Illustrative Mathematics
LEARN MATH FOR LIFE

GRADE 7

Teacher Guide

UNIT

2

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Section A: Representing Proportional Relationships with Tables

Goals

- Determine the constant of proportionality for a proportional relationship represented in a table, and interpret it in context.
- Explain how to calculate unknown values in a table that represents a proportional relationship.

Section Narrative

In this section, students work with proportional relationships that are represented in tables. Students begin by reviewing situations that involve ratios and identifying which ratios are equivalent. Next, they learn to view a table of equivalent ratios as representing a proportional relationship. Students learn that all entries in one column of the table can be obtained by multiplying entries in the other column by the *same* number. This number is called the constant of proportionality. Students use tables to solve problems involving proportional relationships.

grape juice (cups)		peach juice (cups)
5	$\cdot 0.4$	2
10	$\cdot 0.4$	4
30	$\cdot 0.4$	12
2.5	$\cdot 0.4$	1

A note about constant of proportionality:

Although a proportional relationship between two quantities represented by a and b is associated with two constants of proportionality, $\frac{a}{b}$ and $\frac{b}{a}$, throughout the unit, the convention is if a and b are, respectively, in the left and right columns of a table, then $\frac{b}{a}$ is the constant of proportionality for the relationship represented by the table. The quantity represented by the right column is said to be proportional to the quantity represented by the left.

Teacher Reflection Questions

- **Math Content and Student Thinking:** In an earlier grade, students learned to find equivalent ratios. How do you see students' understanding of equivalent ratios helping them to reason about the unknown values in tables of proportional relationships in this section?
- **Pedagogy:** Reflect on times you observed students listening to one another's ideas so far this year. What norms would help each student better attend to their classmates' ideas in future lessons?
- **Access and Equity:** Lesson 2, Activity 3: Making Coco Bread is the first time that the *Three Reads* math language routine is suggested as a support in this course. How did this math language routine help all students make sense of the situation and what the question was asking?

Section A Checkpoint

1

Goals Assessed

- Determine the constant of proportionality for a proportional relationship represented in a table, and interpret it in context.
- Explain how to calculate unknown values in a table that represents a proportional relationship.

Student Task Statement

The table represents mixtures of black and white paint that produce the same shade of gray.

black paint (cups)	white paint (cups)
$\frac{1}{2}$	4
3	24
1	
	16

Complete the table as you answer the questions.

a. To make the same shade of gray, how many cups of white paint will they need to mix with 1 cup of black paint? Explain or show your reasoning.

b. How many cups of black paint will they need to mix with 16 cups of white paint? Explain or show your reasoning.

c. Make up a new pair of numbers that would make the same shade of gray.

d. What is the constant of proportionality?

e. What does the constant of proportionality mean in this situation?

Solution

a. 8 cups. Sample reasoning: From the row showing $\frac{1}{2}$ and 4, I multiplied both by 2 to get 1 and 8.

b. 2 cups. Sample reasoning: I divided 16 by 8 to get 2.

c. Sample response: 10 cups of black paint with 80 cups of white paint. (Any pair of numbers that makes a ratio equivalent to $1 : 8$ is acceptable.)

d. 8

e. For every 1 cup of black paint, mix in 8 cups of white paint.

black paint (cups)	white paint (cups)
$\frac{1}{2}$	4
3	24
1	8
2	16
10	80

Responding To Student Thinking

Press Pause
By this point in the unit, there should be some student mastery of working with a table that represents a proportional relationship and identifying the constant of proportionality. If students struggle with this, make time to examine related work in the lessons referred to here. The Course Guide provides additional ideas for revisiting earlier work.

Grade 7, Unit 2, Lesson 2 Introducing Proportional Relationships with Tables
Grade 7, Unit 2, Lesson 3 More about Constant of Proportionality

Unit 2, Lesson 1

One of These Things Is Not Like the Others

Goals

- Choose and create representations to compare ratios in the context of recipes or scaled copies.
- Coordinate (orally) different representations of a situation involving equivalent ratios, e.g., discrete diagrams, tables, or double number line diagrams.
- Determine which recipes or geometric figures involve equivalent ratios, and justify (orally, in writing, and through other representations) that they are equivalent.

Learning Targets

- I can use equivalent ratios to describe scaled copies of shapes.
- I know that two recipes will taste the same if the ingredients are in equivalent ratios.

Lesson Narrative

In this lesson, students examine situations that involve ratios. They identify features that can be described with **equivalent ratios**. There are opportunities to review work from grade 6 in representing ratios with tables and diagrams. This is intended to support initial, informal conversations about the key ideas in proportional relationships before those ideas are formally introduced.

The tasks are intentionally not well-posed, that is, they do not have exact solutions. They are designed to prompt students to think about how we can use mathematical models to make sense of common perceptual experiences, such as things that taste or look the same or different (MP4). The focus is on examination of a feature that can be represented as a unit rate.

In the first activity, students are given the relevant measurements needed to compare the situations. In the second activity, they are asked to think about how to quantify what they see, in particular, what measurements might help describe the picture. The work of describing observations qualitatively and quantitatively encourages students to communicate with precision (MP6).

Math Community

Today's math community building time has two goals. The first is for students to make a personal connection to the math actions chart and to share on their *Cool-down* the math action that is most important to them. The second is to introduce the idea that the math actions that students have identified will be used to create norms for their mathematical community in upcoming lessons.

Standards

Building On	**6.RP.A**
Addressing	**7.G.A.1**
Building Towards	**7.RP.A**

Instructional Routines

- MLR2: Collect and Display
- MLR8: Discussion Supports

Required Materials

Materials To Gather

- Math Community Chart: Activity 1
- Colored pencils: Activity 2
- Drink mix: Activity 2
- Measuring cup: Activity 2
- Measuring spoons: Activity 2
- Mixing containers: Activity 2
- Small disposable cups: Activity 2
- Water: Activity 2
- Geometry toolkits: Activity 3

Activity 2:

Prepare to show the three drink mixtures, either by making them yourself or by displaying the video.

To make three mixtures:

- 1 cup of water with $1\frac{1}{2}$ teaspoons of powdered drink mix
- 2 cups of water with $\frac{1}{2}$ teaspoon of powdered drink mix
- 1 cup of water with $\frac{1}{4}$ teaspoon of powdered drink mix

Students will need three small cups each; they just need a few sips of the mixture in each cup.

Activity 3:

For the digital version of the activity, acquire devices that can run the applet.

Student Facing Learning Goals

Let's remember what equivalent ratios are.

1.1 Remembering Double Number Lines

Warm-up

5 mins

Activity Narrative

This activity prompts students to reason about equivalent ratios on a double number line and think of reasonable scenarios for these ratios. This is a review of their work in grade 6.

Standards

Building On **6.RP.A**

Instructional Routines

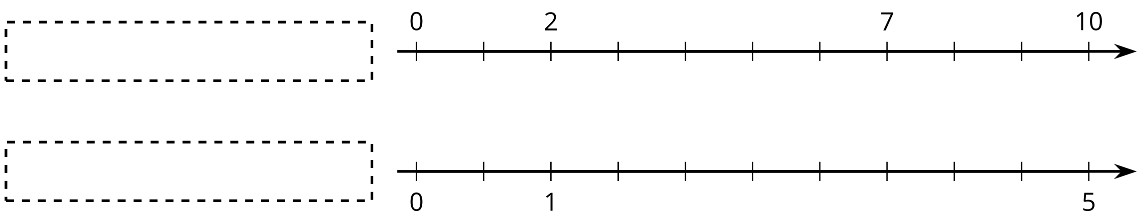
- MLR8: Discussion Supports

Launch

Arrange students in groups of 2. Give students 2 minutes of quiet work time followed by partner discussion. As students discuss their answers and reasoning with their partner, select students to share during the whole-class discussion.

Student Task Statement

1. Complete the double number line diagram with the missing numbers.

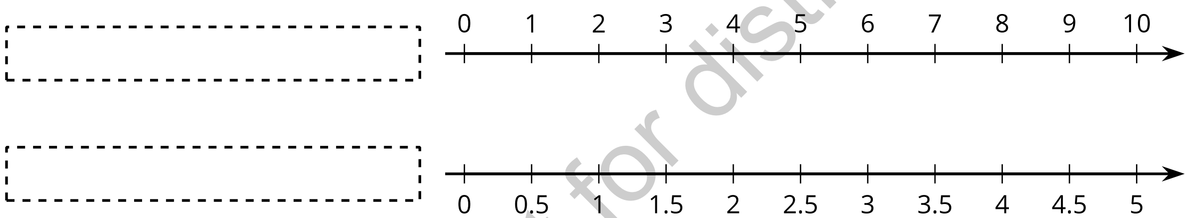


2. What could each of the number lines represent? Invent a situation and label the diagram.

   Make sure your labels include appropriate units of measure.

Student Response

1. The top number line counts by 1 while the bottom number line counts by $\frac{1}{2}$ or equivalent.



2. Sample responses:
   a. number of wheels and number of bicycles
   b. pints of sauce and quarts of sauce
   c. chocolate powder (tablespoons) and milk (cups)

Building on Student Thinking

Students may struggle thinking of a scenario with a $1:2$ ratio. For those students, ask them if they can draw a picture that would represent that ratio and label each line accordingly.

Activity Synthesis

Display the double number line for all to see with correct values filled in. It does not matter whether the bottom line is labeled with fractions, decimals, or mixed numbers.

Invite selected students to share the situations they came up with and the units for each quantity. After each student shares, invite others to agree or disagree with the reasonableness of the diagram representing that situation. For example, is it really reasonable to say that 7 wheels make $3\frac{1}{2}$ bicycles?

Access for English Language Learners

MLR8 Discussion Supports. Provide students with the opportunity to rehearse what they will say with a partner before they share with the whole class.
Advances: Speaking

Math Community
After the *Warm-up*, display the revisions to the class Math Community Chart that were made from student suggestions in an earlier exercise. Tell students that over the next few exercises, this chart will help the class decide on community norms—how they as a class hope to work and interact together over the year. To get ready for making those decisions, students are invited at the end of today's lesson to share which "Doing Math" action on the chart is most important to them personally.

1.2 Mystery Mixtures

15 mins

Activity Narrative

The purpose of this activity is to remind students of strategies for representing and comparing ratios. The taste of the mixture depends on the ratio between the amount of water and the amount of drink mix used to make the mixture. As students describe how they know which mixture would taste different, they attend to precision (MP6).

Ideally, students come into the class knowing how to draw and use diagrams or tables of equivalent ratios to analyze contexts like the one in the task. If the diagnostic assessment suggests that some students can and some students can't, make strategic pairings of students for this task.

Monitor for students who:

- Create discrete diagrams, double number line diagrams, or tables to represent the different mixtures.
- Find ratios that are equivalent to the amounts given to help compare the recipes.
- Calculate unit rates to help compare the recipes.

Access for English Language Learners

This activity uses the *Collect and Display* math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.

Standards

Building On	**6.RP.A**
Building Towards	**7.RP.A**

Instructional Routines

- MLR2: Collect and Display

Launch

First, show students the three drink mixtures. Some options for how to accomplish this include:

- Demonstrate making the three drink mixtures yourself.
- Show the video:
  Video 'Three Mixtures' available here: https://vimeo.com/827656129/f1a4117d43.
- Display the unlabeled image of the drinks:

Tell students that two of these mixtures taste the same, and one tastes different. Ask students which mixture would taste different and why. Give students 1 minute of quiet think time and then time to share their thinking with their partner.

If possible, ask one student volunteer to take a small taste of each drink mixture and then describe to the rest of the class how the flavors compare.

Ask students:

- “What does it mean to say that the drink tastes stronger?” (It has more drink mix for the same amount of water, or it has less water for the same amount of drink mix.)
- “The two glasses that taste the same have different amounts of powdered drink mix in them. Why don’t the mixtures taste different? (The amount of water and amount of drink mix were both scaled by the same factor. The ratios of drink mix to water are equivalent.)

Give students 4–5 minutes of quiet work time to answer the questions. If students finish quickly, consider asking them to find the amount of drink mix per cup of water in each recipe, thus emphasizing the unit rate.

Use *Collect and Display* to create a shared reference that captures students’ developing mathematical language. Collect the language students use to describe how the amount of water and the amount of drink mix affects the taste of the mixture. Display words and phrases such as: “more drink mix,” “more water,” “tastes stronger,” “tastes weaker,” “ratio,” “unit rate,” “per,” etc.

Student Task Statement

Your teacher will show you three mixtures. Two taste the same, and one is different.

1. Which mixture would taste different? Why?

2. Here are the recipes that were used to make the three mixtures:

1 cup of water with $\frac{1}{4}$ teaspoon of powdered drink mix

1 cup of water with $1\frac{1}{2}$ teaspoons of powdered drink mix

2 cups of water with $\frac{1}{2}$ teaspoon of powdered drink mix

Which of these recipes is for the stronger tasting mixture? Explain how you know.

Student Response

1. The first mixture would taste different—stronger than the other two. It looks darker, because it has more drink mix in it.
2. The recipe with $1\frac{1}{2}$ teaspoons of drink mix and 1 cup of water is the strongest one. There is more drink mix per cup of water.

Are You Ready for More?

Salt and sugar give two distinctly different tastes, one salty and the other sweet. In a mixture of salt and sugar, it is possible for the mixture to be salty, sweet or both. Will any of these mixtures taste exactly the same?

- Mixture A: 2 cups water, 4 teaspoons salt, 0.25 cup sugar
- Mixture B: 1.5 cups water, 3 teaspoons salt, 0.2 cup sugar
- Mixture C: 1 cup water, 2 teaspoons salt, 0.125 cup sugar

Extension Student Response

Mixture A and Mixture C will taste exactly the same. Mixture B will taste equally salty, but will be a little bit sweeter.

Activity Synthesis

The key takeaway from this activity is that the flavor depends on the ratio of the amounts of drink mix and water in the mixture. For a given amount of water, the more drink mix you add, the stronger the mixture tastes. Likewise, for a given amount of drink mix, the more water you add, the weaker the mixture tastes.

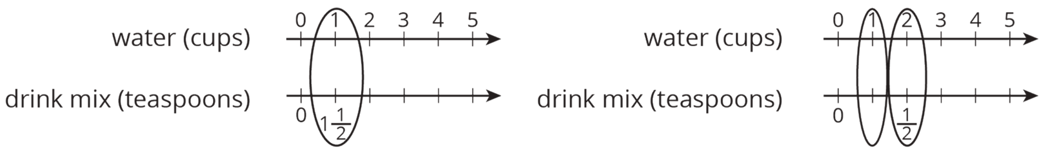
When comparing mixtures where both the amounts of drink mix and the amounts of water are different, we can find equivalent ratios for one or more of the mixtures. This method enables us to compare the amounts of drink mix for the same amount of water or to compare the amounts of water for the same amount of drink mix. Computing a unit rate for each situation is a particular instance of this strategy.

Direct students' attention to the reference created using *Collect and Display*. Ask students to share how they knew which recipe was strongest. Invite students to borrow language from the display as needed and update the reference to include additional phrases as they respond.

If students do not create diagrams, consider showing how the reasoning they describe could be represented visually. However, it is not necessary to show every type of diagram.

Discrete diagrams:

Double number line diagrams:



Tables:

water (cups)	drink mix (teaspoons)
1	$1\frac{1}{2}$
2	3

water (cups)	drink mix (teaspoons)
2	$\frac{1}{2}$
1	$\frac{1}{4}$

Identify correspondences between the recipes and various diagrams. For example, ask questions like "On the double number line diagram we see the 1 to $1\frac{1}{2}$ relationship at the first tick mark. Where do we see that relationship in the discrete diagrams? Where do we see it in the tables?"

Access for Students with Disabilities

Representation: Internalize Comprehension. Use color coding and annotations to highlight connections between representations in a problem. For example, use the same color to illustrate correspondences between the number line diagrams and ratio tables for each mixture.
Supports accessibility for: Visual-Spatial Processing

1.3 Crescent Moons

15 mins

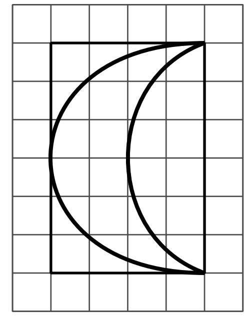
Activity Narrative

There is a digital version of this activity.

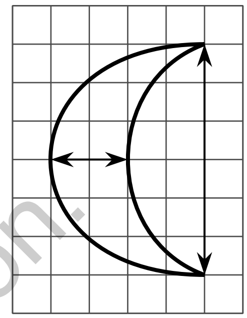
The purpose of this activity is to build on students' recent study of scale drawings to informally introduce the concept of proportional relationships.

Initially, students may describe the difference between Moons A, B, C, and D in qualitative terms, such as "D is more squished than the others." They may also describe Moons A, B, and C as "scaled copies," even though the curved sides make it difficult to compare corresponding side lengths and angle measurements. The activity prompts students to articulate what they mean in quantitative terms. One possibility would be talking about the height relative to the width (after defining "height" and "width" of a moon in some appropriate way).

For example, students can note that the height of the enclosing rectangle is always $1\frac{1}{2}$ times its width for Moons A, B, and C, but not for D.



Alternately, they might note that the distance tip to tip is 3 times the width of the widest part of the moon for Moons A, B, and C, but not for D.



Monitor for different ways students choose to measure the figures. Given the structure of scaled copies, as students work to describe quantitatively what Moons A, B, and C have in common, they are modeling with mathematics (MP4).

In the digital version of the activity, students use an applet to compare corresponding parts in multiple images. The applet allows students to add points and measure distances. The digital version may help students measure quickly and accurately so they can focus more on the mathematical analysis.

Standards

Building On	**6.RP.A**
Addressing	**7.G.A.1**
Building Towards	**7.RP.A**

Instructional Routines

- MLR8: Discussion Supports

Launch

Arrange students in groups of 2. Give students 3 minutes of quiet work time followed by 3 minutes of partner discussion.

Based on student conversations, you may want to have a whole-class discussion to ensure that they see a way to measure lengths associated with the moons. Consider asking questions like:

- "What does it mean to be 'smashed down'? What measurements might you make to show that this is true?" "Is there anything else that A, B, and C have in common that you can identify?"
- "What things might we measure about these moons to be able to talk about what makes them different in a more precise way?"

Then ask students to complete the last question with their partner.

