Illustrative Mathematics®
LEARN MATH FOR LIFE

GRADE 8

Teacher Guide

UNIT 3

Certified by Illustrative Mathematics®

MS_vIV

20231010

Table of Contents

Unit 3: Linear Relationships

Unit Narrative

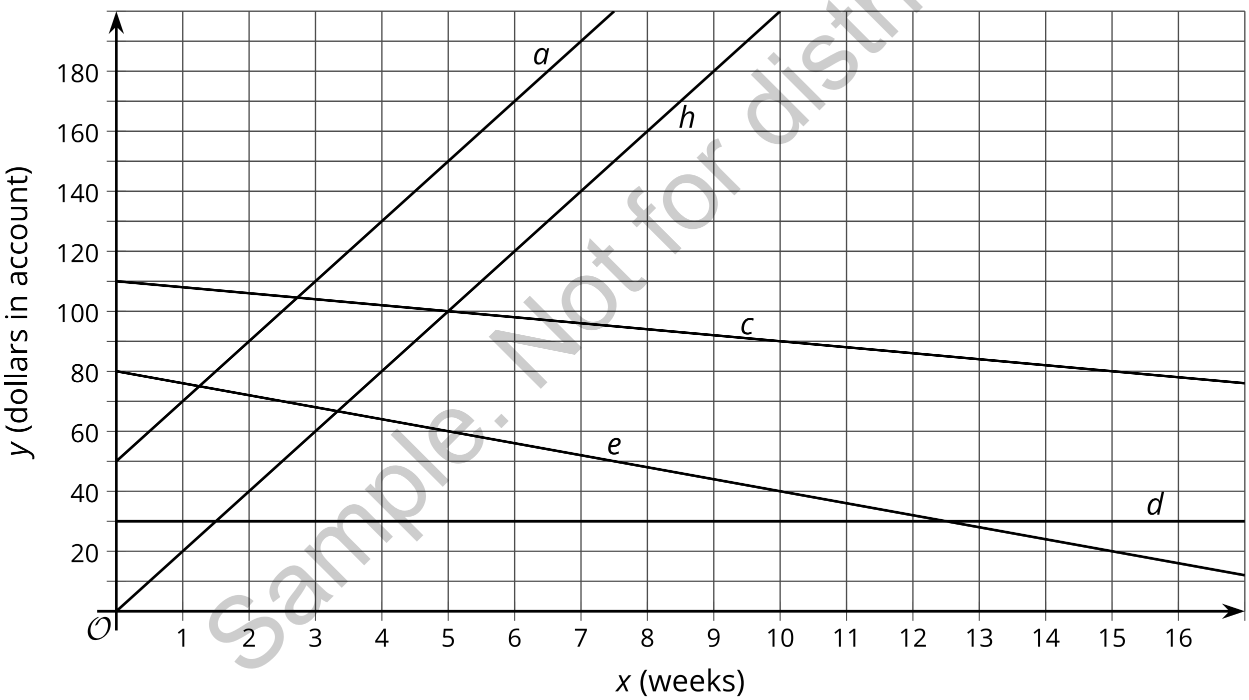
This unit introduces students to nonproportional linear relationships by building on earlier work with rates and proportional relationships from grade 7, and on earlier grade 8 work around similarity and slope.

The unit begins by revisiting different representations of proportional relationships. Students create graphs, tables, and equations in order to interpret the constant of proportionality in a context. They see the constant of proportionality between two variables as the **rate of change** of one variable with respect to the other.

Next, students analyze a relationship that is linear but not proportional. In this context, students see that the rate of change has a numerical value that is the same as the slope of the line that represents the relationship. Students also view the graph of a line in the coordinate plane as the vertical translation of a proportional relationship.

In the following section, students are introduced to lines with non-positive slopes and vertical intercepts. They consider situations represented by linear relationships with negative rates of change and establish a way to compute the slope of a line from any two distinct points on the line. Students also write equations of horizontal and vertical lines.

In the last section, students consider what it means for a pair of values to be a solution to an equation and the correspondence between coordinates of points on a graph and solutions of an equation.



Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes such as representing, generalizing, and explaining. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Represent

- Situations involving proportional relationships (Lesson 1).

- Constants of proportionality in different ways (Lesson 3).
- Slope using expressions (Lesson 10).
- Linear relationships using graphs, tables, equations, and verbal descriptions (Lesson 5).
- Situations using negative slopes and slopes of zero (Lesson 9).
- Situations by graphing lines and writing equations (Lesson 13).
- Situations involving linear relationships (Lesson 15).

