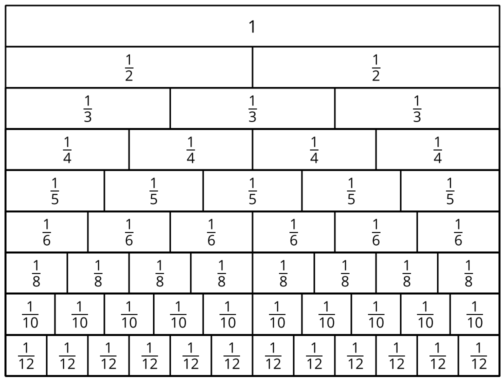
Activity Synthesis

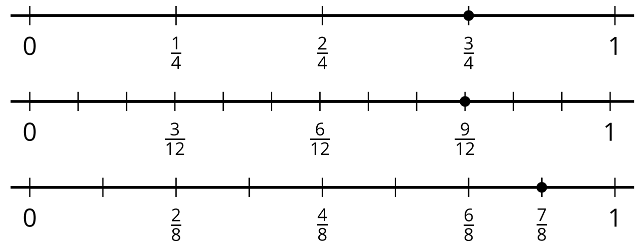
- Invite previously selected students to share their responses and how they knew that $\frac{9}{12}$ is equivalent to $\frac{3}{4}$ but $\frac{7}{8}$ is not.
- To facilitate their explanations, ask student to display their work, or display blank number lines for them to annotate.
- Consider asking:
    - "Who reasoned the same way but would explain it differently?"
    - "Who thought about it differently but arrived at the same conclusion?"

Lesson Synthesis

"Today we represented fractions on number lines and reasoned about equivalent fractions."

Display a labeled diagram of fraction strips and the labeled number lines from the last problem in today’s activities.





“Where in the diagram of fraction strips do we see equivalent fractions?” (Parts that have the same length are equivalent.)

“Where on the number lines do we see equivalent fractions?” (Points that are in the same location on the number line, or are the same distance from 0, are equivalent.)

“Suppose you’d like to help someone see that $\frac{1}{5}$ is equivalent to $\frac{10}{50}$. Would you use a number line or a fraction strip? Why?” (Sample response: Use a number line, because it’s not necessary to show all the tick marks. If using fraction strips, it would mean partitioning each fifth into 10 fiftieths, which would be a lot of work.)

Suggested Centers

- Get Your Numbers in Order (1–5), Stage 3: Denominators 2, 3, 4, 6 (Addressing)
- Number Line Scoot (2–4), Stage 3: Halves, Thirds, Fourths, Sixths, and Eighths (Supporting)

Cool-down

5 mins

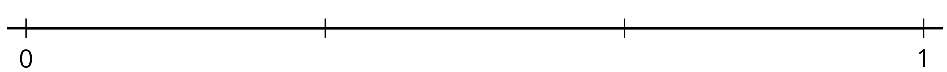
Two of the Same

Standards

Addressing **4.NF.A.1**

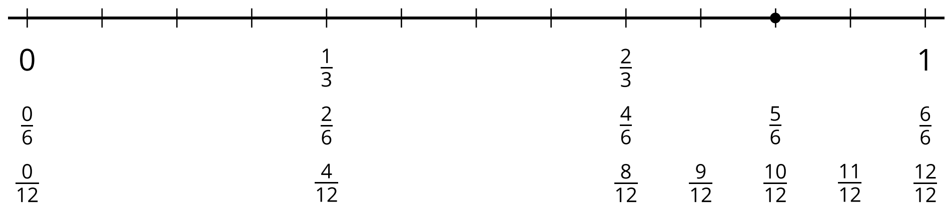
Student Task Statement

Show $\frac{5}{6}$ on the number line. Be sure to include labels. Then explain or show that the fraction $\frac{10}{12}$ is equivalent to $\frac{5}{6}$.



Student Response

Sample response:



Each third can be partitioned into 2 sixths, and each sixth into 2 twelfths. There are 10 twelfths in 5 sixths.

Responding To Student Thinking

Students label $\frac{5}{6}$ but do not explain or show the equivalence of $\frac{5}{6}$ and $\frac{10}{12}$.

Next Day Supports

Before the *Warm-up*, ask students to work with a partner to discuss a correct response to this *Cool-down*. Encourage them to use both fraction strips and number lines to support their reasoning.

Unit 2, Lesson 6

Relate Fractions to Benchmarks

Standards

Building On	**3.NF.A.2, 3.NF.A.3**
Addressing	**4.NF.A.2**
Building Towards	**4.NF.A.2**

Instructional Routines

- Card Sort
- Notice and Wonder

Goals

- Justify (using words or other representations) that a fraction is greater than or less than $\frac{1}{2}$ or 1.