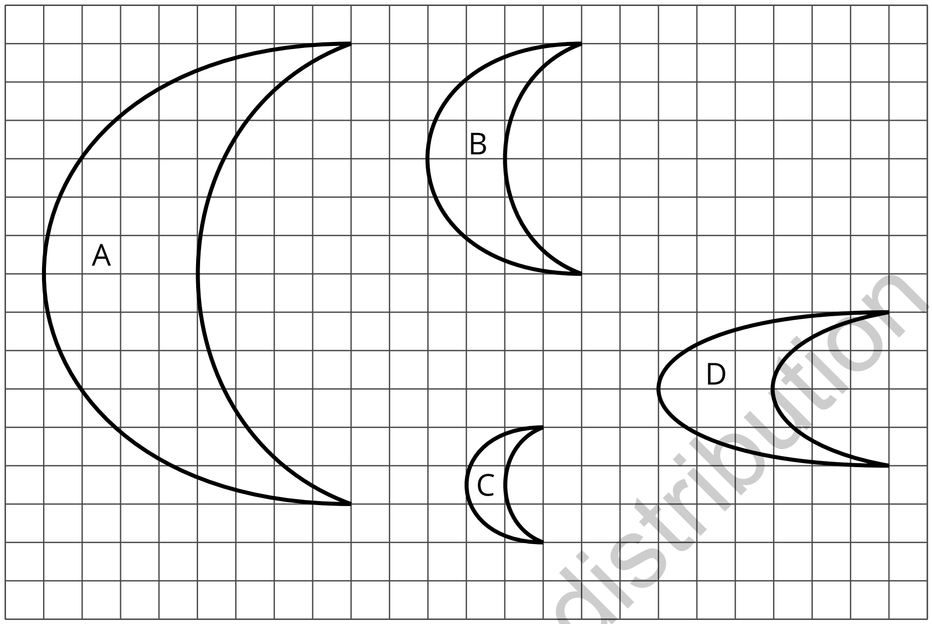
Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Chunk this task into more manageable parts. Consider having students begin by comparing just two crescent moon shapes, like A and B. Check in with students to provide

feedback and encouragement after each chunk. Look for students who create a rectangle around each moon and compare the width-height ratios.
Supports accessibility for: Attention, Social-Emotional Functioning

Student Task Statement

Here are four different crescent moon shapes.

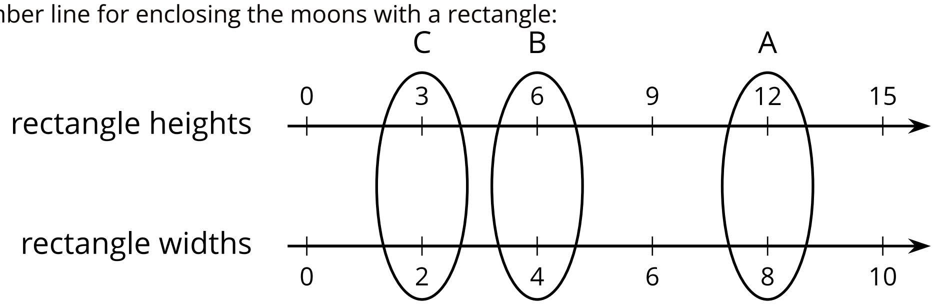


1. What do Moons A, B, and C all have in common that Moon D doesn't?
2. Use numbers to describe how Moons A, B, and C are different from Moon D.

   Pause here so your teacher can review your work.
3. Use a table or a double number line to show how Moons A, B, and C are different from Moon D.

Student Response

1. Sample response: Moon D is smashed down more than Moons A, B, and C. Moons A, B, and C are all taller than they are wide while Moon D is wider than it is tall.
2. Sample responses:
   - We could enclose each moon with a rectangle and compare the ratio of height to width for each moon.
   - We could measure the height of the moon from tip to tip and measure the width of the widest part.
3. Sample responses:
   - A double number line for enclosing the moons with a rectangle:



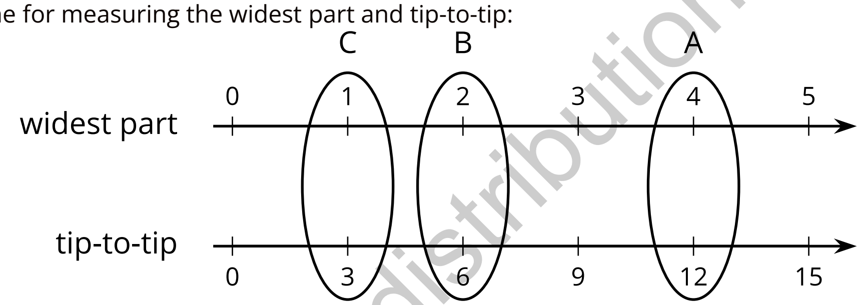
Moon D doesn't fit on this double number line, because $4 : 6$ is not equivalent to these ratios. If we add a tick mark at 4 on the top line, this does not line up with 6 on the bottom line.

- A table for enclosing the moons with a rectangle:

moon	height (units)	width (units)	height $\div$ width
A	12	8	$12 \div 8 = 1.5$
B	6	4	$6 \div 4 = 1.5$
C	3	2	$3 \div 2 = 1.5$
D	4	6	$4 \div 6 = 0.\overline{6}$

For Moon D, the height divided by the width does not equal 1.5, like it does for the other 3 moons.

- A double number line for measuring the widest part and tip-to-tip:



Moon D doesn't fit on this double number line, because $3 : 4$ is not equivalent to these ratios. The tick mark at 3 on the top line does not line up with 4 on the bottom line.

- A table for measuring the widest part and tip-to-tip:

moon	widest part (units)	tip-to-tip (units)	height $\div$ width
A	4	12	$4 \div 12 = \frac{1}{3}$
B	2	6	$2 \div 6 = \frac{1}{3}$
C	1	3	$1 \div 3 = \frac{1}{3}$
D	3	4	$3 \div 4 = \frac{3}{4}$

For Moon D, the width divided by the height does not equal $\frac{1}{3}$, like it does for the other 3 moons.

Building on Student Thinking

For question 2, students might attempt to find the area of each moon by counting individual square units. Suggest that they create a rectangle around each moon instead and compare the width-height ratios.

For question 3, if students are not sure how to set up these representations, providing a template may be helpful.

Activity Synthesis

The goal of this discussion is to show various representations of quantities that are in a proportional relationship for Moons A, B, and C but not for Moon D. However, the term "proportional relationship" is not introduced at this time.

Invite groups to share their representations and reasoning. To involve more students in the conversation, consider asking:

- "Who can restate ________'s reasoning in a different way?"
- "Did anyone use the same strategy but would explain it differently?"
- "Did anyone solve the problem in a different way?"
- "Does anyone want to add on to ________'s strategy?"
- "Do you agree or disagree? Why?"
- "What connections to previous problems do you see?"

Access for English Language Learners

MLR8 Discussion Supports. Display sentence frames to support whole-class discussion: "All _____ have _____ except _____." and "What makes _____ different from the others is _____."
Advances: Speaking, Representing

Lesson Synthesis

Share with students, "Today we examined situations involving ratios. In each case, the ratios of the quantities were equivalent for all but one of the things, and the other thing was different in an important way."

To emphasize ratio and rate language, consider asking students:

- "In what important way were the drink mixtures the same or different?" (the amount of drink powder per cup of water)
- "How could we tell using ratios that these were the same and different?" (The mixtures that tasted the same were described by equivalent ratios, $1 : \frac{1}{4}$ and $2 : \frac{1}{2}$. The ratio for the mixture that tasted different was not equivalent.)
- "In what important way were the moons the same and different?" (the relationship between the width and the height)
- "How could we tell using numbers that these were the same and different?" (The moons that looked the same had a height to width ratio of $3 : 1$. The moon that was different had a ratio of $4 : 3$.)

1.4

Orangey-Pineapple Juice

Cool-down

5 mins

Standards

Building On	**6.RP.A**
Building Towards	**7.RP.A**

Launch

Math Community

Before distributing the *Cool-downs*, display the Math Community Chart and the community building question "Which 'Doing Math' action is most important to you, and why?" Ask students to respond to the question after completing the *Cool-down* on the same sheet.

After collecting the *Cool-downs*, review student responses to the community building question. Use the responses to draft a student norm and a teacher norm to use as an example in Exercise 6. For example, if "sharing ideas" is a common choice for students, a possible norm is "We listen as others share their ideas."

For the teacher norms section, if "questioning vs. telling" from the "Doing Math" section is key for your teaching practice, then one way to express that as a norm is "Ask questions first to make sure I understand how someone is thinking."

Student Task Statement

Here are three different recipes for Orangey-Pineapple Juice. Two of these mixtures taste the same and one tastes different.

- Recipe 1: Mix 4 cups of orange juice with 6 cups of pineapple juice.
- Recipe 2: Mix 6 cups of orange juice with 9 cups of pineapple juice.
- Recipe 3: Mix 9 cups of orange juice with 12 cups of pineapple juice.

Which two recipes will taste the same, and which one will taste different? Explain or show your reasoning.

Student Response

Recipes 1 and 2 will taste the same. Sample reasoning: Recipe 3 is different because it requires $1\frac{1}{3}$ cups of pineapple juice for every 1 cup of orange juice. Recipes 1 and 2 both require $1\frac{1}{2}$ cups of pineapple juice for every 1 cup of orange juice.

recipe 1

orange juice (cups)	pineapple juice (cups)
4	6
2	3
1	$1\frac{1}{2}$

recipe 2

orange juice (cups)	pineapple juice (cups)
6	9
2	3
1	$1\frac{1}{2}$

recipe 3

orange juice (cups)	pineapple juice (cups)
9	12
3	4
1	$1\frac{1}{3}$

Double number line diagrams can be used to compare the recipes, for instance, by noting that for Recipes 1 and 2, you use 2 cups of orange juice for every 3 cups of pineapple juice, whereas with Recipe 3, you use $2\frac{1}{4}$ cups of orange juice for 3 cups of pineapple juice.

Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Lesson 1 Summary

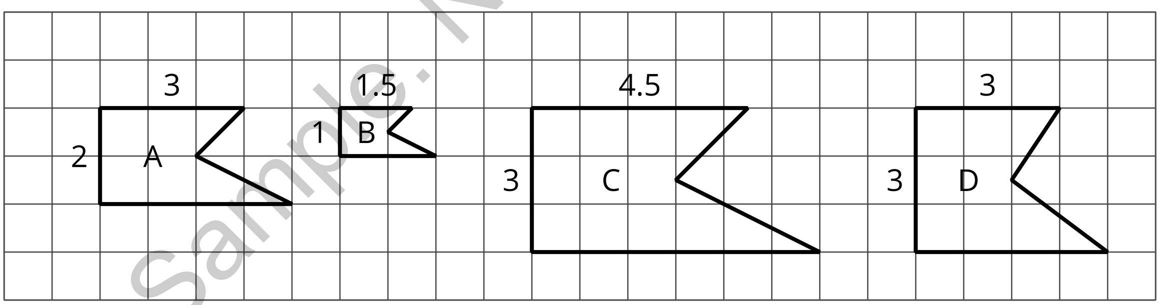
When two different situations can be described by **equivalent ratios**, that means they are alike in some important way.

An example is a recipe. If two people make something to eat or drink, the taste will only be the same as long as the ratios of the ingredients are equivalent. For example, all of the mixtures of water and drink mix in this table taste the same, because the ratios of cups of water to scoops of drink mix are all equivalent ratios.

water (cups)	drink mix (scoops)
3	1
12	4
1.5	0.5

If a mixture were not equivalent to these, for example, if the ratio of cups of water to scoops of drink mix were $6 : 4$, then the mixture would taste different.

Notice that the ratios of pairs of corresponding side lengths are equivalent in Figures A, B, and C. For example, the ratios of the length of the top side to the length of the left side for Figures A, B, and C are equivalent ratios. Figures A, B, and C are *scaled copies* of each other.
This is the important way in which they are alike.



If a figure has corresponding sides that are not in a ratio equivalent to these, like Figure D, then it's not a scaled copy. In this unit, you will study relationships like these that can be described by a set of equivalent ratios.

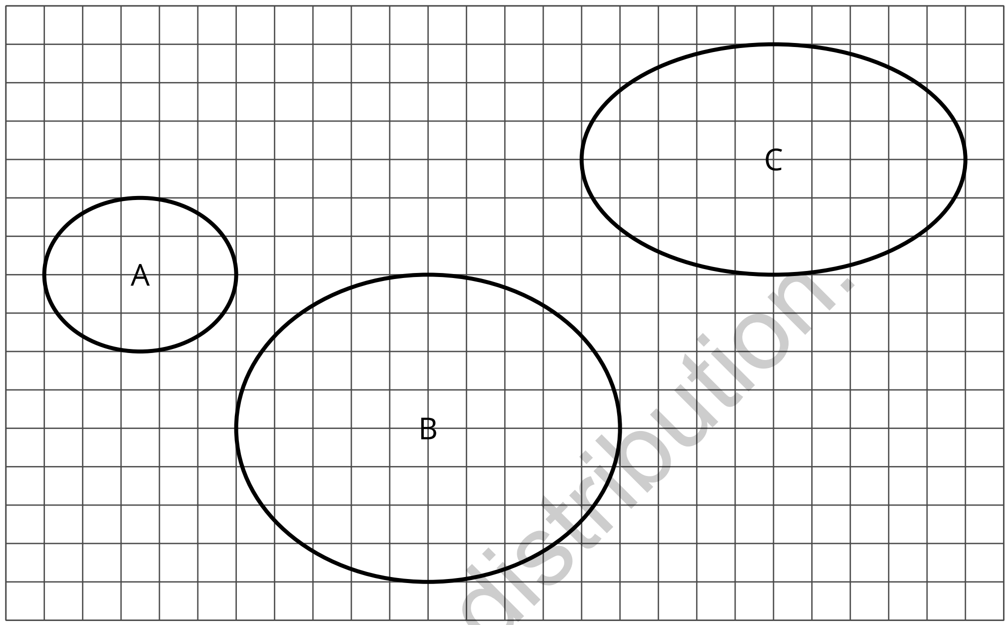
Glossary

- equivalent ratios

Practice Problems

1 Student Task Statement

Which one of these shapes is not like the others? Explain what makes it different by representing each width and height pair with a ratio.



Solution

C is different from A and B. For both A and B, the width:height ratio is 5:4. However, for C, the width is 10 units and the height is 6 units, so the width:height ratio is 5:3.

2 Student Task Statement

In one version of a trail mix, there are 3 cups of peanuts mixed with 2 cups of raisins. In another version of trail mix, there are 4.5 cups of peanuts mixed with 3 cups of raisins. Are the ratios equivalent for the two mixes? Explain your reasoning.

Solution

Yes, since 3 times 1.5 is 4.5 and 2 times 1.5 is 3

3 from Unit 1, Lesson 12

Student Task Statement

For each object, choose an appropriate scale for a drawing that fits on a regular sheet of paper. Not all of the scales on the list will be used.

Objects

a. A person
b. A football field (120 yd by $53\frac{1}{3}$ yd)
c. The state of Washington (about 240 mi by 360 mi)
d. The floor plan of a house
e. A rectangular farm (6 mi by 2 mi)

Scales

- 1 in : 1 ft
- 1 cm : 1 m
- 1 : 1,000
- 1 ft : 1 mile
- 1 : 100,000
- 1 mm : 1 km
- 1 : 10,000,000

Solution

Sample responses:

a. 1 in : 1 ft
b. 1 : 1,000
c. 1 : 10,000,000
d. 1 cm : 1 m
e. 1 : 100,000

4 from Unit 1, Lesson 11

Student Task Statement

Which scale is equivalent to 1 cm to 1 km?

A. 1 to 1000

B. 10,000 to 1

C. 1 to 100,000

D. 100,000 to 1

E. 1 to 1,000,000

Solution

C

5 from an earlier course

Student Task Statement

a. Find 3 different ratios that are equivalent to $7 : 3$.

b. Explain why these ratios are equivalent.

Solution

Sample response:

a. $14 : 6, 21 : 9, 28 : 12$

b. 7 and 3 are each multiplied by 2, 3, and 4, respectively.

Unit 2, Lesson 2

Introducing Proportional Relationships with Tables

Goals

- Comprehend that the phrase “proportional relationship” (in spoken and written language) refers to when two quantities are related by multiplying by a “constant of proportionality.”
- Describe (orally and in writing) relationships between rows or between columns in a table that represents a proportional relationship.
- Explain (orally) how to calculate missing values in a table that represents a proportional relationship.

Learning Targets

- I can use a table to reason about two quantities that are in a proportional relationship.
- I understand the terms proportional relationship and constant of proportionality.

Lesson Narrative

The purpose of this lesson is to introduce the concept of a **proportional relationship** by looking at tables of equivalent ratios. Students learn that all entries in one column of the table can be obtained by multiplying entries in the other column by the same number. This number is called the **constant of proportionality**. The activities use contexts that make using the constant of proportionality the more convenient approach, rather than reasoning about equivalent ratios.

The activities encourage students to look for and make use of structure in a proportional relationship (MP7). The last activity is optional because it provides an opportunity for additional practice with a new context.

In these materials, when we say “b is proportional to a” we usually put a in the left hand column and b in the right hand column, so that multiplication by the constant of proportionality always goes from left to right. This is not a hard and fast rule, but it prepares students for later work on functions, where they will think of x as the independent variable and y as the dependent variable.

Standards

Building On	**6.RP.A.3**
Addressing	**7.RP.A.2, 7.RP.A.2.b**
Building Towards	**7.RP.A.2.b**

Instructional Routines

- 5 Practices
- MLR6: Three Reads
- MLR8: Discussion Supports
- Notice and Wonder

Required Materials

Materials To Gather

- Measuring cup: Activity 3
- Measuring spoons: Activity 3

Student Facing Learning Goals

Let’s solve problems involving proportional relationships using tables.

2.1 Notice and Wonder: Paper Towels by the Case

5 mins

Warm-up

Activity Narrative

The purpose of this *Warm-up* is to elicit the idea of using a table to see patterns between related quantities, which will be useful when students discuss how to use tables to analyze proportional relationships in a later activity. While students may notice and wonder many things about these images, the relationships between the rows and the relationship between the columns are the important discussion points.

This prompt gives students opportunities to see and make use of structure (MP7). The specific structures they might notice are:

- Scale factors between any pair of rows (In other words, multiplying both values in one row by the same number gives the values in another row.)
- A unit rate between the two columns (For example, multiplying any value in the first column by 12 gives the corresponding number in the second column.)