Generalize

- Categories for graphs (Lesson 2).
- About equations and linear relationships (Lesson 7).
- In order to make predictions about the slope of lines (Lesson 10).

Explain

- How to graph proportional relationships (Lesson 3).
- How to use a graph to determine information about a linear situation (Lessons 5 and 6).
- How to graph linear relationships (Lesson 10 and 11).
- How slope relates to changes in a situation (Lesson 11).

In addition, students are expected to describe observations about the equation of a translated line. Students will also have opportunities to use language to interpret situations involving proportional relationships, interpret graphs using different scales, interpret slopes and intercepts of linear graphs, justify reasoning about linear relationships, justify correspondences between different representations, and justify which equations correspond to graphs of horizontal and vertical lines.

The table shows lessons where new terminology is first introduced, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms from the glossary appear bolded. Teachers should continue to support students' use of a new term in the lessons that follow the one in which it was first introduced.

lesson	new terminology	
	receptive	productive
8.3.1	represent scale label	constant of proportionality
8.3.2	equation	
8.3.3	**rate of change**	equation
8.3.5	**linear relationship** constant rate **rate of change**	slope
8.3.6	**vertical intercept** y-intercept	
8.3.7	initial (value or amount)	constant rate
8.3.8	relate	
8.3.9	horizontal intercept x-intercept	
8.3.10		**rate of change** **vertical intercept** y-intercept
8.3.12	constraint	horizontal line vertical line
8.3.13	**solution to an equation with two variables** variable combination set of solutions	

Materials To Gather

- Colored pencils
- Geometry toolkits
  For grade 6: Tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

  For grades 7 and 8: Everything listed for grade 6, plus a ruler and a protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

  Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially available "patty paper" is 5 inches by 5 inches and is ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6–8, they are listed as separate Required Material.
- Graduated cylinders
- Graph paper
- Math Community Chart
- Straightedges
  A rigid edge that can be used for drawing line segments. Sometimes a ruler is okay to use as a straightedge, but sometimes it is preferable to use an unruled straightedge, like a blank index card.
- Teacher’s collection of objects
- Tools for creating a visual display
  Any way for students to create work that can be easily displayed to the class. Examples: Chart paper and markers, whiteboard space and markers, shared online drawing tool, access to a document camera.
- Water

Materials Needed

Lesson	Materials to Gather	Materials to Copy
Lesson 1		
Lesson 2	• Straightedges: Lesson • Straightedges: Activity 3	• Proportional Relationships Cards (1 copy for every 4 students): Activity 2
Lesson 3		• Graphing Proportional Relationships Cards (1 copy for every 2 students): Activity 2
Lesson 4	• Math Community Chart: Activity 2 • Tools for creating a visual display: Activity 2	
Lesson 5	• Graph paper: Activity 2 • Straightedges: Activity 2, Activity 3	
Lesson 6		• Slopes, Vertical Intercepts, and Graphs Cards (1 copy for every 2 students): Activity 3
Lesson 7	• Graduated cylinders: Activity 2 • Straightedges: Activity 2 • Teacher's collection of objects: Activity 2 • Water: Activity 2	

Lesson 8	• Geometry toolkits: Activity 1	• Translating a Line Cards (1 copy for every 2 students): Activity 3
Lesson 9	• Straightedges: Activity 2	
Lesson 10		
Lesson 11	• Straightedges: Activity 2, Activity 3	• Making Designs Cards (1 copy for every 2 students): Activity 2
Lesson 12	• Straightedges: Activity 2, Activity 3	
Lesson 13	• Colored pencils: Activity 3 • Graph paper: Activity 3 • Straightedges: Activity 3	
Lesson 14		• I'll Take an X Please Cards (1 copy for every 2 students): Activity 3
Lesson 15		

Check Your Readiness (A)

Teacher Instructions

Students will need a straightedge for this assessment.

1

Standards

Addressing **7.RP.A.2.a**

Narrative

The content assessed in this problem is first encountered in Lesson 3: Representing Proportional Relationships.

In this unit, students review previous work with proportional relationships as a lead-in to linear equations.

If most students struggle with this item, plan to use this problem or a similar one as an additional warm-up activity. During Lessons 1 and 2 plan to emphasize multiple ways to identify whether a relationship is proportional, such as finding a constant of proportionality using a table of values and using coordinates of points on the graph.

Student Task Statement

Select **all** the tables that could represent proportional relationships.