Student Facing Learning Goals

Let's compare the size of fractions to $\frac{1}{2}$ and to 1.

Lesson Purpose

The purpose of this lesson is for students to locate fractions on the number line and compare their size to $\frac{1}{2}$ and to 1.

Narrative

In this lesson, students continue to identify fractions on the number line by reasoning about known distances or intervals. They also consider the size of fractions in relation to $\frac{1}{2}$ and 1, by examining the position and distance of fractions from these benchmarks on the number line.

Although students consider the distance between a point on the number line and either $\frac{1}{2}$ or 1, finding differences of two fractions is not the focus of this lesson. (That mathematical work will take place in a future unit.) What is important here is for students to reason about the relative sizes of fractions using number lines, their knowledge of equivalent fractions and familiar benchmarks, and the meaning of a fraction's numerator and denominator. Activity 2 is optional and allows an opportunity for students to use the relationships between a fraction's numerator and denominator and between different denominators without requiring them to use the number line.

Access For Students with Disabilities

- Representation

Access For English Learners

- MLR8

Required Materials

Materials To Copy

- Card Sort Where Do They Belong? Cards (1 copy for every 2 students): Activity 2

Lesson Timeline

Warm-up	10 mins
Activity 1	20 mins
Activity 2	20 mins
Activity 3	15 mins
Synthesis Estimate	10 mins
Cool-down	5 mins

Teacher Reflection Questions

What question asked today seemed to promote students' reasoning about benchmarks to compare fractions?

Warm-up

10 mins

Notice and Wonder: A Point on a Number Line

Standards

Building On **3.NF.A.2**

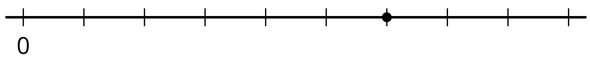
Instructional Routines

- Notice and Wonder

The purpose of this *Warm-up* is for students to recognize that two values of reference are needed to determine the number that a point on the number line represents. The numbers 0 and 1 are commonly used when the numbers of interest are small. With only one number shown (for example, only a 0 or a 1), we can't tell what number a point represents, though we can tell if the number is greater or less than the given number. These understandings will be helpful later in the lesson, as students determine the size of fractions relative to $\frac{1}{2}$ and 1.

Student Task Statement

What do you notice? What do you wonder?



Student Response

Students may notice:

- A number line has a point on it.
- The point is on the 6th tick mark from 0.
- There are no other numbers on the line other than 0.
- There are 10 tick marks on the line.
- The point shows a number greater than 0.

Students may wonder:

- Why isn't there a label for 1?
- Is it possible to tell what number the point represents if there are no other labels?
- Can we assume that the last tick mark represents 1?

Launch

- Groups of 2
- Display the image.
- "What do you notice? What do you wonder?"
- 1 minute: quiet think time

Activity

- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Share and record responses.

Activity Synthesis

- "How would we know what number the point represents? What's missing and needs to be there?" (A label for one of the tick marks so that we'd know what each space between tick marks represents.)

Activity 1

20 mins

Greater than or Less than 1?

Standards

Building On	**3.NF.A.2**
Building Towards	**4.NF.A.2**

The purpose of this activity is for students to identify fractions using known benchmarks on the number line and to compare them to 1. Given a point on a number line, the location of 0, and one other benchmark value, students decide if the point represents a number greater or less than 1. They also quantify the distance of that number from 1. Students do so by relying on what they know about the number of fractional parts in 1 whole, as well as by looking for and making use of structure (MP7).

The work here develops students' ability and flexibility in using number lines to reason about fractions. In later lessons, students will work with number lines that are increasingly more abstract to help them reason about fractions in more sophisticated ways.

Access for English Language Learners

MLR8 Discussion Supports. Synthesis: For each response that is shared, invite students to turn to a partner and restate what they heard using precise mathematical language.
Advances: Listening, Speaking

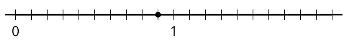
Student Task Statement

For each number line:

a. Name a fraction that the point represents.

b. Is that fraction greater than or less than 1?

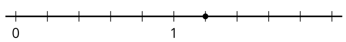
c. How far is it from 1?

1. 

   a.

   b.

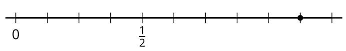
   c.

2. 

   a.

   b.

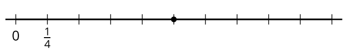
   c.

3. 

   a.

   b.

   c.

4. 

   a.

   b.

   c.