Standards

Building Towards **7.RP.A.2.b**

Instructional Routines

- Notice and Wonder

Launch

Arrange students in groups of 2. Display the table for all to see and read the problem stem aloud. Ask students to think of at least one thing they notice and at least one thing they wonder. Give students 1 minute of quiet think time, and then 1 minute to discuss the things they notice and wonder with their partner.

Student Task Statement

Here is a table that shows how many rolls of paper towels a store receives when they order different numbers of cases.

	number of cases they order	number of rolls of paper towels	
	1	12	
	3	36	
	5	60	
$\cdot 2$	10	120	$\cdot 2$

What do you notice? What do you wonder?

Student Response

Things students may notice:

- To go from one row to another, multiply both columns by the same number.
- To find the number of rolls, multiply the number of cases by 12.
- There are 12 rolls in a case.

Things students may wonder:

- How much does a case cost?
- How many paper towels are on a roll?
- Why would you need 120 rolls of paper towels?

Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses for all to see, without editing or commentary. If possible, record the relevant reasoning on or near the image. Next, ask students, “Is there anything on this list that you are wondering about now?” Encourage students to respectfully disagree, ask for clarification, or point out contradicting information.

If the relationship between the number of cases and the number of paper towels does not come up during the conversation, ask students to discuss this idea.

15 mins

2.2 Feeding a Crowd

Activity Narrative

The purpose of this task is to introduce students to the idea of a proportional relationship. Students examine two different tables of equivalent ratios and use them to solve problems. In the *Activity Synthesis* they learn that there is a proportional relationship between two quantities when the quantities are characterized by a set of equivalent ratios.

The contexts and numbers used in this activity are intended to be accessible to all students. This way they can focus on its mathematical structure (MP7) and the new terms introduced in the lesson without being distracted.

Monitor for groups who use these different strategies to complete the first table:

- Create a drawing or diagram that depicts 15 cups of rice and 45 people, organized into 3 people per cup
- Use a scale factor to scale from one row of the table to another (For example, since $2 \cdot 5 = 10$, then we can do $6 \cdot 5$ to get 30.)
- Calculate and use a unit rate (In other words, one cup of rice serves 3 people, so 10 cups of rice must serve 30 people.)
- Apply the constant relationship between the columns of the table (In other words, multiply the number in the left column by 3 to get the number in the right column.)

Plan for students to present in this order, from concrete to more abstract.

Standards

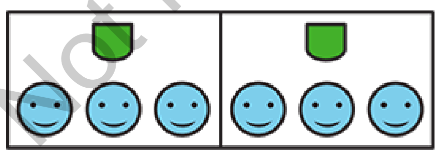
Addressing **7.RP.A.2**

Instructional Routines

- 5 Practices

Launch

Explain that this task looks at two food items: rice and spring rolls. Say, "Rice is a big part of the traditions and cultures of many families. Does your family cook rice, and if so, how?" Invite a student to describe the process (measure rice, measure water, simmer for a while). You use more rice for more people and less rice for fewer people. If students have trouble understanding or representing the context, show them the measuring cup so that they have a sense of its size, or draw a literal diagram that looks something like this:



Similarly, ask students if they have ever eaten a spring roll and invite them to describe what they are. While some spring rolls can be very large, the ones referred to in this activity are smaller.

Keep students in the same groups. Give students 5–6 minutes of partner work time. Select students with different strategies for completing the first table (such as those described in the *Activity Narrative*) to share later.

Access for Students with Disabilities

Engagement: Provide Access by Recruiting Interest. Invite students to share their experience of making rice at home. How does their family cook rice? Based on their experience, about many people does one cup of dry rice feed?
Supports accessibility for: Conceptual Processing, Memory

Student Task Statement

1. A recipe says that 2 cups of dry rice will serve 6 people. Complete the table as you answer the questions. Be prepared to explain your reasoning.

a. How many people will 10 cups of rice serve?

b. How many cups of rice are needed to serve 45 people?

cups of rice	number of people
2	6
3	9
10	
	45

2. A recipe says that 6 spring rolls will serve 3 people. Complete the table.

number of spring rolls	number of people
6	3
30	
40	
	28

Student Response

1. a. 30

   b. 15

cups of rice	number of people
2	6
3	9
10	30
15	45

2.

number of spring rolls	number of people
6	3
30	15
40	20
56	28

Activity Synthesis

The purpose of this discussion is to introduce the concept of a proportional relationship. Ask previously selected groups to share their reasoning for completing the first table. Sequence the discussion of the strategies in the order listed in the

Activity Narrative. If possible, record and display their work for all to see.
Connect the different responses to the learning goals by asking questions such as:

- "Why do the different approaches lead to the same outcome?"
- "What do you notice about the columns in the completed table?"
- "How does the unit rate (people per cup of rice) show up in each method?"

After students have shared their reasoning, introduce the term **proportional relationship**. For example, say, "Whenever we have a situation like this where two quantities are always in the same ratio, we say there is a proportional relationship between the quantities." Display these statements for all to see:

- "The relationship between the number of cups of rice and the number of people is a proportional relationship."
- "The number of people is proportional to the number of cups of rice."
- "There are 3 people for every 1 cup of rice."

Next, ask students to work with their partner to write a sentence that describes the proportional relationship in the second table. Then, invite students to share their sentences. Record and display their sentences for all to see.
Sample responses:

- The relationship between the number of spring rolls and the number of people is a proportional relationship.
- The number of people is proportional to the number of spring rolls.
- There are 2 spring rolls for each person.
- There are 0.5 people per spring roll. (The unit rate 0.5 people per spring roll may sound strange, but it means that 1 spring roll only halfway satisfies a person.)

2.3 Making Coco Bread

10 mins

Activity Narrative

The purpose of this activity is to introduce the term constant of proportionality. Students continue to find missing values in a table representing a proportional relationship. However, the numbers are purposefully chosen so that it is difficult to identify scale factors that scale one row to another. This encourages students to look for a unit rate relationship between the columns.

This is the first time Math Language Routine 6: Three Reads is suggested in this course. In this routine, students are supported in reading a mathematical text, situation, or word problem three times, each with a particular focus. During the first read, students focus on comprehending the situation. During the second read, students identify quantities. During the third read, the final prompt is revealed and students brainstorm possible starting points for answering the question. The intended question is withheld until the third read so students can make sense of the whole context before rushing down a solution path. The purpose of this routine is to support students' reading comprehension as they make sense of mathematical situations and information through conversation with a partner.

Access for English Language Learners

This activity uses the *Three Reads* math language routine to advance reading and representing as students make sense of what is happening in the text.

Standards

Addressing **7.RP.A.2**

Instructional Routines

- MLR6: Three Reads

Launch

Keep students in the same groups. Use *Three Reads* to support reading comprehension and sense-making about this problem. Display only the problem stem and without revealing the questions.

- In the first read, students read the problem with the goal of comprehending the situation.

  For the first read, read the problem aloud while everyone else reads along, and then ask, "What is this situation about? What is going on here?" Allow 1 minute to discuss with a partner and then share with the whole class. A typical response may be, "A bakery uses a recipe to make bread. The recipe includes coconut milk and flour." Listen for and clarify any questions about the context.

- In the second read, students analyze the mathematical structure of the story by naming quantities.

  Invite students to read the problem aloud with their partner, or select a student to read to the class, then prompt students by asking, "What can be counted or measured in this situation?" Give students 30 seconds of quiet think time, followed by another 30 seconds to share with their partner. A typical response may be: "milliliters of coconut milk; cups of flour; size of the batches."

- In the third read, students brainstorm possible starting points for answering the questions.

  Invite students to read the problem aloud with their partner, or select a different student to read to the class. After the third read, reveal the first question on finding the amount of flour to mix with 1,000 milliliters of coconut milk and ask, "What are some ways we might get started on this?" Instruct students to think of ways to approach the questions without actually solving. Give students 1 minute of quiet think time followed by another minute to discuss with their partner. Invite students to name some possible strategies referencing quantities from the second read.

  Provide these sentence frames as partners discuss: "To figure out how much flour is needed for a given amount of coconut milk . . ." "One way a diagram could help is . . ." "To calculate the unit rate we can . . . ."

  As partners are discussing their solution strategies, select 1–2 students to share their ideas with the whole class. As students are presenting their strategies to the whole class, create a display that summarizes starting points for each question. (Stop students as needed before they share complete solutions or answers.)

Give students time to complete the rest of the activity followed by a whole-class discussion.

Student Task Statement

Coco bread is a popular food in Jamaica. To make coco bread, a bakery uses 200 milliliters of coconut milk for every 360 grams of flour. Some days they bake bigger batches and some days they bake smaller batches, but they always use the same ratio of coconut milk to flour.

Complete the table as you answer the questions. Be prepared to explain your reasoning.

1. How many grams of flour do they use with 500 milliliters of coconut milk?
2. How many grams of flour do they use with 235 milliliters of coconut milk?
3. How many milliliters of coconut milk do they use with 450 grams of flour?
4. What is the proportional relationship represented by this table?

coconut milk (millimeters)	flour (grams)
200	360
500	
375	
	450

Student Response

1. 900 grams of flour
2. 675 grams of flour
3. 250 milliliters of coconut milk
4. Sample responses:
   a. The relationship between the milliliters of coconut milk and the grams of flour is proportional.
   b. The relationship between the amount of coconut milk and the amount of flour is a proportional relationship.
   c. The table represents a proportional relationship between the amount of coconut milk and the amount of flour.
   d. The amount of flour is proportional to the amount of coconut milk.
   e. The bakery uses 1.8 grams of flour for every 1 milliliter of coconut milk.

Activity Synthesis

The key takeaway from this discussion is that the constant of proportionality for this proportional relationship is 1.8.

Invite students to share their strategies for completing the table, especially the row with 375 milliliters of coconut milk. Highlight strategies that make use of the relationship between the columns of the table, that is, any value in the first column can be multiplied by 1.8 to get the corresponding value in the second column. Tell students that 1.8 is the constant of proportionality for this proportional relationship. Note that, like unit rate, the constant of proportionality can always be determined by finding how much of the second quantity there is per one of the first quantity.

Ask students to interpret the constant of proportionality in the context: "What does the 1.8 tell us about the situation?" (There are 1.8 grams of flour per milliliter of coconut milk.)

If time permits, consider inviting students to share their strategies for the question about the amount of coconut milk to use with 450 grams of flour. Possible strategies include:

- Multiplying the values in the second row of the table by the scale factor $\frac{1}{2}$
- Dividing the given amount of flour by the constant of proportionality, 1.8

Highlight the structure that dividing any value in the second column by the constant of proportionality gives the corresponding value in the first column.

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Encourage and support opportunities for peer collaboration. When students share their work with a partner, display sentence frames to support conversation such as: "First, I _____ because . . . ." "I noticed _____, so I . . . ." "Why did you . . . ?" or "I agree/disagree because . . . ."

Supports accessibility for: Language, Social-Emotional Functioning

2.4 Quarters and Dimes

Optional

10 mins

Activity Narrative

The purpose of this activity is to practice finding and using the constant of proportionality in another context.

Standards

Addressing **7.RP.A.2.b**

Instructional Routines

- MLR8: Discussion Supports

Launch

If needed, remind students of the value of a quarter and a dime before starting the activity.

Student Task Statement

4 quarters are equal in value to 10 dimes.

1. How many dimes equal the value of 6 quarters?
2. How many dimes equal the value of 14 quarters?
3. What value belongs next to the 1 in the table? What does it mean in this context?

number of quarters	number of dimes
1	
4	10
6	
14	

Student Response

1. 15
2. 35
3. 2.5, which means that 2.5 dimes are worth the same amount as 1 quarter

number of quarters	number of dimes
1	2.5
4	10
6	15
14	35

Are You Ready for More?

Pennies made before 1982 are 95% copper and weigh about 3.11 grams each. (Pennies made after that date are primarily made of zinc). Some people claim that the value of the copper in one of these pennies is greater than the face value of the penny. Find out how much copper is worth right now, and decide if this claim is true.

Extension Student Response

The cost of copper fluctuates, so the answer depends on the current value of copper. Assuming that copper costs about $5.00 per kg, the copper in a penny would be worth about $0.95 \cdot 0.00311 \cdot 5 \approx 0.01.5$ or about 1.5 cents. As long as copper is worth at least $3.39 per kilogram, then the copper in a pre-1982 penny will be worth more than 1 cent.

Activity Synthesis

Ask students for the value that belongs next to the 1 in the table. Invite several students to explain the significance of this number.

Access for English Language Learners

MLR8 Discussion Supports. Display sentence frames to support whole-class discussion: "_____ is equal in value to _____, because . . . ."
Advances: Speaking, Representing

Lesson Synthesis

Share with students, "Today we looked at examples of proportional relationships, which are situations that are characterized by equivalent ratios. We found unknown values in the table."

Briefly revisit the two activities, demonstrating the use of the new terms. It would be helpful to display a completed table for each activity to facilitate the conversation. Consider asking students:

- "In the first activity, what were the proportional relationships?" (The number of people served was proportional to the amount of rice or the number of spring rolls.)
- "What was the constant of proportionality in each situation?" (3 for the rice, and $\frac{1}{2}$ for the spring rolls)
- "In the second activity, what was the proportional relationship?" (The amount of flour was proportional to the amount of coconut milk.)
- "What was the constant of proportionality in that situation?" (1.8)

2.5 Green Paint

Cool-down

5 mins

Standards

Building On 6.RP.A.3

Addressing 7.RP.A.2

Student Task Statement

When you mix two colors of paint in equivalent ratios, the resulting color is always the same. Complete the table as you answer the questions.

1. How many cups of yellow paint should you mix with 1 cup of blue paint to make the same shade of green? Explain or show your reasoning.
2. Make up a new pair of numbers that would make the same shade of green. Explain how you know they would make the same shade of green.
3. What is the proportional relationship represented by this table?
4. What is the *constant of proportionality*? What does it represent?

cups of blue paint	cups of yellow paint
2	10
1	

Student Response

1. You need 5 cups of yellow paint for 1 cup of blue paint. You can see this by multiplying the first row by a factor of $\frac{1}{2}$. Alternatively, you have to multiply 2 by $\frac{10}{2} = 5$ to get 10. Multiplying 1 by 5 gives 5.
2. Any amounts equivalent to the ratio of 1 cup of blue paint to 5 cups of yellow paint. Sample response: 3 cups of blue paint mixed with 15 cups of yellow paint will also make the same shade of green. This can be obtained by multiplying the second row by a factor of 3 or choosing 3 for blue and then multiplying that by 5.
3. The relationship between the amount of blue paint and the amount of yellow paint is the proportional relationship represented by this table.
4. The constant of proportionality is 5. It represents the cups of yellow paint needed for 1 cup of blue paint.

Responding To Student Thinking

More Chances

Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Lesson 2 Summary

If the ratios between two corresponding quantities are always equivalent, the relationship between the quantities is called a **proportional relationship**.

This table shows different amounts of milk and chocolate syrup. The ingredients in each row, when mixed together, would make a different total amount of chocolate milk, but these mixtures would all taste the same.

Notice that each row in the table shows a ratio of tablespoons of chocolate syrup to cups of milk that is equivalent to $4 : 1$.

About the relationship between these quantities, we could say:

tablespoons of chocolate syrup	cups of milk
4	1
6	$1\frac{1}{2}$
8	2
$\frac{1}{2}$	$\frac{1}{8}$
12	3
1	$\frac{1}{4}$

- The relationship between the amount of chocolate syrup and the amount of milk is proportional.
- The table represents a proportional relationship between the amount of chocolate syrup and amount of milk.
- The amount of milk is proportional to the amount of chocolate syrup.

We could multiply any value in the chocolate syrup column by $\frac{1}{4}$ to get the value in the milk column. We might call $\frac{1}{4}$ a *unit rate*, because $\frac{1}{4}$ cup of milk is needed for 1 tablespoon of chocolate syrup. We also say that $\frac{1}{4}$ is the **constant of proportionality** for this relationship. It tells us how many cups of milk we would need to mix with 1 tablespoon of chocolate syrup.

Glossary

- constant of proportionality
- proportional relationship

Practice Problems

1 Student Task Statement

When Han makes chocolate milk, he mixes 2 cups of milk with 3 tablespoons of chocolate syrup. Here is a table that shows how to make batches of different sizes. Use the information in the table to complete the statements. Some terms are used more than once.

cups of milk	tablespoons of chocolate syrup
2	3
8	12
1	$\frac{3}{2}$
10	15

a. The table shows a proportional relationship between ____________ and ____________.

b. The scale factor shown is ____________.

c. The constant of proportionality for this relationship is____________.

d. The units for the constant of proportionality are ____________ per ____________.

Bank of Terms: tablespoons of chocolate syrup, 4, cups of milk, cup of milk, $\frac{3}{2}$

Solution

a. cups of milk; tablespoons of chocolate syrup

b. 4

c. $\frac{3}{2}$

d. tablespoons of chocolate syrup; cup of milk

2 Student Task Statement

A certain shade of pink is created by adding 3 cups of red paint to 7 cups of white paint.

a. How many cups of red paint should be added to 1 cup of white paint?

cups of white paint	cups of red paint
1	
7	3

b. What is the constant of proportionality?

Solution

a. $\frac{3}{7}$ cups of red paint

b. $\frac{3}{7}$

3 from Unit 1, Lesson 12

Student Task Statement

A map of a rectangular park has a length of 4 inches and a width of 6 inches. It uses a scale of 1 inch for every 30 miles.

a. What is the actual area of the park? Show how you know.

b. The map needs to be reproduced at a different scale so that it has an area of 6 square inches and can fit in a brochure. At what scale should the map be reproduced so that it fits on the brochure? Show your reasoning.