A.

x	y
2	3
5	7.5
10	15

B.

x	y
0	0
3	7
6	14

C.

x	y
0	2
2	4
4	6

Solution

A, B

2

Standards

Addressing **6.RP.A.3**

Narrative

The content assessed in this problem is first encountered in Lesson 1: Understanding Proportional Relationships .

Students move from scale factors to proportional relationships in preparation for linear relationships. If most students struggle with this item, before beginning Lesson 1, do Grade 6, Unit 3, Lesson 7, Activity 3 to practice the concept of generating equivalent ratios.

Student Task Statement

To mix a particular shade of purple paint, red paint and blue paint are mixed in the ratio $5 : 3$. To make 20 gallons of this shade of purple paint, how many gallons of red and blue paint should be used?

Solution

12.5 gallons red, 7.5 gallons blue

3

Standards

Addressing **7.RP.A.2.c**

Narrative

The content assessed in this problem is first encountered in Lesson 1: Understanding Proportional Relationships .

In grade 7, students wrote equations to describe proportional relationships. The graphs of these equations are lines through the origin. In this unit, students will write equations for proportional relationships as well as other linear relationships.

If most students struggle with this item, plan to do Lesson 1, Activity 3. During the *Activity Synthesis* spend some extra time sharing student equations and making connections to the tick-mark diagram.

Student Task Statement

At one gas station, gas costs \$2.75 per gallon. Write an equation that relates the total cost, C, to the number of gallons of gas purchased, g.

Solution

$C = 2.75g$ (or equivalent)

4

Standards

Addressing **8.EE.B**

Narrative

The content assessed in this problem is first encountered in Lesson 1: Understanding Proportional Relationships .

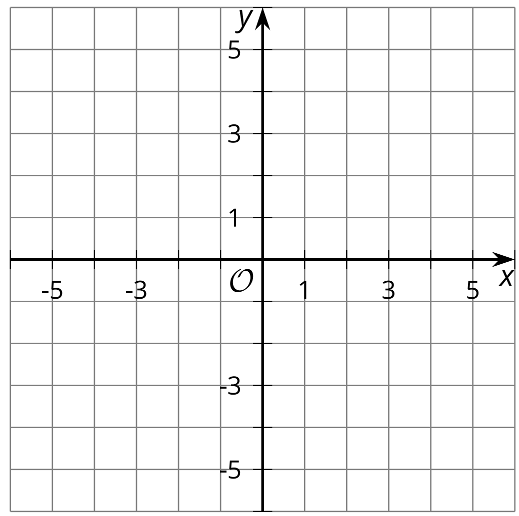
Students will need to be familiar with the coordinate plane to graph lines.

If most students struggle with this item, plan to pause students as they are working on Lesson 1, Activity 2, Question 4 to ensure that they can plot and mark points once they have identified the bug's location at the given time. If students need additional practice, refer to Grade 6, Unit 7, Lesson 11, Activity 1.

Student Task Statement

a. Plot and label 3 different points with x-coordinate 3.

b. Plot and label 3 different points with y-coordinate -5.



Solution

a. Any 3 points with x-coordinate 3 plotted and labeled. Sample responses: $(3, 0)$, $(3, -2)$, $(3, 4)$

b. Any 3 points with y-coordinate -5 plotted and labeled. Sample responses: $(0, -5)$, $(3, -5)$, $(-4, -5)$

5

Standards

Addressing **8.G.A.1, 8.G.A.1.c**

Narrative

The content assessed in this problem is first encountered in Lesson 8: Translating to $y = mx + b$.

In this unit, students are presented with various forms of linear equations and various ways of thinking about those forms. One interpretation is to consider nonproportional linear equations as vertical translations of the line $y = mx$.