Student Response

1. a. $\frac{9}{10}$
   b. less
   c. $\frac{1}{10}$
2. a. $\frac{6}{5}$
   b. greater
   c. $\frac{1}{5}$
3. a. $\frac{9}{8}$
   b. greater
   c. $\frac{1}{8}$
4. a. $\frac{5}{4}$

Launch

- Groups of 2–4
- "Tell your partner a fraction that is greater than 1 and a fraction that is less than 1. Explain how you know."
- 1 minute: partner discussion
- Share responses and ask how they used 1 whole to choose their fractions.
- Read the *Task Statement* as a class. Make sure students understand that they are to do three things for each number line diagram.

Activity

- "Take a few minutes to work independently on at least two diagrams before discussing with your group."
- 5 minutes: independent work time
- 5–7 minutes: group work time
- Monitor for students who:
  - Label one or more tick marks with unit fractions.
  - Locate the number 1 on the number line when it is not given.

Activity Synthesis

- Invite previously selected students to share their responses. Display their work, or display the number lines from the task for them to annotate as they explain.
- "How did you know what fraction each point represents?" (Figure out what each space between tick marks represents, and then count the number of spaces.)
- "How did you know if it's more or less than 1?" (It is more than 1 if the point is to the right of 1, or if the numerator is greater than the denominator.)

b. greater

c. $\frac{1}{4}$

Activity 2: Optional

20 mins

Card Sort: Where Do They Belong?

Standards

Building On	**3.NF.A.3**
Building Towards	**4.NF.A.2**

Instructional Routines

- Card Sort

In this optional activity, students sort a set of fractions into groups based on whether they are less than, equal to, or greater than $\frac{1}{2}$. This sorting task enables students to estimate or to reason informally about the size of fractions relative to this benchmark before they go on to do so more precisely. In the next activity, students reason about fractions represented by unlabeled points on the number line and their distance from $\frac{1}{2}$.

As students discuss and justify their decisions, they share a mathematical claim and the thinking behind it (MP3).

This activity is optional because it asks students to reason about fractions without the support of the number line.

Required Materials

Materials To Copy

- Card Sort Where Do They Belong? Cards (1 copy for every 2 students): Activity 2

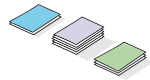
Required Preparation

- Create a set of fraction cards from the blackline master for each group.

Student Task Statement

Your teacher will give you a set of cards that show fractions.

1. Sort the cards into 3 groups: less than $\frac{1}{2}$, equal to $\frac{1}{2}$, and greater than $\frac{1}{2}$. Be ready to explain your reasoning.



Discuss your sorting with another group. Then record the fractions in the table.

less than $\frac{1}{2}$	equal to $\frac{1}{2}$	greater than $\frac{1}{2}$

2. Discuss your sorting with the class. Then complete the sentences.
    - A fraction is less than $\frac{1}{2}$ when . . .
    - A fraction is greater than $\frac{1}{2}$ when . . .
    - A fraction is between $\frac{1}{2}$ and 1 when . . .

Student Response

1. Groups:

less than $\frac{1}{2}$	equal to $\frac{1}{2}$	greater than $\frac{1}{2}$
$\frac{1}{3}, \frac{1}{5}, \frac{1}{12}, \frac{2}{10}, \frac{3}{8}, \frac{3}{12}, \frac{4}{12}, \frac{5}{12}$	$\frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \frac{6}{12}$	$\frac{2}{3}, \frac{3}{5}, \frac{4}{2}, \frac{4}{5}, \frac{5}{6}, \frac{5}{8}, \frac{6}{3}, \frac{7}{10}, \frac{8}{4}, \frac{9}{10}, \frac{11}{12}$

Launch

- Groups of 2–4
- Give each group a set of cards.

Activity

- "This set of cards includes fractions. Sort the cards into three groups: fractions that are less than $\frac{1}{2}$, fractions that are equal to $\frac{1}{2}$, and fractions that are greater than $\frac{1}{2}$. Work with your group to explain your reasoning."
- "When you are done, compare your sorting results with another group."
- "If the two groups disagree about where a fraction belongs, discuss your thinking until you reach an agreement."
- 7–8 minutes: group work time
- 3–4 minutes: Discuss results with another group.
- "Record your sorting results after you have discussed them."