Solution

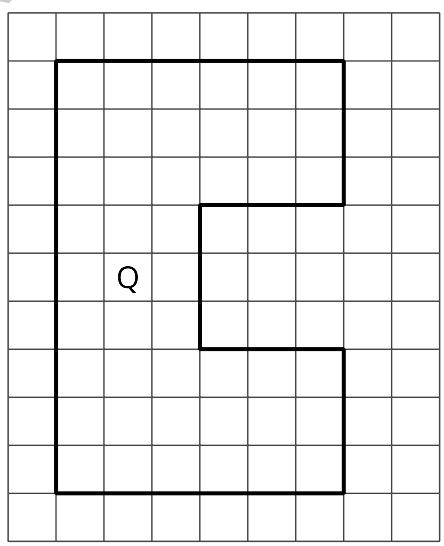
a. 21,600 square miles. Sample reasoning: The area on the map is 24 square inches. 1 square inch represents 900 square miles, since $30 \cdot 30 = 90$. The actual area is $24 \cdot 900$, which equals 21,600 square miles.

b. 1 inch to 60 miles. Sample reasonings:
- If 21,600 square miles need to be represented by 6 square inches, each square inches needs to represent 3,600 square miles: $21,600 \div 6 = 3,600$. This means each 1-inch side of the square needs to be 60 miles.
- The area of this new map is $\frac{1}{4}$ of the first map, since 6 is $\frac{1}{4}$ of 24. This means each 1 inch square in this map has to represent 4 times as much area as in the first map. $900 \cdot 4 = 3,600$. If each square inch represents 3,600 square miles, every 1 inch represents 60 miles.

4 from Unit 1, Lesson 6

Student Task Statement

Noah drew a scaled copy of Polygon P and labeled it Polygon Q.



If the area of Polygon P is 5 square units, what scale factor did Noah apply to Polygon P to create Polygon Q? Explain or show how you know.

Solution

The area of polygon Q is 45 square units, so the area has scaled by a factor of 9, since $5 \cdot 9 = 45$. Since the area of a scaled copy varies from the original area by the square of the scale factor, the scale factor is 3.

5 from an earlier course

Student Task Statement

Select **all** the ratios that are equivalent to each other.

A. $4 : 7$

B. $8 : 15$

C. $16 : 28$

D. $2 : 3$

E. $20 : 35$

Solution

A, C, E

Unit 2, Lesson 3

More about Constant of Proportionality

Goals

- Compare, contrast, and critique (orally and in writing) different ways to express the constant of proportionality for a relationship.
- Explain (orally) how to determine the constant of proportionality for a proportional relationship represented in a table.
- Interpret the constant of proportionality for a relationship in the context of constant speed.

Learning Targets

- I can find missing information in a proportional relationship using a table.
- I can find the constant of proportionality from information given in a table.

Lesson Narrative

In this lesson, students continue to work with proportional relationships represented by tables. They identify the constant of proportionality and use it to answer questions about the context. The contexts are familiar from previous grades: unit conversion and constant speed. When students recognize that the conversion factor or the speed are the constants of proportionality for the relationships, they are reasoning abstractly and quantitatively (MP2). Although students might continue to reason with equivalent ratios to solve problems, the contexts are designed so that it is more efficient to use the constant of proportionality.

This lesson also introduces students to the idea that there are two ways of viewing any proportional relationship. In other words, if y is proportional to x, then x is also proportional to y. The two constants of proportionality are **reciprocals**, $\frac{y}{x}$ and $\frac{x}{y}$, respectively. This idea will be developed more in future lessons.

Math Community
In today's activities, students are introduced to the idea of math norms as expectations that help everyone in the room feel safe, comfortable, and productive doing math together. Students then consider what norms would connect and support the math actions the class recorded so far in the Math Community Chart.

Standards

Building On	**5.MD.A.1**
Addressing	**7.RP.A.2.a, 7.RP.A.2.b**

Instructional Routines

- Math Talk
- MLR2: Collect and Display
- MLR5: Co-Craft Questions
- MLR8: Discussion Supports

Required Materials

Materials To Gather

- Chart paper: Activity 1
- Math Community Chart: Activity 1

Student Facing Learning Goals

Let's solve more problems involving proportional relationships using tables.

3.1 Math Talk: Division

mins

Warm-up

Activity Narrative

This *Math Talk* focuses on division that results in a decimal. It encourages students to think about how they can use the result of one division problem to find the answer to a similar problem with a different, but related, divisor or dividend. The understanding elicited here will be helpful later in the lesson when students calculate constants of proportionality.

To recognize how a divisor has been scaled and predict how the quotient will be affected, students need to look for and make use of structure (MP7).

Standards

Building On **5.MD.A.1**

Instructional Routines

- Math Talk
- MLR8: Discussion Supports

Launch

Tell students to close their books or devices (or to keep them closed). Reveal one problem at a time. For each problem:

- Give students quiet think time and ask them to give a signal when they have an answer and a strategy.
- Invite students to share their strategies and record and display their responses for all to see.
- Use the questions in the activity synthesis to involve more students in the conversation before moving to the next problem.

Keep all previous problems and work displayed throughout the talk.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. To support working memory, provide students with access to sticky notes or mini whiteboards.
Supports accessibility for: Memory, Organization

Student Task Statement

Find the value of each expression mentally.

- $645 \div 10$
- $645 \div 100$
- $645 \div 50$
- $64.5 \div 50$

Student Response

- 64.5. Sample reasoning: 64.5 is one-tenth of 645. This is seen from the location of the decimal point: it is between the 4 and 5 instead of after the 5.
- 6.45. Sample reasoning: Since the current divisor, 100, is ten times the previous divisor, 10, the current quotient will be one-tenth the previous quotient. 6.45 is one-tenth of 64.5. This is seen from the location of the decimal point: it is between the 6 and 4 instead of after the 4.
- 12.9. Sample reasoning: Since the current divisor, 50, is half the previous divisor, 100, the current quotient will be double the previous quotient. $6.45 \cdot 2 = 12.9$
- 1.29. Sample reasoning: Since the current dividend, 64.5, is one-tenth of the previous dividend, 645, the current quotient will be one-tenth of the previous quotient. $12.9 \div 10 = 1.29$

Activity Synthesis

To involve more students in the conversation, consider asking:

- "Who can restate ________'s reasoning in a different way?"
- "Did anyone use the same strategy but would explain it differently?"
- "Did anyone solve the problem in a different way?"
- "Does anyone want to add on to ________'s strategy?"
- "Do you agree or disagree? Why?"
- "What connections to previous problems do you see?"

The key takeaway to highlight is the effects of multiplying or dividing numbers by powers of 10.

Access for English Language Learners

MLR8 Discussion Supports. Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because . . . ." or "I noticed _____ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Advances: Speaking, Representing

Math Community

At the end of the *Warm-up*, display the Math Community Chart. Tell students that norms are expectations that help everyone in the room feel safe, comfortable, and productive doing math together. Using the Math Community Chart, offer an example of how the "Doing Math" actions can be used to create norms. For example, "In the last exercise, many of you said that our math community sounds like 'sharing ideas.' A norm that supports that is 'We listen as others share their ideas.' For a teacher norm, 'questioning vs telling' is very important to me, so a norm to support that is 'Ask questions first to make sure I understand how someone is thinking.'"

Invite students to reflect on both individual and group actions. Ask, "As we work together in our mathematical community, what norms, or expectations, should we keep in mind?" Give 1–2 minutes of quiet think time and then invite as many students as time allows to share either their own norm suggestion or to "+1" another student's suggestion. Record student thinking in the student and teacher "Norms" sections on the Math Community Chart.

Conclude the discussion by telling students that what they made today is only a first draft of math community norms and that they can suggest other additions during the *Cool-down*. Throughout the year, students will revise, add, or remove norms based on those that are and are not supporting the community.

3.2 Centimeters and Millimeters

10 mins

Activity Narrative

This activity has two purposes. First, it involves an important context that can be represented by proportional relationships, namely, measurement conversion. Second, it introduces the idea that there are two constants of proportionality and that they are reciprocals (also known as multiplicative inverses). Students start to use "is proportional to" language to distinguish between the two constants of proportionality:

- The length measured in millimeters is proportional to the length measured in centimeters, and the constant of proportionality is 10.
- The length measured in centimeters is proportional to the length measured in millimeters, and the constant of proportionality is $\frac{1}{10}$, or 0.1.

Access for English Language Learners

This activity uses the *Collect and Display* math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.

Standards

Addressing **7.RP.A.2.b**

Instructional Routines

- MLR2: Collect and Display

Launch

Use *Collect and Display* to direct attention to words collected and displayed from an earlier lesson. Collect the language students use to describe the proportional relationships. Display words and phrases such as "is proportional to," "constant of proportionality," "reciprocal," "inverse," etc.

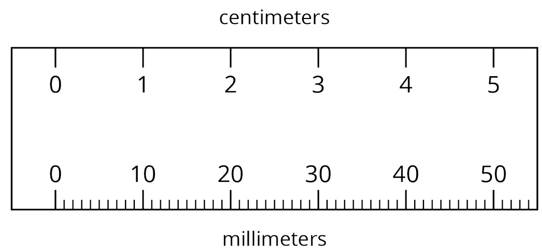
Give students quiet work time followed by partner and whole-class discussion.

Access for Students with Disabilities

Action and Expression: Provide Access for Physical Action. Activate or supply background knowledge. Provide a display showing equivalence between dividing by a number and multiplying by its reciprocal for students to use as a reference.
Supports accessibility for: Memory, Organization

Student Task Statement

There is a proportional relationship between any length measured in centimeters and the same length measured in millimeters.



There are two ways of thinking about this proportional relationship.

1. If the length of something in centimeters is known, its length in millimeters can be calculated.
   a. Complete the table.
   b. What is the constant of proportionality?

length (centimeters)	length (millimeters)
9	
12.5	
50	
88.49	

2. If the length of something in millimeters is known, its length in centimeters can be calculated.
   a. Complete the table.
   b. What is the constant of proportionality?

length (millimeters)	length (centimeters)
70	
245	
4	
699.1	

3. How are these two constants of proportionality related to each other?
4. Complete each sentence:
   a. To convert from centimeters to millimeters, the value in centimeters is multiplied by _______.
   b. To convert from millimeters to centimeters, the value in millimeters is divided by _______, or multiplied by _______.

Student Response

1. a.

length (centimeters)	length (millimeters)
9	90
12.5	125
50	500
88.49	884.9

   b. 10

2. a.

length (millimeters)	length (centimeters)
70	7
245	24.5
4	0.4
699.1	69.91

   b. $\frac{1}{10}$ or 0.1.

3. 10 and $\frac{1}{10}$ are reciprocals.

4. a. 10

   b. 10; $\frac{1}{10}$.

Building on Student Thinking

Some students may say that the constants of proportionality are both 10 since you can divide by 10 in the second table. Tell students, "The constant of proportionality is what you *multiply* by. Can you find a way to *multiply* the numbers in the first column to get the numbers in the second column?"

Are You Ready for More?

1. How many square millimeters are there in a square centimeter?
2. How do you convert square centimeters to square millimeters? How do you convert the other way?

Extension Student Response

1. 100
2. Multiply by 100; multiply by $\frac{1}{100}$

Activity Synthesis

The goal of this discussion is to help students recognize how the structure of proportional relationships applies to this situation involving measurement conversion.

Direct students' attention to the reference created using *Collect and Display*. Ask students to share what they noticed about the two constants of proportionality, borrowing language from the display as needed. Next, ask students to suggest ways to update the display: "Are there any new words or phrases that you would like to add?" "Is there any language you would like to revise or remove?"

The key takeaway is that the two constants of proportionality are reciprocals. If needed, remind students that dividing by 10 is the same as multiplying by its reciprocal, $\frac{1}{10}$. One way to explain why these two constants of proportionality are multiplicative inverses is to imagine starting with a measurement in centimeters, 15 centimeters for example. To convert 15 centimeters to millimeters, we multiply 15 by 10. So 15 centimeters = 150 millimeters. If we convert the measurement in millimeters back to centimeters, it will be 15, so the constant of proportionality we need to multiply by is $\frac{1}{10}$.

15 mins

3.3 Pittsburgh to Phoenix

Activity Narrative

This activity focuses on making connections between constant speed and proportional relationships, with special attention to the constant of proportionality. The numbers are chosen so that students are more likely to use unit rate rather than scale factors. The goal is for students to understand that when speed is constant, time elapsed and distance traveled are proportional. The constant of proportionality indicates the magnitude of the speed. For example, if distance is given in miles and time in hours, then the constant of proportionality indicates the speed in miles per hour. As students make these connections, they are reasoning abstractly and quantitatively (MP2).

Students might wonder why the route is not a straight line. If they ask about it, teachers can share some reasons why airplane routes are complex, for example the need to avoid congested areas and the fact that the shortest distances on the curved surface of Earth do not always correspond to lines on a map.

This is the first time *Math Language Routine 5: Co-Craft Questions* is suggested in this course. In this routine, students are given a context or situation, often in the form of a problem stem (for example, a story, image, video, or graph) with or without numerical values. Students develop mathematical questions that can be asked about the situation. A typical prompt is: "What mathematical questions could you ask about this situation?" The purpose of this routine is to allow students to make sense of a context before feeling pressure to produce answers, and to develop students' awareness of the language used in mathematics problems.

Access for English Language Learners

This activity uses the *Co-Craft Questions* math language routine to advance reading and writing as students make sense of a context and practice generating mathematical questions.

Standards

Addressing **7.RP.A.2.b**

Instructional Routines

- MLR5: Co-Craft Questions

Launch

Arrange students in groups of 2. Use *Co-Craft Questions* to give students an opportunity to familiarize themselves with the context, and to practice producing the language of mathematical questions.

- Display only the problem stem and related image, without revealing the questions.

  Ask students, “What mathematical questions could you ask about this situation?”

- Give students 1–2 minutes to write a list of mathematical questions that could be asked about the situation before comparing questions with a partner.

  As partners discuss, support students in using conversation and collaboration skills to generate and refine their questions, for instance, by revoicing a question, seeking clarity, or referring to their written notes.

  Listen for how students use language about speed, distance traveled, and elapsed time.

- Invite several students to share one question with the class and record for all to see. Ask the class to make comparisons among the shared questions and their own. Ask, “What do these questions have in common? How are they different?” Listen for and amplify questions that focus on the relationship between speed, distance traveled, and elapsed time.

- Reveal the table and questions and give students 1–2 minutes to compare it to their own question and those of their classmates. Invite students to identify similarities and differences by asking:
  - “Which of your questions is most similar to or different from the ones provided? Why?”
  - “Is there a main mathematical concept that is present in both your questions and those provided? If so, describe it.”

Ask students to complete the questions.

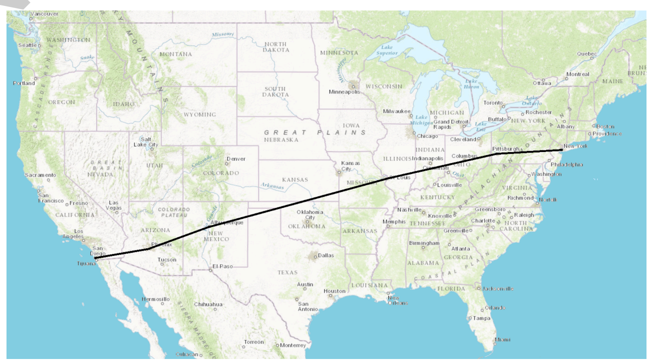
Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Provide tools to facilitate information processing or computation, enabling students to focus on key mathematical ideas. For example, allow students to use calculators to support their reasoning.
Supports accessibility for: Memory, Conceptual Processing

Student Task Statement

On its way from New York to San Diego, a plane flew over Pittsburgh, Saint Louis, Albuquerque, and Phoenix traveling at a constant speed.

Complete the table as you answer the questions. Be prepared to explain your reasoning.



segment	time	distance	speed
Pittsburgh to Saint Louis	1 hour	550 miles	
Saint Louis to Albuquerque	1 hour 42 minutes		
Albuquerque to Phoenix		330 miles	

1. What is the distance between Saint Louis and Albuquerque?
2. How many minutes did it take to fly between Albuquerque and Phoenix?
3. What is the proportional relationship represented by this table?
4. Diego says the constant of proportionality is 550. Andre says the constant of proportionality is $9\frac{1}{6}$. Do you agree with either of them? Explain your reasoning.

Student Response

segment	time	distance	speed
Pittsburgh to Saint Louis	1 hour	550 miles	550 miles per hour
Saint Louis to Albuquerque	1 hour 42 minutes	935 miles	550 miles per hour
Albuquerque to Phoenix	36 minutes	330 miles	550 miles per hour

1. 935 miles. 42 minutes is $\frac{42}{60}$ hours, or $\frac{7}{10}$ of an hour. $\frac{7}{10}$ of 550 miles is 385 miles, and $385 + 550 = 935$.
2. 36 minutes. 330 miles is $\frac{3}{5}$ of 550 miles, and $\frac{3}{5}$ of 60 minutes is 36 minutes.
3. The distance traveled is proportional to the elapsed time.
4. Diego uses miles per hour, and Andre uses miles per minute.

segment	time	distance	speed
Pittsburgh to Saint Louis	1 hour	550 miles	550 miles per hour
Saint Louis to Albuquerque	1.7 hours	935 miles	550 miles per hour
Albuquerque to Phoenix	0.6 hours	330 miles	550 miles per hour

segment	time	distance	speed
Pittsburgh to Saint Louis	60 minutes	550 miles	$9\frac{1}{6}$ miles per minute
Saint Louis to Albuquerque	102 minutes	935 miles	$9\frac{1}{6}$ miles per minute
Albuquerque to Phoenix	36 minutes	330 miles	$9\frac{1}{6}$ miles per minute

Building on Student Thinking

Students who need support in understanding the context can trace the segments on the map, labeling the distances they know and putting question marks for unknown distances. An empty double number line could also be a useful tool in helping students reason about the context.

Activity Synthesis

The goal of the discussion is for students to understand that when speed is constant, then distance traveled is proportional to elapsed time. The constant of proportionality is the speed. For example, if distance is given in miles and time in hours, then the constant of proportionality indicates the speed in miles per hour.

Begin by having a student share a table that is all in hours, miles, and miles per hour. Use this example to point out that every number in the first column can be multiplied by the speed to get the number in the second column. Ask:

- "Which quantities are in a proportional relationship? How do you know?"
- "What is the constant of proportionality in this case?"

If one or more students use minutes for time and miles per minute for speed in the table, the same questions can be asked for these units if time allows, but it is not required. In any case, summarize by making it explicit that when time and distance are in a proportional relationship, the constant of proportionality is the speed (or pace).

Lesson Synthesis

Share with students, "Today we looked at more tables of proportional relationships and found the constant of proportionality for each one."