Activity Synthesis

- Invite groups to share how they sorted the fractions.
- "How did the numerator and denominator of each fraction tell you how a fraction relates to $\frac{1}{2}$?" (Sample responses:
    - We already know fractions that are equivalent to $\frac{1}{2}$, so we could compare any fraction to one of those equivalent fractions that has the same denominator.
    - A fraction that is equal to $\frac{1}{2}$ has a denominator that is twice the numerator.
    - If a numerator is less than half of the denominator, the fraction is less than $\frac{1}{2}$. If the numerator is greater than half of the denominator, it is more than $\frac{1}{2}$.
    - If a numerator is 1 or is much less than the denominator, then the fraction is small and less than $\frac{1}{2}$ (except for $\frac{1}{2}$ itself).
    - If a numerator is really close to the denominator, then the fraction is close to 1, which means it is greater than $\frac{1}{2}$ (except for $\frac{1}{2}$ itself).

Sample response: $\frac{11}{12}$ is greater than $\frac{1}{2}$ because it's only $\frac{1}{12}$ from 1.

2. Sample responses:
    - A fraction is less than $\frac{1}{2}$ when the numerator is less than half of the denominator.
    - A fraction is greater than $\frac{1}{2}$ when the numerator is more than half of the denominator.
    - A fraction is between $\frac{1}{2}$ and 1 when the numerator is really close to the denominator.

- Give students 2–3 minutes of quiet time to complete the sentence frames in the activity.

Activity 3

15 mins

Greater than or Less than $\frac{1}{2}$?

Standards

Building On	**3.NF.A.2, 3.NF.A.3**
Building Towards	**4.NF.A.2**

Previously, students located fractions on number lines and considered their distance and relative position to 1. Here, they think about fractions in relation to $\frac{1}{2}$. The purpose of this activity is to prompt students to use another benchmark value to determine the relative size of a fraction.

While students may be able to visually tell if a point on the number line represents a number that is greater or less than $\frac{1}{2}$, finding its distance to $\frac{1}{2}$ is less straightforward than finding its distance to 1. The former requires thinking about $\frac{1}{2}$ in terms of equivalent fractions.

In three cases, the fraction $\frac{1}{2}$ and the point of interest are each on a tick mark on the number line. This makes it possible for students to quantify the distance without further partitioning the number line. In the last diagram, $\frac{1}{2}$ is not on a tick mark, prompting students to subdivide the given intervals, relying on their understanding of equivalence and relationships between fractions.

The work here encourages students to look for and make use of structure (MP7) and will be helpful later in the unit when students compare fractions by reasoning about their distance from benchmark values.

Access for Students with Disabilities

Representation: Internalize Comprehension. Synthesis: Invite students to identify which details were necessary to solve the problem. Display the sentence frame, "The next time I compare a fraction to $\frac{1}{2}$, I will look for . . . ."
Supports accessibility for: Language, Attention, Conceptual Processing

Student Task Statement

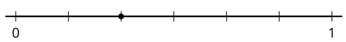
For each number line:

a. Name a fraction that the point represents.

b. Is that fraction greater than or less than $\frac{1}{2}$?

c. What fraction describes how far the point is from $\frac{1}{2}$?

1.

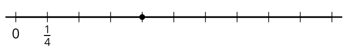


a.

b.

c.

2.

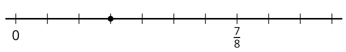


a.

b.

c.

3.

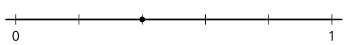


a.

b.

c.

4.



a.

b.

c.

Student Response

1. a. $\frac{2}{6}$
   b. less
   c. $\frac{1}{6}$
2. a. $\frac{4}{4}$
   b. greater
   c. $\frac{2}{4}$
3. a. $\frac{3}{8}$
   b. less
   c. $\frac{1}{8}$
4. a. $\frac{2}{5}$
   b. less

Launch

- Groups of 2–4
- "Let's identify a few more fractions on number lines, but this time, let's find out how they relate to $\frac{1}{2}$."

Activity

- "Work independently for a few minutes. Work through at least two diagrams before discussing with your group."
- 5 minutes: independent work time
- 5 minutes: group work time
- Monitor for students who:
  - Locate 1 and $\frac{1}{2}$ on the number line.
  - Label the point for $\frac{1}{2}$ with an equivalent fraction whose denominator matches the number of intervals between 0 and 1. (Example: Label the middle tick mark on the first number line with $\frac{3}{6}$.)
  - On the last number line, subdivide the intervals of fifths into tenths in order to locate $\frac{1}{2}$ at $\frac{5}{10}$.