Briefly revisit the two contexts, reinforcing the use of new terms. Consider asking students:

- "In the first activity, we examined the proportional relationship between millimeters and centimeters from two different perspectives and found two constants of proportionality. What were they?" (10 and $\frac{1}{10}$)
- "What is the relationship between the two constants of proportionality?" (They are reciprocals.)
- "In the second activity, we examined a proportional relationship between the time a plane flies and the distance it travels. What was the constant of proportionality?" (550 or $9\frac{1}{6}$)
- "What did the constant of proportionality represent in terms of the situation?" (the plane's speed)

3.4 Fish Tank

Cool-down

5 mins

Standards

Addressing **7.RP.A.2.a, 7.RP.A.2.b**

Launch

Math Community
Before distributing the *Cool-downs*, display the Math Community Chart and the norms question "Which norm has not already been listed that you'd like to add to our chart?" Ask students to respond to the question after completing the *Cool-down* on the same sheet.

After collecting the *Cool-downs*, identify themes from the norms question. Use that information to add to the initial draft of the "Norms" sections of the Math Community Chart.

Student Task Statement

Mai is filling her fish tank. Water flows into the tank at a constant rate. Complete the table as you answer the questions.

1. How many gallons of water will be in the fish tank after 3 minutes? Explain your reasoning.
2. How long will it take to fill the tank with 40 gallons of water? Explain your reasoning.
3. What is the constant of proportionality?

time (minutes)	water (gallons)
0.5	0.8
1	
3	
	40

Student Response

1. 4.8. If the first row is doubled (scale by 2), there are 1.6 gallons after 1 minute. If the second row is tripled (scale by 3), there are 4.8 gallons after 3 minutes. Or the first row could be scaled by 6 to get 4.8 gallons after 3 minutes.
2. 25 minutes. One way to find a scale factor to use is to divide 40 by 0.8. $\frac{40}{0.8} = 50$ and $50 \cdot 0.5 = 25$.
3. 1.6 (or equivalent). You can observe the amount of water that corresponds with 1 minute, or you can divide any value in the right column with its corresponding value in the left column.

Responding To Student Thinking

Points to Emphasize
If students struggle with finding the constant of proportionality, focus on this as opportunities arise over the next several lessons. For example, in the activity referred to here, invite multiple students to share their thinking about how they found the constant of proportionality.

Grade 7, Unit 2, Lesson 4, Activity 3 Denver to Chicago

Lesson 3 Summary

When something is traveling at a constant speed, there is a proportional relationship between the time it takes and the distance traveled. The table shows the distance traveled and elapsed time for a bug crawling on a sidewalk.

distance traveled (cm)	elapsed time (sec)
$\frac{3}{2}$	1
1	$\frac{2}{3}$
3	2
10	$\frac{20}{3}$

$\cdot \frac{2}{3}$

We can multiply any number in the first column by $\frac{2}{3}$ to get the corresponding number in the second column. We can say that the elapsed time is proportional to the distance traveled, and the constant of proportionality is $\frac{2}{3}$. This means that the bug's *pace* is $\frac{2}{3}$ seconds per centimeter.

This table represents the same situation, except the columns are switched.

elapsed time (sec)	distance traveled (cm)
1	$\frac{3}{2}$
$\frac{2}{3}$	1
2	3
$\frac{20}{3}$	10

$\cdot \frac{3}{2}$

We can multiply any number in the first column by $\frac{3}{2}$ to get the corresponding number in the second column. We can say that the distance traveled is proportional to the elapsed time, and the constant of proportionality is $\frac{3}{2}$. This means that the bug's *speed* is $\frac{3}{2}$ centimeters per second.

Notice that $\frac{3}{2}$ is the reciprocal of $\frac{2}{3}$. When two quantities are in a proportional relationship, there are two constants of proportionality, and they are always reciprocals of each other. When we represent a proportional relationship with a table, we say the quantity in the second column is proportional to the quantity in the first column, and the corresponding constant of proportionality is the number we multiply values in the first column by to get the values in the second.

Practice Problems

1 Student Task Statement

Noah is running a portion of a marathon at a constant speed of 6 miles per hour.

Complete the table to predict how long it would take him to run different distances at that speed, and how far he would run in different time intervals.

time in hours	miles traveled at 6 miles per hour
1	
$\frac{1}{2}$	
$1\frac{1}{3}$	
	$1\frac{1}{2}$
	9
	$4\frac{1}{2}$

Solution

time in hours	miles traveled at 6 miles per hour
1	6
$\frac{1}{2}$	3
$1\frac{1}{3}$	8
$\frac{1}{4}$	$1\frac{1}{2}$
$1\frac{1}{2}$	9
$\frac{3}{4}$	$4\frac{1}{2}$

2 Student Task Statement

One kilometer is 1000 meters.

a. Complete the tables. What is the interpretation of the constant of proportionality in each case?

meters	kilometers
1,000	1
250	
12	
1	

kilometers	meters
1	1,000
5	
20	
0.3	

The constant of proportionality tells us that:

The constant of proportionality tells us that:

b. What is the relationship between the two constants of proportionality?

Solution

a. i.

meters	kilometers
1,000	1
250	0.25
12	0.012
1	0.001

0.001 kilometers per meter

ii.

kilometers	meters
1	1,000
5	5,000
20	20,000
0.3	300

1000 meters per kilometer

b. 0.001 and 1000 are reciprocals of each other. This is easier to see if 0.001 is written as $\frac{1}{1000}$.

3 Student Task Statement

Jada and Lin are comparing inches and feet. Jada says that the constant of proportionality is 12. Lin says it is $\frac{1}{12}$. Do you agree with either of them? Explain your reasoning.

Solution

Both can be correct. Jada is saying that there are 12 inches for every 1 foot. Lin is saying that there is $\frac{1}{12}$ foot for every 1 inch.

4 from Unit 1, Lesson 12

Student Task Statement

The area of the Mojave desert is 25,000 square miles. A scale drawing of the Mojave desert has an area of 10 square inches. What is the scale of the map?

Solution

1 inch to 50 miles

5 from Unit 1, Lesson 11

Student Task Statement

Which of these scales is equivalent to the scale 1 cm to 5 km? Select **all** that apply.

A. 3 cm to 15 km

B. 1 mm to 150 km

C. 5 cm to 1 km

D. 5 mm to 2.5 km

E. 1 mm to 500 m

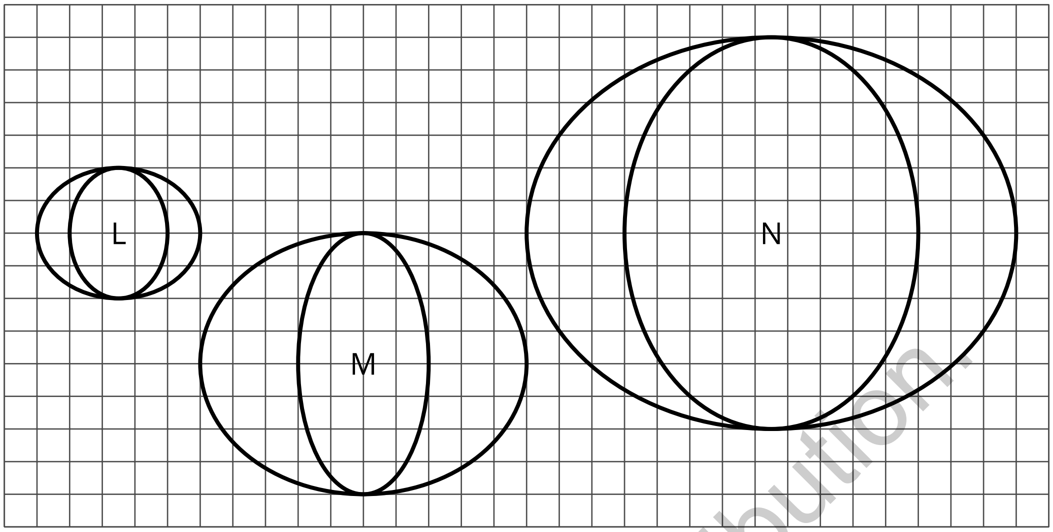
Solution

A, D, E

6 from Unit 2, Lesson 1

Student Task Statement

Which one of these pictures is not like the others? Explain what makes it different using ratios.



Solution

M is different from L and N. The width:height ratios for the outsides of the pictures are all equivalent to 5:4. However, the width:height ratios of the insides of L and N both have a 3:4 ratio of width:height, while the inside of M has a width of 4 units and a height of 8 units, making its ratio 1:2.

Alternatively, the ratio of height to thickness at the widest part for L and N are both 4:1. But M has a height of 8 units and a thickness of 3 units, making that ratio 8:3.

Section B: Representing Proportional Relationships with Equations

Goals

- Use an equation to solve problems involving a proportional relationship.
- Write an equation of the form $y = kx$ to represent a proportional relationship, given a table or a description of the situation.

Section Narrative

In this section, students use equations to represent proportional relationships and solve problems. They learn that any proportional relationship can be represented by an equation of the form $y = kx$, where k is the constant of proportionality. Students begin by revisiting some of the same contexts that they previously examined with tables. They think about how the repeated calculations can be expressed with an equation.

Next, students see that there are two different equations that represent each situation, depending on which quantity is regarded as being proportional to the other. The two constants of proportionality in those two equations are reciprocals of each other. Then students write equations and use them to solve problems involving proportional relationships in new contexts.

$b = 4r$

red paint (parts)	blue paint (parts)
3	12
1	4
7	28
$\frac{1}{4}$	1
r	$4r$

$r = \frac{1}{4}b$

blue paint (parts)	red paint (parts)
12	3
4	1
28	7
1	$\frac{1}{4}$
b	$\frac{1}{4}b$

Teacher Reflection Questions

- **Math Content and Student Thinking:** In what ways did using tables to represent the proportional relationship between two variables support students in writing the equation of this relationship? What understandings supported students in writing the equation when the table was no longer available?
- **Pedagogy:** In what ways do you determine whether students make a connection between the math work they do in the lesson and the learning goals? Which curricular resources support you in making these determinations?
- **Access and Equity:** What makes someone want to persist through a difficult math problem? In what ways are you making assumptions about which of your students are resilient, motivated, or persistent with their mathematics?

Section B Checkpoint

1

Goals Assessed

- Write an equation of the form $y = kx$ to represent a proportional relationship, given a table or a description of the situation.

Student Task Statement

Elena is riding her bike around the park at a constant pace. She completes 5 laps in 20 minutes.

Write an equation that shows the time in minutes it takes Elena to complete x laps at the same pace. (If you get stuck, consider completing the table.)

number of laps	time in minutes
5	20
10	
11	
1	
x	

Solution

$y = 4x$ (where y represents time in minutes)

Responding To Student Thinking

Points to Emphasize
If most students struggle with writing an equation that represents a proportional relationship, as opportunities arise over the next several lessons, revisit the structure and meaning of an equation of the form $y = kx$. For example, in the activity referred to here, invite multiple students to share their thinking about the structure of the equations.

Grade 7, Unit 2, Lesson 8, Activity 3 Total Edge Length, Surface Area, and Volume

2

Goals Assessed

- Use an equation to solve problems involving a proportional relationship.

Narrative

This item assesses whether students can use the equation of a proportional relationship and rely on its structure to make sense of a novel situation and solve problems.

While the units of muku and anana are likely unfamiliar, students have converted other units of length. To help orient students to what the item is asking, it may be helpful to remind them of some other units of length, such as meters and centimeters, used throughout the unit.

Student Task Statement

Hawaiians have a unique system for measuring lengths. Two of their units are called “muku” and “anana.”

The equation $a = 0.75m$ gives the relationship between a length measured in muku, m, and the same length measured in anana, a.

a. How many anana are in 24 muku?
b. How many muku are in 24 anana?

Solution

a. 18 anana
b. 32 muku

Responding To Student Thinking

Points to Emphasize
If most students struggle with using an equation to find unknown values, revisit this concept as opportunities arise over the next several lessons. For example, in the activity referred to here, invite multiple students to share their thinking about substituting values and evaluating expressions.

Grade 7, Unit 2, Lesson 8, Activity 2 More Conversions

Unit 2, Lesson 4

Proportional Relationships and Equations

Goals

- Generalize a process for finding unknown values in a proportional relationship, and justify (orally) why this can be abstracted as $y = kx$, where k is the constant of proportionality.
- Generate an equation of the form $y = kx$ to represent a proportional relationship in a familiar context.
- Write the constant of proportionality to complete a row in a table representing a proportional relationship where the value for the first quantity is 1.

Learning Targets

- I can write an equation of the form $y = kx$ to represent a proportional relationship shown in a table or described in a story.
- I can write the constant of proportionality as an entry in a table.

Lesson Narrative

In this lesson, students represent proportional relationships using equations of the form $y = kx$. The activities revisit various contexts from earlier in the unit and continue presenting values in tables. Students see that the relationship in each table can be represented by an equation of the form $y = kx$, where k is the constant of proportionality that relates the two quantities. As students calculate values in the tables and write equations relating the quantities, they practice looking for and expressing regularity in repeated reasoning (MP8).

The last activity is optional because it provides an opportunity for additional practice by revisiting another familiar context.

Standards

Building On **5.NBT.B.7**

Addressing **7.RP.A.2, 7.RP.A.2.c**

Instructional Routines

- 5 Practices
- MLR3: Critique, Correct, Clarify
- MLR7: Compare and Connect
- MLR8: Discussion Supports
- Which Three Go Together?

Student Facing Learning Goals

Let’s write equations describing proportional relationships.

4.1 Which Three Go Together: Expressions

Warm-up

5 mins

Activity Narrative

This *Warm-up* prompts students to compare four expressions. In making comparisons, students have a reason to use language precisely (MP6). The activity also enables the teacher to hear the terminologies students know and how they talk about characteristics of algebraic expressions.

Standards

Building On **5.NBT.B.7**

Instructional Routines

- Which Three Go Together?

Launch

Arrange students in groups of 2–4. Display the expressions for all to see. Give students 1 minute of quiet think time and ask them to indicate when they have noticed three expressions that go together and can explain why. Next, tell students to share their response with their group, and then together find as many sets of three as they can.

Student Task Statement

Which three go together? Why do they go together?

A $5 \cdot 2$

B $4 + ? = 20$

C $x + 5$

D $5x$

Student Response

Sample responses:

A, B, and C go together because:

- They have a symbol that represents the operation.

A, B, and D go together because:

- They all represent multiplication.
- They all involve multiplying by 5.

A, C, and D go together because:

- They are expressions, but not equations.
- They include the numeral 5.

B, C, and D go together because:

- They have a symbol representing an unknown number.

Activity Synthesis

Invite each group to share one reason why a particular set of three go together. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which three go together, attend to students' explanations and ensure the reasons given are correct.

During the discussion, prompt students to explain the meaning of any terminology they use, such as "sum," "product," "factor," "term," "expression," "equation," "variable," or "unknown," and to clarify their reasoning as needed. Consider asking:

- "How do you know . . . ?"
- "What do you mean by . . . ?"
- "Can you say that in another way?"

4.2 Feeding a Crowd, Revisited

10 mins

Activity Narrative

In this activity, students revisit two contexts seen previously and ultimately find equations for the proportional relationships. As students find missing values in the table, they should see that they can always multiply the number of food items by the constant of proportionality to find the number of people served. When students see this pattern and represent the number of people served by x cups of rice as $3x$ (or by n spring rolls as $\frac{1}{2}n$), they are expressing regularity in repeated reasoning (MP8).

Monitor for students who use these strategies to complete the tables:

- Find the unit rate, interpret it, and use it to calculate other values.
- Use a scale factor to find the values for the first row and then scale those values to find other rows.
- Find the constant of proportionality that relates the left column to the right column.

Regardless of whether students reason based on the meaning of the unit rate in context or based on the structure of the table, the key takeaway is the constant multiplicative relationship.

Access for English Language Learners

This activity uses the *Compare and Connect* math language routine to advance representing and conversing as students use mathematically precise language in discussion.

Standards

Addressing **7.RP.A.2.c**

Instructional Routines

- MLR7: Compare and Connect

Launch

Arrange students in groups of 2–3. Tell students that they will revisit the situation about rice and spring rolls from an earlier activity in this unit.

Select work from students with different strategies, such as those described in the *Activity Narrative*, to share later.

Student Task Statement

1. A recipe says that 2 cups of dry rice will serve 6 people. Complete the table as you answer the questions. Be

prepared to explain your reasoning.

a. How many people will 1 cup of rice serve?

b. How many people will 3 cups of rice serve? 12 cups? 43 cups?

c. How many people will x cups of rice serve?

cups of dry rice	number of people
1	
2	6
3	
12	
43	
x	

2. A recipe says that 6 spring rolls will serve 3 people. Complete the table as you answer the questions. Be prepared to explain your reasoning.

a. How many people will 1 spring roll serve?

b. How many people will 10 spring rolls serve? 16 spring rolls? 25 spring rolls?

c. How many people will n spring rolls serve?

number of spring rolls	number of people
1	
6	3
10	
16	
25	
n	

3. How was completing the table about spring rolls different from completing the table about rice? How was it the same?

Student Response

1. a. 3

   b. 9; 36; 129

   c. $3x$

cups of dry rice	number of people
1	3
2	6
3	9
12	36
43	129
x	$3x$

2. a. $\frac{1}{2}$.
   b. 5 ; 8; 2.5
   c. $\frac{1}{2}n$

number of spring rolls	number of people
1	$\frac{1}{2}$
6	3
10	5
16	8
25	12.5
n	$\frac{1}{2}n$

3. Sample response: Both problems involved multiplying (or dividing). The constant of proportionality for the rice was greater than 1, so the number of people is larger than the number of cups. The constant of proportionality for the spring rolls was less than 1, so the number of people is less than the number of spring rolls.

Building on Student Thinking

If students have trouble representing each relationship with an expression, encourage them to draw diagrams or to describe the relationship in words.

Activity Synthesis

The goal of this discussion is to show how an equation can be used to represent the proportional relationship shown in each table. Display 2–3 approaches to the rice problem from previously selected students for all to see. Use *Compare and Connect* to help students compare, contrast, and connect the different representations. Here are some questions for discussion:

- "What do the approaches have in common? How are they different?"
- "How does the constant of proportionality, 3, show up in each method?"
- "Are there any benefits or drawbacks to one method compared to another?"

The key takeaway is that any value in the right column can be found by multiplying the corresponding value in the left column by 3. For the last row, we can represent x times 3 as $3x$.

Next, suggest to students that we let y represent the number of people who can be served by x cups of rice. Ask students to write an equation that gives the relationship between x and y. Display the equation $y = 3x$ and help students interpret its meaning in the context: "To find y, the number of people served, we can multiply the number of cups of rice, x, by 3."

Lastly, ask students to write an equation that represents the relationship for the spring rolls. Record and display students' equations. Ask them to interpret what the equations tell us about the situation. (To find the number of people served, y, we can divide the number of spring rolls, n, by 2, or multiply it by $\frac{1}{2}$ or 0.5.)

10 mins

4.3 Denver to Chicago

Activity Narrative

In this activity, students write an equation to represent a proportional relationship between distance and time. The context of airplane flight is similar to that of a previous activity, but not exactly the same. This activity prompts students to move back and forth between the abstract representation and the context (MP2) as they create an equation and then use it to find other values that aren't in the table. This task increases the level of difficulty by having so much missing information and by using decimals in the table.

There are various ways students may approach the last question. Monitor for students who:

- Multiply the elapsed times by the rate 610 miles per hour, without referencing the table or equation.
- Use the structure of the table, adding rows for 3 and 3.5 hours.
- Use the equation to find the distances for 3 and 3.5 hours.

Plan to have students present in this order to support moving them from arithmetic methods towards algebraic methods.

Standards

Addressing **7.RP.A.2.c**

Instructional Routines

- 5 Practices
- MLR8: Discussion Supports

Launch

Tell students that this activity revisits the context of flying in an airplane but it is not exactly the same situation as the airplane activity in the earlier lesson.

Select students with different strategies, such as those described in the *Activity Narrative*, to share later. Aim to elicit both key mathematical ideas and a variety of student voices, especially from students who haven't shared recently.

Access for English Language Learners

MLR8 Discussion Supports. Prior to solving the problems, invite students to make sense of the situations and take turns sharing their understanding with their partner. Listen for and clarify any questions about the context.

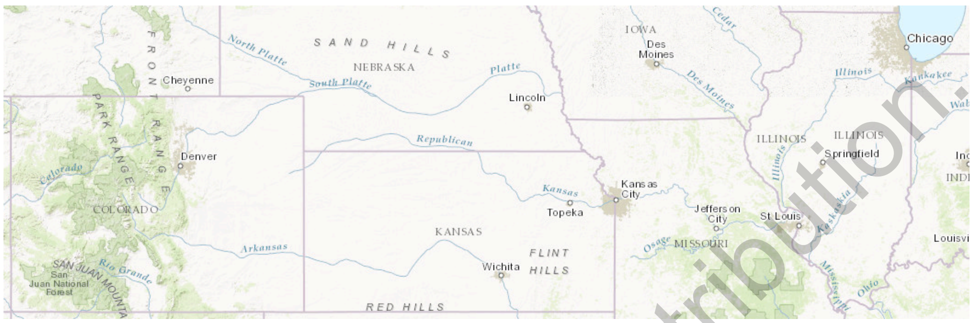
Advances: Reading, Representing

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Provide tools to facilitate information processing or computation, enabling students to focus on key mathematical ideas. For example, allow students to use calculators to support their reasoning.
Supports accessibility for: Memory, Conceptual Processing

Student Task Statement

A plane flew at a constant speed between Denver and Chicago. It took the plane 1.5 hours to fly 915 miles.



1. Complete the table.

time (hours)	distance (miles)
1	
1.5	915
2	
2.5	
t	

2. How far does the plane fly in 1 hour?
3. How far would the plane fly in t hours at this speed?
4. If d represents the distance that the plane flies at this speed for t hours, write an equation that relates t and d.
5. How far would the plane fly in 3 hours at this speed? in 3.5 hours? Explain or show your reasoning.

Student Response

1.

time (hours)	distance (miles)
1	610
1.5	915
2	1,220
2.5	1,525
t	$610t$

2. 610 miles since $915 \div 1.5 = 610$ for the speed, and at a speed of 610 miles per hour, it would travel 610 miles after 1 hour.
3. $610t$ miles
4. $d = 610t$ (or equivalent)
5. 1,830 miles; 2,135 miles. Sample reasoning: I multiplied each number of hours by 610.

Building on Student Thinking

Students who are having trouble understanding the task can draw a segment between Denver and Chicago on the map and label it with the distance and the time. Consider prompting them to find the speed in miles per hour (the distance the plane travels in 1 hour at this speed). Then ask students how this number can help them complete the table and answer the questions.

Are You Ready for More?

A rocky planet orbits Proxima Centauri, a star that is about 1.3 parsecs from Earth. This planet is the closest planet outside of our solar system.

1. How long does it take light from Proxima Centauri to reach Earth? (A parsec is about 3.26 light years. A light year is the distance light travels in one year.)
2. Imagine there are two twins. One twin leaves Earth on a spaceship and travels to a planet near Proxima Centauri. The spaceship travels at 90% of the speed of light. The other twin stays home on Earth. How much does the twin on Earth age while the other twin travels to Proxima Centauri? (Do you think the answer would be the same for the twin on the spaceship? Consider researching “The Twin Paradox” to learn more.)

Extension Student Response

1. $1.3 \cdot 3.26 \approx 4.24$ or about 4.24 years
2. $4.24 \div 0.9 \approx 4.7$ or about 4.7 years

Activity Synthesis

The purpose of this discussion is to show the value of finding an equation that represents a proportional relationship.

Ask previously selected students to share their solutions to the last question (the distances the plane would travel in 3 and 3.5 hours). Sequence the discussion of the strategies in the order listed in the *Activity Narrative*. If possible, record and display their work for all to see.

Connect the different responses to the learning goals by asking questions such as:

- "How are these methods the same? How are they different?"
- "How does the constant of proportionality show up in each representation?"
- "Are there any benefits or drawbacks to one strategy compared to another?"

If not mentioned by students, display the equation $d = 610t$ for all to see, and ask students to interpret its meaning in the context of the situation. (To find d, the distance traveled by the plane in miles, multiply the hours of travel, t, by the plane's speed in miles per hour, 610.)

The key takeaways are:

- An equation is an efficient way to represent a proportional relationship.
- The equation shows the constant of proportionality, and the variables indicate how the quantities are related.
- You can substitute a value in for one of the variables and then multiply (or divide) to find the value for the other variable.

4.4 Revisiting Coco Bread

Optional

10 mins

Activity Narrative

This activity gives students more practice writing an equation that represents a proportional relationship. It revisits a context examined in a previous lesson—the amounts of coconut milk and flour in a bread recipe. Students can then use their equation to answer additional questions about the situation.

Access for English Language Learners

This activity uses the *Critique, Correct, Clarify* math language routine to advance representing and conversing as students critique and revise mathematical arguments.

Standards

Addressing **7.RP.A.2.c**

Instructional Routines

- MLR3: Critique, Correct, Clarify

Launch

Tell students that this activity revisits the context of making coco bread from an earlier lesson. Give students quiet work time followed by partner discussion.

Student Task Statement

To bake coco bread, a bakery uses 200 milliliters of coconut milk for every 360 grams of flour. Some days they bake bigger batches and some days they bake smaller batches, but they always use the same ratio of coconut milk to flour.

1. Complete the table.
2. Use f to represent the grams of flour needed for c milliliters of coconut milk. Write an equation that relates f and c.
3. How much flour is needed for 680 milliliters of coconut milk? 945 milliliters? Explain or show your reasoning.

coconut milk (milliliters)	flour (grams)
100	
200	350
450	
c	

Student Response

1.

coconut milk (milliliters)	flour (grams)
100	180
200	360
450	810
c	$1.8c$

2. $f = 1.8c$ or equivalent.
3. 1,224 grams; 1,701 grams; Sample reasoning: I multiplied each number by 1.8.

Activity Synthesis

Ask students to compare answers with their partner and discuss their reasoning until they reach an agreement. Then invite students to share with the class how they used their equation from the second question to answer the third question.

Use *Critique, Correct, Clarify* to give students an opportunity to improve a sample written response about how to get the equation for this relationship by correcting errors, clarifying meaning, and adding details.

- Display this first draft:
  "First I wrote the equation $200c = 360f$. Then I wanted to solve it for one of the variables, so I divided each side by 200 to get the equivalent equation $c = 1.8f$."
  Ask, "What parts of this response are unclear, incorrect, or incomplete?" As students respond, annotate the display with 2–3 ideas to indicate the parts of the writing that could use improvement.
- Give students 2–4 minutes to work with a partner to revise the first draft.
- Here is an example of a second draft:
  "For each row of the table, the grams of flour is 1.8 times the milliliters of coconut milk. For example $200 \cdot 1.8 = 360$. An equation that represents this relationship is $f = 1.8c$.
- Select 1–2 individuals or groups to read their revised draft aloud slowly enough to record for all to see. Scribe as each student shares, then invite the whole class to contribute additional language and edits to make the final draft

even more clear and more convincing.

Lesson Synthesis

Share with students "Today we used the constant of proportionality to write an equation to represent each proportional relationship."

To review the structure of an equation that represents a proportional relationship, consider asking students:

- "What was the constant of proportionality for the relationship between cups of rice and people served?" (3)
    - "What equation did we write for this situation?" ($y = 3x$)
- "What was the constant of proportionality for the relationship between number of spring rolls and people served?" ($\frac{1}{2}$)
    - "What equation did we write for this situation?" ($p = \frac{1}{2}n$ or equivalent)
- "What was the constant of proportionality for the relationship between the distance the plane traveled and the elapsed time?" (610)
    - "What equation did we write for this situation?" ($d = 610t$)
- "What do these equations have in common?" (They show a multiplicative relationship between two quantities. They have a structure like $y = kx$, where k is the constant of proportionality.)

4.5 It's Snowing in Syracuse

Cool-down

5 mins

Standards

Addressing **7.RP.A.2**

Student Task Statement

Snow is falling steadily in Syracuse, New York. After 2 hours, 4 inches of snow has fallen.

1. If it continues to snow at the same rate, how many inches of snow would you expect after 6.5 hours? If you get stuck, you can use the table to help.
2. Write an equation that gives the amount of snow that has fallen after x hours at this rate.
3. How many inches of snow will fall in 24 hours if it continues to snow at this rate?

time (hours)	snow (inches)
	1
1	
2	4
6.5	
x	

Student Response

1. 13 inches (Two inches fell in 1 hour, 6.5 is $1 \cdot (6.5)$, and $2 \cdot (6.5) = 13$.)
2. Sample response: $y = 2x$, where x is the number of hours that have passed and y is the inches of snow that has fallen.
3. 48 inches ($24 \cdot 2 = 48$)

Responding To Student Thinking

Points to Emphasize
If students struggle with writing an equation to represent a proportional relationship, focus on this as opportunities arise over the next several lessons. For example, in the activity refered to here, invite multiple students to share their thinking about how they wrote their equations.

Grade 7, Unit 2, Lesson 5, Activity 2 Meters and Centimeters

Lesson 4 Summary

In this lesson, we wrote equations to represent proportional relationships described in words and shown in tables.

This table shows the amount of red paint and blue paint needed to make a certain shade of purple paint, called Venusian Sunset.

Note that "parts" can be *any* unit for volume. If we mix 3 cups of red with 12 cups of blue, you will get the same shade as if we mix 3 teaspoons of red with 12 teaspoons of blue.

red paint (parts)	blue paint (parts)
3	12
1	4
7	28
$\frac{1}{4}$	1
r	$4r$

The last row in the table shows that if we know the amount of red paint, r, we can always multiply it by 4 to find the amount of blue paint needed to make Venusian Sunset. If b is the amount of blue paint, we can say this more succinctly with the equation $b = 4r$. So, the amount of blue paint is proportional to the amount of red paint, and the constant of proportionality is 4.

We can also look at this relationship the other way around.

If we know the amount of blue paint, b, we can always multiply it by $\frac{1}{4}$ to find the amount of red paint, r, needed to make Venusian Sunset. So, the equation $r = \frac{1}{4}b$ also represents the relationship. The amount of red paint is proportional to the amount of blue paint, and the constant of proportionality $\frac{1}{4}$.

blue paint (parts)	red paint (parts)
12	3
4	1
28	7
1	$\frac{1}{4}$
b	$\frac{1}{4}b$

In general, when y is proportional to x, we can always multiply x by the same number k—the constant of proportionality—to get y. We can write this much more succinctly with the equation $y = kx$.

Practice Problems

1 Student Task Statement

A ceiling is made up of tiles. Every square meter of the ceiling requires 10.75 tiles. Fill in the table with the missing values.

square meters of ceiling	number of tiles
1	
10	
	100
a	

Solution

square meters of ceiling	number of tiles
1	10.75
10	107.5
9.3	100
a	$10.75 \cdot a$

2 Student Task Statement

On a flight from New York to London, an airplane travels at a constant speed. An equation relating the distance traveled in miles, d, to the number of hours flying, t, is $t = \frac{1}{500}d$. How long will it take the airplane to travel 800 miles?

Solution

1.6 hours, since $\frac{1}{500} \cdot 800 = 1.6$

3 Student Task Statement

Each table represents a proportional relationship. For each, find the constant of proportionality, and write an equation that represents the relationship.

s	P
2	8
3	12
5	20
10	40

Constant of proportionality:

Equation: $P =$

d	C
2	6.28
3	9.42
5	15.7
10	31.4

Constant of proportionality:

Equation: $C =$

Solution

a. Constant of proportionality: 4
   Equation: $P = 4s$

b. Constant of proportionality: 3.14
   Equation: $C = 3.14d$

4 from Unit 1, Lesson 11

Student Task Statement

A map of Colorado says that the scale is 1 inch to 20 miles or 1 to 1,267,200. Are these two ways of reporting the scale the same? Explain your reasoning.

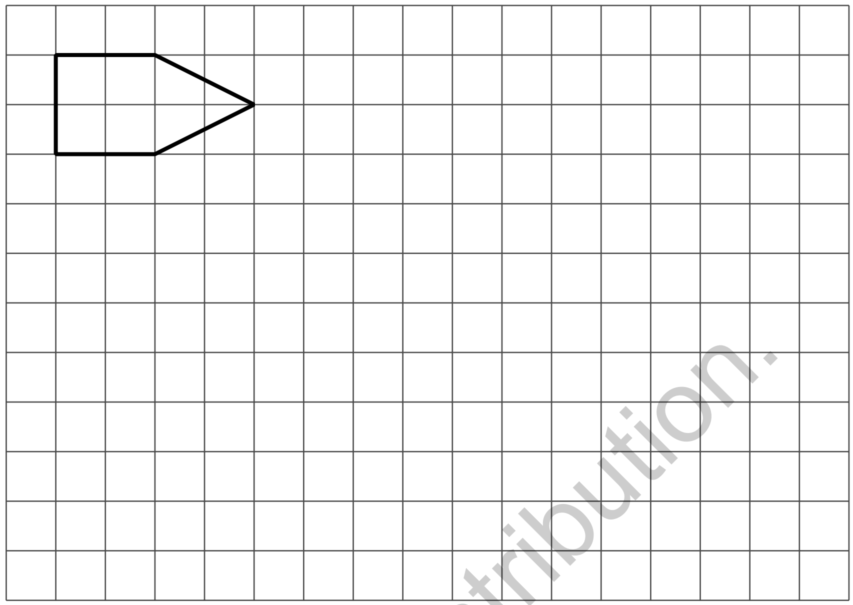
Solution

Yes. Sample reasoning: There are 12 inches in a foot and 5280 feet in 1 mile, so that's 63,360 inches in a mile and 1,267,200 inches in 20 miles.

5 from Unit 1, Lesson 3

Student Task Statement

Here is a polygon on a grid.



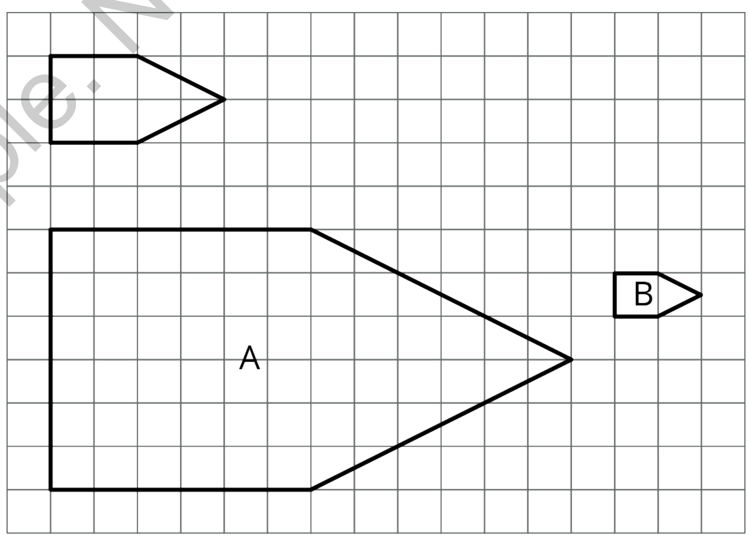
a. Draw a scaled copy of the polygon using a scale factor 3. Label the copy A.

b. Draw a scaled copy of the polygon with a scale factor $\frac{1}{2}$. Label it B.

c. Is Polygon A a scaled copy of Polygon B? If so, what is the scale factor that takes B to A?

Solution

a.



b. Yes, A is a scaled copy of B with a scale factor of 6.

Unit 2, Lesson 5

Two Equations for Each Relationship

Goals

- Use the word “reciprocal” to explain (orally and in writing) that there are two related constants of proportionality for a proportional relationship.
- Write two equations that represent the same proportional relationship, i.e., $y = kx$ and $x = (\frac{1}{k})y$, and explain (orally) what each equation means.

Learning Targets

- I can find two constants of proportionality for a proportional relationship.
- I can write two equations representing a proportional relationship described by a table or story.

Lesson Narrative

In this lesson, students practice writing two different equations for the same proportional relationship. This is accomplished by switching which quantity is regarded as being proportional to the other. Students see why the constants of proportionality associated with the two equations are reciprocals of each other.

For example, if a person runs at a constant speed and travels 12 miles in 2 hours, then the distance traveled is proportional to the time elapsed, with constant of proportionality 6, because $\text{distance} = 6 \boldsymbol{\cdot} \text{time}$. The time elapsed is proportional to distance traveled with constant of proportionality $\frac{1}{6}$, because $\text{time} = \frac{1}{6} \boldsymbol{\cdot} \text{distance}$.

The activities in this lesson use familiar contexts, but not identical situations from previous lessons: measurement conversions and water flowing at a constant rate. Students are expected to use methods developed earlier: organize data in a table, write and solve an equation to determine the constant of proportionality, and generalize from repeated calculations to arrive at an equation (MP8). Students also practice reasoning quantitatively and abstractly as they write or use an equation and then interpret their answers in the context of the situation (MP2). The last activity is optional because it provides an opportunity for additional practice with a new context.