Activity Synthesis

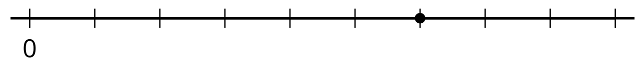
- "How did you know where $\frac{1}{2}$ is on each number line?" (Find out where 1 is, and then locate the halfway point. Use fractions that are equivalent to $\frac{1}{2}$, such as $\frac{3}{6}$, $\frac{4}{8}$, and $\frac{5}{10}$.)
- "What was different about the last number line compared to the others?" (There was no tick mark to represent $\frac{1}{2}$ on the number line. The number line had an odd number of intervals.)
- "What did you have to do differently to figure out how far away the fraction is from $\frac{1}{2}$ on the last number line?" (First split each fifth into tenths and then locate $\frac{5}{10}$.)

c. $\frac{1}{10}$

Lesson Synthesis

"Today we identified fractions on a number line and compared them to $\frac{1}{2}$ and 1."

Display the number line from the *Warm-up* (or ask students to refer to the diagram there).



Label one of the tick marks (other than the one with the point) with "$\frac{1}{2}$".

"Suppose a classmate is absent today, and you are asked to explain how to figure out the fraction that the point represents and how far away it is from $\frac{1}{2}$. What would you say?" (I'd see how far away $\frac{1}{2}$ is from 0 and then double that distance to know where 1 is, which would tell me the size of each space between tick marks. If $\frac{1}{2}$ is 4 spaces away from 0, then 1 must be 8 spaces away, and each space must represent $\frac{1}{8}$. I'd count the spaces from 0 to know the fraction. I'd count the spaces between the point and $\frac{1}{2}$ to know its distance from $\frac{1}{2}$.)

Suggested Centers

- Get Your Numbers in Order (1–5), Stage 3: Denominators 2, 3, 4, 6 (Addressing)
- Number Line Scoot (2–4), Stage 3: Halves, Thirds, Fourths, Sixths, and Eighths (Supporting)

Cool-down

5 mins

Greater than or Less than . . .?

Standards

Addressing **4.NF.A.2**

Student Task Statement

For each question, explain or show your reasoning. Use a number line if it is helpful.

1. Is $\frac{6}{10}$ greater or less than $\frac{1}{2}$?
2. Is $\frac{11}{12}$ greater or less than 1?

Student Response

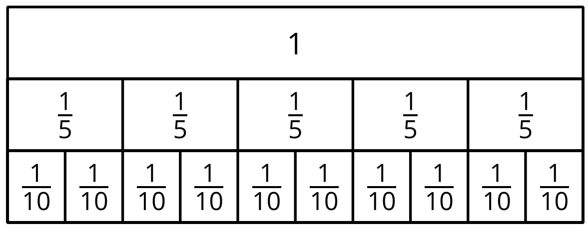
1. Greater than $\frac{1}{2}$. Sample response: I know that $\frac{5}{10}$ is equivalent to $\frac{1}{2}$ and $\frac{6}{10}$ is greater than $\frac{5}{10}$.
2. Less than 1. Sample response: I know that $\frac{12}{12}$ is 1 and $\frac{11}{12}$ is less than $\frac{12}{12}$.

Responding To Student Thinking

Students respond that $\frac{6}{10}$ is less than $\frac{1}{2}$ or $\frac{11}{12}$ is more than 1 whole.

Next Day Supports
In a small group, give students access to pre-made fraction strips. Ask them to list fractions that have the same size as $\frac{1}{2}$ and to notice patterns in the numbers in those fractions.

Section A Summary



We used fraction strips to represent fractions with denominators of 2, 3, 4, 5, 6, 8, 10, and 12. We also used the strips to reason about relationships between fractions.

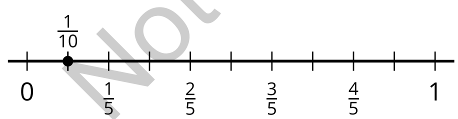
Example:

1 whole split into 5 equal parts = 5 fifths

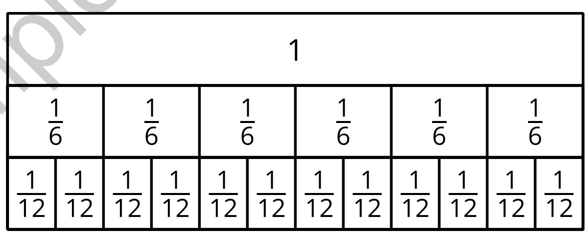
each fifth split into 2 equal parts = 10 equal parts = 10 tenths

When the denominator is larger, there are more parts in a whole.

We used what we learned about fraction strips to partition number lines and represent different fractions.



We also used fraction strips to reason about the sizes of fractions.



Same Denominator: The size of the parts is the same. So, the fraction with more parts is greater.

Example: $\frac{5}{6}$ is greater than $\frac{2}{6}$.

Same Numerator: The number of parts is the same. So, the fraction with larger parts is greater.

Example: $\frac{5}{6}$ is greater than $\frac{5}{12}$.