Standards

Building On	**5.OA.B**
Addressing	**7.RP.A, 7.RP.A.2, 7.RP.A.2.b, 7.RP.A.2.c**

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- MLR5: Co-Craft Questions
- MLR8: Discussion Supports
- Which Three Go Together?

Student Facing Learning Goals

Let’s investigate equations that represent proportional relationships.

5.1 Which Three Go Together: Tiles

Warm-up

5 mins

Activity Narrative

This *Warm-up* prompts students to compare four geometric patterns. It gives students a reason to use language precisely (MP6). It gives the teacher an opportunity to hear how students use terminology and talk about characteristics of the items in comparison to one another.

Standards

Building On **5.OA.B**

Instructional Routines

- Which Three Go Together?

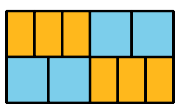
Launch

Arrange students in groups of 2–4. Display the images for all to see. Give students 1 minute of quiet think time and ask them to indicate when they have noticed three images that go together and can explain why. Next, tell students to share their response with their group, and then together find as many sets of three as they can.

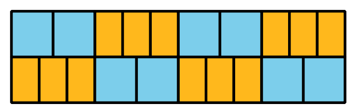
Student Task Statement

Which three go together? Why do they go together?

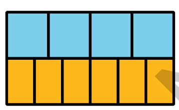
A



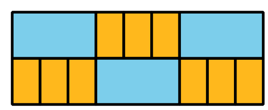
B



C



D



Student Response

Sample responses:

A, B, and C go together because:

- The ratio of the number of blue tiles to the number of yellow tiles is $2 : 3$.
- They have an even number of blue tiles and an even number of yellow tiles.
- You can use vertical cuts to make groups of 5 tiles (with 2 blue and 3 yellow in each group).

A, B, and D go together because:

- They have both colors on the top and on the bottom.
- They have some blue tiles on the bottom.
- They have some yellow tiles on the top.

A, C, and D go together because:

- The pattern ends (on the right) with blue on top and yellow on bottom.

- They have fewer than 15 total tiles.

B, C, and D go together because:

- The pattern starts (on the left) with blue on top and yellow on bottom.
- Each image has 6 yellow tiles on the bottom.

Activity Synthesis

Invite each group to share one reason why a particular set of three go together. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which three go together, attend to students' explanations and ensure the reasons given are correct.

During the discussion, prompt students to explain the meaning of any terminology they use, such as "row," "group," "partition," "even," "odd," "horizontal," "vertical," "ratio," or "area," and to clarify their reasoning as needed. Consider asking:

- "How do you know . . . ?"
- "What do you mean by . . . ?"
- "Can you say that in another way?"

If time allows, invite 2–3 students to briefly share what they notice all of the figures have in common. For example:

- They are all rectangles composed of smaller rectangles.
- Half their area is blue and half is yellow.
- They all have an even number of total tiles.

The purpose of this concluding share out is to reinforce the importance of using precise terminology. For example, saying "the ratio of blue to yellow" is not specific enough. The ratio of blue area to yellow area is $1 : 1$ for all of the figures, while the ratio of blue pieces to yellow pieces is either $2 : 3$ or $1 : 3$.

5.2 Meters and Centimeters

15 mins

Activity Narrative

In this activity, students practice writing an equation to represent a proportional relationship given in a table. This activity revisits the idea that there are two reciprocal constants of proportionality between two related quantities. Students use repeated reasoning (MP8) to arrive at the equations and to identify the constants of proportionality as reciprocals.

We return to the context of measurement conversion, again examining the same distances measured in two different units. Previously students compared centimeters with millimeters and saw that the constants of proportionality were 10 and $\frac{1}{10}$. Here, students compare meters with centimeters and see that the constants of proportionality are 100 and $\frac{1}{100}$. The similarities between this activity and the earlier one may cause this activity to go very quickly.

Standards

Addressing **7.RP.A.2.b, 7.RP.A.2.c**

Instructional Routines

- MLR8: Discussion Supports

Launch

Remind students that in an earlier lesson, they examined the relationship between millimeters and centimeters. Tell them that today, they will examine the relationship between centimeters and meters.

Student Task Statement

There are 100 centimeters (cm) in every meter (m).

length (m)	length (cm)
1	100
0.94	
1.67	
57.24	
x	

length (cm)	length (m)
100	1
250	
78.2	
123.9	
y	

1. Complete the tables.
2. For each table, find the constant of proportionality.
3. Describe the relationship between these two constants of proportionality.
4. For each table, write an equation for the proportional relationship. Let x represent a length measured in meters and y represent the same length measured in centimeters.

Student Response

1. Tables:

length (m)	length (cm)
1	100
0.94	94
1.67	167
57.24	5,724
x	$100x$

length (cm)	length (m)
100	1
250	2.5
78.2	0.782
123.9	1.239
y	$0.01y$

2. The constant of proportionality for the first table is 100, and for the second table it is 0.01 or $\frac{1}{100}$.
3. Sample response: The constants of proportionality are reciprocals.
4. $y = 100x$ and $x = 0.01y$ or $x = \frac{1}{100}y$

Are You Ready for More?

1. How many cubic centimeters are there in 1 cubic meter?
2. How do you convert cubic centimeters to cubic meters?
3. How do you convert the other way?

Extension Student Response

1. 1,000,000
2. Sample response: Multiply by $\frac{1}{1,000,000}$ (or 0.0.000001).
3. Sample response: Multiply by 1,000,000.

Activity Synthesis

The purpose of this discussion is to highlight how the two equations illustrate the reciprocal relationship. Invite students to share how they found the equation for each table. Consider asking:

- "Where does the constant of proportionality occur in each table and equation?" (In the table, the constant of proportionality is in the right column on the row that has 1 in the left column. In the equation, the constant of proportionality is right next to the variable that represents the quantity that was in the left column of the table.)
- "What is the relationship between the two constants of proportionality? How can you use the equations to see why this should be true?" (They are reciprocals. This makes sense because the variables are in opposite places.)

Display and discuss this sequence of equivalent equations to help students see why the constants of proportionality are reciprocals:

$$y = 100x$$

$$\left(\frac{1}{100}\right) y = \frac{1}{100} \cdot (100x)$$

$$\left(\frac{1}{100}\right) y = x$$

$$x = \left(\frac{1}{100}\right) y$$

This line of reasoning should be accessible to students, because it builds on grade 6 work with expressions and equations.

Ask students to interpret the meaning of the equations in the context: "What do the equations tell us about the conversion from meters to centimeters and back?"

- Given the length in meters, to find the length in centimeters we can multiply the number of meters by 100.
- Given the length in centimeters, to find the length in meters we can multiply the number of centimeters by $\frac{1}{100}$.

Access for English Language Learners

MLR8 Discussion Supports. For each response that is shared, invite students to turn to a partner and restate what they heard using precise mathematical language.
Advances: Listening, Speaking

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Encourage and support opportunities for peer collaboration. When students share their work with a partner, display sentence frames to support conversation such as: "First, I _____ because . . . ." "I noticed _____, so I . . . ." "Why did you . . . ?" and "I agree/disagree because . . . ."
Supports accessibility for: Language, Social-Emotional Functioning

5.3 Filling a Water Cooler

15 mins

Activity Narrative

In this activity, students make sense of the two rates associated with a given proportional relationship (MP2). Students are asked to identify the two equations that represent a situation, working with both the number of gallons per minute and the number of minutes per gallon. This activity is the first time that no table is given to help students make sense of the proportional relationship, though students may find it helpful to create a table.

Monitor for students who use different ways to decide if the cooler was filling faster before or after the flow rate was changed.

Standards

Addressing **7.RP.A.2.c**

Instructional Routines

- MLR1: Stronger and Clearer Each Time

Launch

Give students 4–5 minutes quiet work time followed by partner and a whole-class discussion.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. To support development of organizational skills in problem-solving, chunk this task into more manageable parts. For example, give students one equation at a time and have them determine if it represents the relationship correctly or not by testing out example values.
Supports accessibility for: Organization, Attention

Student Task Statement

It took Priya 5 minutes to fill a cooler with 8 gallons of water from a faucet that was flowing at a steady rate. Let w be the number of gallons of water in the cooler after t minutes.

1. Which of the following equations represent the relationship between w and t? Select **all** that apply.
   a. $w = 1.6t$
   b. $w = 0.625t$
   c. $t = 1.6w$
   d. $t = 0.625w$
2. What does 1.6 tell you about the situation?
3. What does 0.625 tell you about the situation?
4. Priya changed the rate at which water flowed through the faucet. Write an equation that represents the relationship of w and t when it takes 3 minutes to fill the cooler with 1 gallon of water.
5. Was the cooler filling faster before or after Priya changed the rate of water flow? Explain how you know.

Student Response

1. $w = 1.6t$ and $t = 0.625w$
2. The water is flowing at 1.6 gallons per minute.
3. It takes 0.625 minutes for 1 gallon of water to flow out of the faucet (or into the cooler).
4. $t = 3w$ or $w = \frac{1}{3}t$
5. Before. Sample reasonings:
   a. Before the change, it took 0.625 minutes to get one gallon, but after the change, it took 3 minutes to get one gallon.
   b. Before the change, she got 1.6 gallons per minute, but after the change, she only got $\frac{1}{3}$ of a gallon per minute.

Building on Student Thinking

For the first question, if students struggle to identify the correct equations, encourage them to create two tables of values for the situation. Encourage them to create rows for both unit rates, in order to foster connections to prior learning.

Activity Synthesis

The goal of this discussion is to connect the meaning of each constant of proportionality with the structure of each equation that represents the relationship. Invite students to share how they decided which equations represent the situation. Ask students to interpret what the equations tell us about the situation.

- To find the amount of water, w, we can multiply the elapsed time in minutes, t, by 1.6.
- To find the elapsed time, t, we can multiply the number of gallons of water, w, by 0.625.

If not mentioned by students, highlight the fact that 1.6 and 0.625 are reciprocals. Since these constants of proportionality are given as decimals in the equations, it may be harder for students to recognize this relationship. Consider asking half the class to calculate $1 \div 1.6$ while the other half calculates $1 \div 0.625$.

Next, invite students to share their responses to the last two questions about Priya changing the rate of water flow. Two possible approaches for the last question are:

- Comparing the new equation $t = 3w$ to the previous equation $t = 0.625w$. In these equations, the constant of proportionality represents the numbers of minutes per gallon. Since 3 is greater than 0.625, that means it takes Priya more time to get the same amount of water after changing the rate of water flow.
- Comparing the new equation $w = \frac{1}{3}t$ to the previous equation $w = 1.6t$. In these equations, the constant of proportionality represents the number of gallons per minute. Since $\frac{1}{3}$ is less than 1.6, that means Priya gets less water in the same amount of time changing the rate of water flow.

It is not necessary to demonstrate every possible approach. The goal is for students to see how keeping in mind the meaning of the numbers and variables is helpful for making sense of the situation.

Access for English Language Learners

MLR1 Stronger and Clearer Each Time. Before the whole-class discussion, give students time to meet with 2–3 partners to share and get feedback on their first draft response to the last question, about whether the cooler was filling faster before or after Priya changed the rate of water flow. Invite listeners to ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing. Give students 3–5 minutes to revise their first draft based on the feedback they receive.
Advances: Writing, Speaking, Listening

5.4 Feeding Shrimp

Optional

10 mins

Activity Narrative

This activity provides an additional opportunity for students to represent a proportional relationship with two related equations in a new context. This situation builds on the earlier work students did with feeding a crowd, but includes more complicated calculations. Students interpret the meaning of the constants of proportionality in the context of the situation and use the equations to answer questions.

Standards

Addressing **7.RP.A.2**

Instructional Routines

- MLR5: Co-Craft Questions

Launch

Arrange students in groups of 2. Introduce the context of feeding animals in an aquarium. Give students 6 minutes of partner work time followed by whole-class discussion.

Access for English Language Learners

MLR5 Co-Craft Questions. Keep books or devices closed. Display only the problem stem, without revealing the questions, and ask students to record possible mathematical questions that could be asked about the situation. Invite students to compare their questions before revealing the task. Ask, "What do these questions have in common? How are they different?" Reveal the intended questions for this task and invite additional connections.
Advances: Reading, Writing

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Differentiate the degree of difficulty or complexity. Begin with more accessible values. For example, start students with an integer amount of grams of food for each feeding and let them solve the problem. Then invite students to solve the original problem.
Supports accessibility for: Conceptual Processing, Memory

Student Task Statement

At an aquarium, a shrimp is fed $\frac{1}{5}$ gram of food each feeding and is fed 3 times each day.

1. How much food does a shrimp get fed in 1 day?
2. Complete the table to show how many grams of food the shrimp is fed over different numbers of days.

number of days	grams of food
1	
7	
30	

3. What is the constant of proportionality? What does it tell us about the situation?
4. If the columns in the table were switched, what would be the constant of proportionality? Explain your reasoning
5. Use d for number of days and f for amount of food in grams that a shrimp is fed to write *two* equations that represent the relationship between d and f.
6. At this rate, how much food does a shrimp get fed in 75 days?
7. At this rate, how many days would 75 grams of shrimp food last? Explain or show your reasoning.

Student Response

1. $\frac{3}{5}$ gram

2.

number of days	grams of food
1	$\frac{3}{5}$
7	$4\frac{1}{5}$
30	18

3. $\frac{3}{5}$; It tells the feeding rate for the shrimp, in grams per day.
4. $\frac{5}{3}$ because the reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$.
5. $f = \frac{3}{5}d$ and $d = \frac{5}{3}f$
6. 45 grams
7. 125 days, because $\frac{5}{3}(75) = 125$

Activity Synthesis

The goal of this discussion is to highlight the structure of the two equations that represent the proportional relationship, including the meaning of the two constants of proportionality. Invite students to share their answers. Ask students which equation was most useful to answer each of the last two questions and to explain their reasoning.

Lesson Synthesis

Share with students, "Today we saw how to write two different equations for the same proportional relationship."

To review the reciprocal relationship between these equations, consider asking students:

- "In the first activity, we examined the proportional relationship between meters and centimeters. What two equations did we write for this relationship?" ($y = 100x$ and $x = \frac{1}{100}y$ or $x = 0.01y$)
- "Why were we able to write two equations for the same relationship?" (One is for converting meters to centimeters, and the other is for converting centimeters to meters.)
- "In the second activity, we examined a proportional relationship where we knew how long it took to fill a water cooler with a certain amount of water. What two equations did we determine would represent this relationship?" ($w = 1.6t$ and $t = 0.625w$)
- "What do these pairs of equations have in common?" (They all have a structure like $y = kx$, where k is the constant of proportionality. In each pair, the variables are reversed, and the constants of proportionality are reciprocals.)

If desired, use this example to review these concepts:

- "One equation that represents a proportional relationship is $w = \frac{1}{4}n$. What is a different equation that represents this same relationship?" ($n = 4w$)

5.5 Flight of the Albatross

Cool-down

5 mins

Standards

Addressing **7.RP.A**

Student Task Statement

An albatross is a large bird that can fly 400 kilometers in 8 hours at a constant speed. Using d for distance in kilometers and t for number of hours, an equation that represents this situation is $d = 50t$.

1. What are two constants of proportionality for the relationship between distance in kilometers and number of hours? What is the relationship between these two values?
2. Write another equation that relates d and t in this context.

Student Response

1. 50 and $\frac{1}{50}$; Sample response: They are reciprocals of each other.
2. $t = \frac{1}{50}d$

Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Lesson 5 Summary

If Kiran rode his bike at a constant 10 miles per hour, his distance in miles, d, is proportional to the number of hours, t, that he rode. We can write the equation
$d = 10t$
to represent the proportional relationship. With this equation, it is easy to find the distance Kiran rode when we know how long it took, because we can just multiply the time by 10.

We can rewrite the equation:

$$d = 10t$$
$$\left(\frac{1}{10}\right) d = t$$
$$t = \left(\frac{1}{10}\right) d$$

This version of the equation tells us that the amount of time Kiran rode is proportional to the distance he traveled, and the constant of proportionality is $\frac{1}{10}$. That form of the equation is easier to use when we know his distance and want to find how long it took, because we can just multiply the distance by $\frac{1}{10}$.

When two quantities x and y are in a proportional relationship, we can write the equation
$y = kx$

and say, "y is proportional to x." In this case, the number k is the corresponding constant of proportionality. We can also write the equation

$$x = \frac{1}{k}y$$

and say, "x is proportional to y." In this case, the number $\frac{1}{k}$ is the corresponding constant of proportionality. Each equation can be useful, depending on the information we have and the quantity we are trying to figure out.

Practice Problems

1 Student Task Statement

The table represents the relationship between a length measured in meters and the same length measured in kilometers.

a. Complete the table.

b. Write an equation for converting the number of meters to kilometers. Use x for the number of meters and y for the number of kilometers.

meters	kilometers
1,000	1
3,500	
500	
75	
1	
x	

Solution

a.

meters	kilometers
1,000	1
3,500	3.5
500	0.5
75	0.075
1	0.001
x	$0.001x$

b. $y = 0.001x$ (or equivalent)

2 Student Task Statement

Concrete building blocks weigh 28 pounds each. Using b for the number of concrete blocks and w for the weight, write two equations that relate the two variables. One equation should begin with $w =$ and the other should begin with $b =$.

Solution

$w = 28b$ and $b = \frac{1}{28}w$

3

Student Task Statement

A store sells rope by the meter. The equation $p = 0.8L$ represents the price, p, in dollars of a piece of nylon rope that is L meters long.

a. How much does the nylon rope cost per meter?

b. How long is a piece of nylon rope that costs $1.00?

Solution

a. $0.80

b. 1.25 meters

4

from Unit 2, Lesson 4

Student Task Statement

The table represents a proportional relationship. Find the constant of proportionality and write an equation to represent the relationship.

a	y
2	$\frac{2}{3}$
3	1
10	$\frac{10}{3}$
12	4

Constant of proportionality: _________

Equation: $y =$

Solution

Constant of proportionality: $\frac{1}{3}$ Equation: $y = \frac{1}{3}a$

5 from Unit 1, Lesson 8

Student Task Statement

On a map of Chicago, 1 cm represents 100 m. Select **all** statements that express the same scale.

A. 5 cm on the map represents 50 m in Chicago.

B. 1 mm on the map represents 10 m in Chicago.

C. 1 km in Chicago is represented by 10 cm on the map.

D. 100 cm in Chicago is represented by 1 m on the map.

Solution

B, C

Unit 2, Lesson 6

Writing Equations to Represent Relationships

Goals

- Generate an equation for a proportional relationship, given a description of the situation but no table.
- Interpret (orally) each part of an equation that represents a proportional relationship in an unfamiliar context.
- Use an equation to solve problems involving a proportional relationship, and explain (orally) the reasoning.

Learning Targets

- I can find missing information in a proportional relationship using the constant of proportionality.
- I can relate all parts of an equation like $y = kx$ to the situation it represents.

Lesson Narrative

In this lesson, students continue to write equations of the form $y = kx$ to represent proportional relationships. They begin to recognize situations where using the equation is a more efficient way of solving problems than other methods they have been using, such as using tables or equivalent ratios.

The activities introduce new contexts without providing tables. Students who still need tables should be given a chance to realize that fact and create tables for themselves. The activities are intended to motivate the usefulness of representing proportional relationships abstractly with equations (MP2). The repeated calculations called for in the activities serve as scaffolding for finding the equations (MP8).

Math Community
Today's activity is for students to individually reflect on the norms generated so far. During the *Cool-down*, students provide feedback on the norms, sharing those they agree with and those they feel need revision or removal. These suggestions will inform the next version of the classroom norms.

Standards

Building On	**5.NBT.B.7, 6.RP.A.2**
Addressing	**7.RP.A.2, 7.RP.A.2.c**

Instructional Routines

- Math Talk
- MLR6: Three Reads
- MLR7: Compare and Connect
- MLR8: Discussion Supports

Required Materials

Materials To Gather

- Math Community Chart: Activity 1

Required Preparation

Lesson:

Calculators can optionally be made available to take the focus off computation.

Student Facing Learning Goals

Let's use equations to solve problems involving proportional relationships.

6.1 Math Talk: Products with Decimal Points

5 mins

Warm-up

Activity Narrative

This *Math Talk* focuses on multiplication by a decimal. It encourages students to think about how they can use the result of one multiplication problem to find the answer to a similar problem with a different, but related, factor. The understanding elicited here will be helpful later in the lesson when students evaluate equations where the constant of proportionality is a decimal.

To recognize how a factor has been scaled and predict how the product will be affected, students need to look for and make use of structure (MP7).

Standards

Building On **5.NBT.B.7**

Instructional Routines

- Math Talk
- MLR8: Discussion Supports

Launch

Tell students to close their books or devices (or to keep them closed). Reveal one problem at a time. For each problem:

- Give students quiet think time, and ask them to give a signal when they have an answer and a strategy.
- Invite students to share their strategies and record and display their responses for all to see.
- Use the questions in the activity synthesis to involve more students in the conversation before moving to the next problem.

Keep all previous problems and work displayed throughout the talk.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. To support working memory, provide students with access to sticky notes or mini whiteboards.
Supports accessibility for: Memory, Organization

Student Task Statement

Find the value of each expression mentally.

- $32 \cdot (1.5)$

- $32 \cdot (0.15)$
- $3{,}200 \cdot (0.15)$
- $3{,}200 \cdot (0.03)$

Student Response

- 48. Sample reasoning: $32 + \frac{1}{2} \cdot 32 = 32 + 16$.
- 4.8. Sample reasoning: 0.15 is $\frac{1}{10}$ of 1.5, and $\frac{1}{10}$ of 48 is 4.8.
- 480. Sample reasoning: 3,200 is 100 times 32, and 100 times 4.8 is 480.
- 96. Sample reasoning:
    - 0.03 is $\frac{1}{5}$ of 0.15, and $\frac{1}{5}$ of 480 is 96.
    - To get from 0.15 to 0.03 you can multiply by 2 and divide by 10. $480 \cdot 2 \div 10 = 96$.

Activity Synthesis

To involve more students in the conversation, consider asking:

- “Who can restate ________’s reasoning in a different way?”
- “Did anyone use the same strategy but would explain it differently?”
- “Did anyone solve the problem in a different way?”
- “Does anyone want to add on to ________’s strategy?”
- “Do you agree or disagree? Why?”
- “What connections to previous problems do you see?”

The key takeaway to highlight is how we can use the structure of place value and properties of operations to find products involving decimals.

Access for English Language Learners

Speaking: MLR8 Discussion Supports.: Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because . . . ." or "I noticed _____ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Advances: Speaking, Representing

Math Community
After the *Warm-up*, display the Math Community Chart. Remind students that norms are agreements that everyone in the class shares responsibility for, so it is important that everyone understands the intent of each norm and can agree with it. Tell students that today’s *Cool-down* includes a question asking for feedback on the drafted norms. This feedback will help identify which norms the class currently agrees with and which norms need revising or removing.

15 mins

6.2 Bottle Deposits

Activity Narrative

In this activity, students practice writing an equation to represent a proportional relationship in a new context. Students reason about quantities and prices, calculating several values before writing an equation. No table is provided for students to organize their thinking in order to encourage them to look for regularity in repeated reasoning (MP8) and to notice the efficiency of using an equation to express the relationship.

This activity also emphasizes the interpretation of the constant of proportionality in the context, as students may choose to express the relationship as 5 cents per bottle, 0.05 dollars per bottle, or 20 bottles per dollar. Students compare these different approaches during the whole-class discussion.

Monitor for students who:

- Write many calculations, without any organization
- Create a table to organize their work
- Write an equation to record their repeated reasoning
- Use 5 as the constant of proportionality
- Use 0.05 as the constant of proportionality
- Use 20 as the constant of proportionality

Access for English Language Learners

This activity uses the *Compare and Connect* math language routine to advance representing and conversing as students use mathematically precise language in discussion.

Standards

Addressing **7.RP.A.2**

Instructional Routines

- MLR7: Compare and Connect

Launch

Provide access to calculators.

Select work from students with different strategies, such as those described in the *Activity Narrative*, to share later.

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Provide tools to facilitate information processing or computation, enabling students to focus on key mathematical ideas. For example, allow students to use calculators to support their reasoning.
Supports accessibility for: Memory, Conceptual Processing

Student Task Statement

Answer the following questions. Be prepared to explain your reasoning.

In Iowa, collection centers pay 5¢ per bottle that is returned.

1.
   a. How much would 30 bottles be worth?
   b. How much would 250 bottles be worth?
   c. How much would 860 bottles be worth?
2.
   a. How many bottles would it take to earn $100?
   b. How many bottles would it take to earn $2,750?
3. Write an equation that relates the number of bottles to the amount of money received when the bottles are returned. What do your variables represent?

Student Response

1.
   a. $1.50. Sample reasoning: $30 \cdot 5 = 150$ and $150 \div 100 = 1.5$
   b. $12.50. Sample reasoning: $250 \cdot 0.05 = 12.5$
   c. $43. Sample reasoning: $860 \div 20 = 43$
2.
   a. 2,000 bottles. Sample reasoning: $100 \cdot 20 = 2{,}000$
   b. 55,000 bottles. Sample reasoning: $2{,}750 \div 0.05 = 55{,}000$
3. Sample responses:
   a. $y = 5x$, where x represents the number of bottles and y represents the amount of money, in cents
   b. $m = 0.05b$, where b represents the number of bottles and m represents the amount of money, in dollars
   c. $b = 20m$, where b represents the number of bottles and m represents the amount of money, in dollars

Activity Synthesis

The two goals of this discussion are:

- To highlight the efficiency of expressing the relationship as an equation
- To contrast different ways the relationship could be expressed as an equation

Invite previously selected students to share how they found the amounts of money and the numbers of bottles. Use *Compare and Connect* to help students compare, contrast, and connect the different approaches. Here are some questions for discussion:

- "How could we represent this reasoning with an equation? What does the constant of proportionality in this equation represent?"
- "Why do the different approaches lead to the same outcome?"
- "Are there any benefits or drawbacks to one approach compared to another?"

Here are some different strategies for finding the value of some number of bottles, along with one or more ways to record that reasoning with an equation.

sample repeated reasoning	possible equations	meaning of the constant of proportionality
multiply by 5 (and then divide by 100)	$y = 5x$	5 cents per bottle
multiply by 0.05	$y = 0.05x$	0.05 dollars per bottle
divide by 20*	$m = \frac{1}{20}b$ or $b = 20m$	$\frac{1}{20}$ dollar per bottle or 20 bottles per dollar

*If students suggest an equivalent equation that uses division, such as $m = b \div 20$, confirm that that equation is correct and also ask if they can think of a way to express that equation in the form $y = kx$.

The key takeaway is that defining variables and writing an equations can be an efficient way to describe the proportional relationship between two quantities. However, there is more than one way to represent a given situation with an equation. It is important to specify what each variable represents so others can interpret the equation.

6.3 Recycling

15 mins

Activity Narrative

This activity is intended to further develop students' ability to write equations to represent proportional relationships. It involves work with decimals and asks for equations that represent proportional relationships of different pairs of quantities, which increases the challenge of the task. As students identify the constants of proportionality between each pair of quantities to represent the relationships with equations, they are reasoning quantitatively and abstractly (MP2).

Students may solve the first two problems in different ways. Monitor for different solution approaches, such as using computations, using tables, finding the constant of proportionality, and writing equations.

Access for English Language Learners

This activity uses the *Three Reads* math language routine to advance reading and representing as students make sense of what is happening in the text.

Standards

Addressing **7.RP.A.2**

Instructional Routines

- MLR6: Three Reads

Launch

Arrange students in groups of 2. Provide access to calculators.

Use *Three Reads* to support reading comprehension and sense-making about this problem. Display only the problem stem, without revealing the questions.

- For the first read, read the problem aloud then ask, "What is this situation about?" (weight of cans and the amount of money made from recycling). Listen for and clarify any questions about the context.

- After the second read, ask students to list any quantities that can be counted or measured. (number of aluminum cans; total weight of aluminum cans, in kilograms; money earned, in dollars)
- After the third read, reveal the questions: "A family threw away 2.4 kg of aluminum in a month. How many cans did they throw away? What would be the dollar value if they recycled those same cans?" and ask, "What are some ways we might get started on this?" Invite students to name some possible starting points, referencing quantities from the second read. (Calculate the weight of aluminum in 1 can and the amount of money earned from 1 can)

Give 5 minutes of quiet work time followed by partner discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. Represent the same information through different modalities by using tables. If students are unsure where to begin, suggest that they draw a table to help organize the information provided.
Supports accessibility for: Conceptual Processing, Visual-Spatial Processing

Student Task Statement

Aluminum cans can be recycled instead of being thrown in the garbage. The weight of 10 aluminum cans is 0.16 kilograms. The aluminum in 10 cans that are recycled has a value of $0.14.

1. A family threw away 2.4 kg of aluminum cans in a month.
   a. How many cans did they throw away? Explain or show your reasoning.
   b. What would be the dollar value if they recycled those same cans? Explain or show your reasoning.
2. Write an equation to represent the relationship between each pair of quantities:
   a. the number of cans c and their weight w, in kilograms
   b. the number of cans c and their recycled value r, in dollars
   c. the weight of cans w and their recycled value r

Student Response

1.
   a. 150 cans. Sample reasoning: $2.4 \div 0.16 = 15$ and $10 \cdot 15 = 150$. There are 15 groups of 10 cans.
   b. $2.10. Sample reasoning: $(0.14) \cdot 15 = 2.1$
2.
   a. $c = 62.5w$ or equivalent
   b. $r = 0.014c$ or equivalent
   c. $r = 0.875w$ or equivalent

Here is one way to organize the given information and solutions in a table:

number of cans (c)	weight in kilograms (w)	recycled value in dollars (r)
10	0.16	0.14
150	2.4	2.10
1	0.016	0.014
62.5	1	0.875

Building on Student Thinking

If students have trouble getting started, encourage them to create representations of the relationships, like a diagram or a table. If they are still stuck, suggest that they first find the weight and dollar value of 1 can.

Are You Ready for More?

The U.S. Environmental Protection Agency (EPA) estimates that in 2018, the average amount of garbage produced in the United States was 4.9 pounds per person per day. At that rate, how long would it take your family to produce a ton of garbage? (A ton is 2,000 pounds.)

Extension Student Response

Sample responses: A family of two would take about 29 weeks. A family of three would take about $19\frac{1}{2}$ weeks. A family of four would take about $14\frac{1}{2}$ weeks.

Activity Synthesis

Invite several students to share their methods for solving the first two problems, such as using computations, using tables, finding the constant of proportionality, and writing equations. If students did not use equations to solve the first two problems, ask them how they can use the equations they found later in the activity to answer the first two questions.

If time permits, highlight connections between the equations generated, illustrated by the following sequence of equations.

$r = 0.014c$

$r = 0.014(62.5w)$

$r = 0.875w$

Lesson Synthesis

Share with students "Today we wrote equations to represent proportional relationships where no tables were given. We saw that it is important to state what the variables in the equation represent."

Briefly revisit some equations from the activities. For each equation, ask students:

- "In this equation, what did each variable represent?"
- "What did the number mean?"

To help students generalize about equations of proportional relationships, consider asking students:

- “What do all these equations have in common?” (They are of the form $y = kx$, where k is the constant of proportionality. They have two variables and one number. The constant of proportionality is being multiplied by one of the variables.)
- “How did writing an equation help you solve the problems?” (It made it easier to see what number you should multiply or divide by to answer each question.)

6.4 More Recycling

Cool-down

5 mins

Standards

Building On **6.RP.A.2**

Addressing **7.RP.A.2.c**

Launch

Math Community

Before distributing the *Cool-downs*, display the Math Community Chart and these questions:

- “What norm(s) should stay the way they are?”
- “What norm(s) do you think should be made more clear? How?”
- “What norms are missing that you would add?”
- “What norm(s) should be removed?”

Ask students to respond to one or more of the questions after completing the *Cool-down* on the same sheet. Make sure students know they can make suggestions for both student and teacher norms.

After collecting the *Cool-downs*, identify themes from the norms questions. There will be many opportunities throughout the year to revise the classroom norms, so focus on revision suggestions that multiple students made to share in the next exercise. One option is to list one addition, one revision, and one removal that the class has the most agreement about.

Student Task Statement

Glass bottles can be recycled. At one recycling center, 1 ton of clear glass is worth \$25. (1 ton = 2,000 pounds)

1. How many pounds of clear glass is worth \$10?
2. How much money is 40 pounds of clear glass worth?
3. Write an equation to represent the relationship between the weight of clear glass and the value of the glass.

Student Response

1. 800 pounds, because $2{,}000 \div 25 = 80$ and $80 \cdot 10 = 800$
2. \$0.50, because $40 = 80 \cdot 0.50$
3. Sample response: If v represents the value, in dollars, of p pounds of clear glass, then the equation could be either $p = 80v$ or $v = 0.0125p$.

Responding To Student Thinking

Points to Emphasize

If students struggle with writing an equation to represent a proportional relationship, focus on this as opportunities arise over the next several lessons. For example, in the activities referred to here, use the tables to help students see a pattern of constant change.

Grade 7, Unit 2, Lesson 8, Activity 2 More Conversions
Grade 7, Unit 2, Lesson 8, Activity 4 All Kinds of Equations

Lesson 6 Summary

Remember that if there is a proportional relationship between two quantities, their relationship can be represented by an equation of the form $y = kx$. Sometimes writing an equation is the easiest way to solve a problem.

For example, we know that Denali, the highest mountain peak in North America, is 20,310 feet above sea level. How many miles is that? There are 5,280 feet in 1 mile. This relationship can be represented by the equation

$$f = 5{,}280m$$

where f represents a distance measured in feet and m represents the same distance measured in miles. Since we know Denali is 20,310 feet above sea level, we can write

$$20{,}310 = 5{,}280m$$

Solving this equation for m gives $m = \frac{20{,}310}{5{,}280} \approx 3.85$, so we can say that Denali is approximately 3.85 miles above sea level.

Practice Problems

1 Student Task Statement

A car is traveling on a highway at a constant speed, described by the equation $d = 65t$, where d represents the distance, in miles, that the car travels at this speed in t hours.

a. What does the 65 tell us in this situation?

b. How many miles does the car travel in 1.5 hours?

c. How long does it take the car to travel 26 miles at this speed?

Solution

a. The car is traveling 65 miles per hour, which is the constant of proportionality.

b. 97.5 miles

c. $\frac{2}{5}$ of an hour, or 24 minutes

2 Student Task Statement

Elena has some bottles of water that each holds 17 fluid ounces.

a. Write an equation that relates the number of bottles of water (b) to the total volume of water (w) in fluid ounces.

b. How much water is in 51 bottles?

c. How many bottles does it take to hold 51 fluid ounces of water?

Solution

a. $w = 17b$ or $b = \frac{1}{17}w$

b. 867 fluid ounces, because $17 \cdot 51 = 867$

c. 3 bottles, because $51 \div 17 = 3$

3 from Unit 2, Lesson 5

Student Task Statement

There are about 1.61 kilometers in 1 mile. Use x to represent a distance measured in kilometers and y to represent the same distance measured in miles. Write two equations that relate a distance measured in kilometers and the same distance measured in miles.

Solution

$x = 1.61y$ and $y = \frac{1}{1.61}x$ or about $y = 0.62x$

4 from Unit 2, Lesson 2

Student Task Statement

In Canadian coins, 16 quarters is equal in value to 2 toonies.

number of quarters	number of toonies
1	
16	2
20	
24	

a. Complete the table.

b. What does the value in the right column that is next to 1 in the left column mean in this situation?

Solution

a.

number of quarters	number of toonies
1	$\frac{1}{8}$
16	2
20	2.5
24	3

b. $\frac{1}{8}$ Sample response: 1 Canadian quarter has the same value as $\frac{1}{8}$ of a toonie.

5 from Unit 2, Lesson 2

Student Task Statement

Each table represents a proportional relationship. For each table:

a. Fill in the unknown values.

b. Draw a circle around the constant of proportionality.

x	y
2	10
	15
7	
1	

a	b
12	3
20	
	10
1	

m	n
5	3
10	
	18
1	

Solution

$\cdot \frac{3}{2}$

x	y
2	10
3	15
7	35
1	5

$\cdot 2$

a	b
12	3
20	5
40	10
1	$\frac{1}{4}$

$\cdot 3$

m	n
5	3
10	6
30	18
1	$\frac{3}{5}$

6 from Unit 1, Lesson 4

Student Task Statement

Describe some things you could observe about two polygons that would help you decide that they were not scaled copies.

Solution

Sample response: I could find an angle measure in one that is not an angle measure of the other. I could find that a different scale factor would have to be used on one part of the pair than on another.