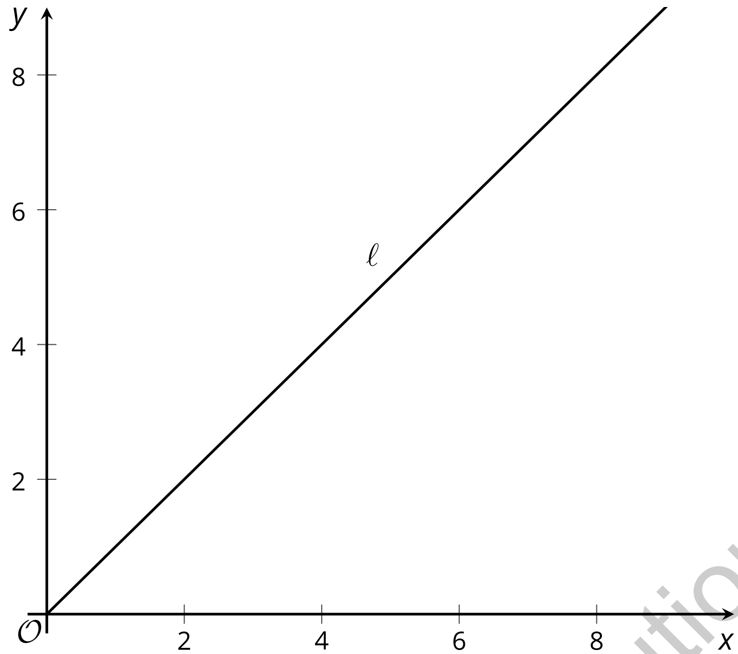
If most students struggle with this item, plan to use Activity 1 in Lesson 8 to review translations. If students need additional practice recalling translations, especially translations of lines, refer to Unit 1, Lesson 9, Activity 2.

Student Task Statement

On the coordinate plane, draw:

a. A line m that is a translation of line ℓ.

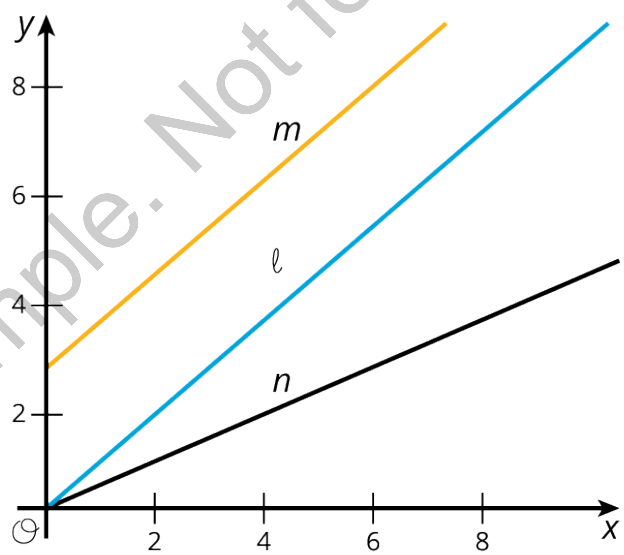
b. A line n that is a rotation of line ℓ, using the origin as the center of rotation.



Solution

a. Any line parallel to ℓ
b. Any line through the origin

Sample response:



6

Standards

Addressing **7.EE.B.3**

Narrative

The content assessed in this problem is first encountered in Lesson 5: Introduction to Linear Relationships.

Another interpretation of a linear equation is to start with a given amount and thereafter increase the amount at a constant rate. Students are asked to engage in repeated reasoning in anticipation of this way of thinking.

If most students struggle with this item, plan to review it with students before beginning Lesson 6, Activity 2 and amplify vocabulary such as "constant of proportionality" and "rate of change" starting in Lesson 3 Activity 1.

Student Task Statement

A store sells ice cream with assorted toppings. They charge \$3.00 for an ice cream, plus \$0.50 per ounce of toppings.

a. How much does an ice cream cost with 4 ounces of toppings?

b. How much does an ice cream cost with 11 ounces of toppings?

c. If Elena's ice cream costs \$1.50 more than Jada's ice cream, how much more did Elena's toppings weigh?

Solution

a. \$5.00
b. \$8.50
c. 3 ounces

Check Your Readiness (B)

Teacher Instructions

Students will need a straightedge for this assessment.

1

Standards

Addressing **7.RP.A.2.a**

Narrative

The content assessed in this problem is first encountered in Lesson 3: Representing Proportional Relationships.

In this unit, students review previous work with proportional relationships as a lead-in to linear equations.

If most students struggle with this item, plan to use this problem or a similar one as an additional warm-up activity. During Lessons 1 and 2 plan to emphasize multiple ways to identify whether a relationship is proportional, such as finding a constant of proportionality using a table of values and using coordinates of points on the graph.

Student Task Statement

There is a proportional relationship between x and y. Complete the table with the missing values.

x	y
4.5	3
18	12
	30
15	

Solution

x	y
4.5	3
18	12
45	30
15	10

2

Standards

Addressing **7.RP.A.3**

Narrative

The content assessed in this problem is first encountered in Lesson 1: Understanding Proportional Relationships .

Students move from scale factors to proportional relationships in preparation for linear relationships. If most students struggle with this item, before beginning Lesson 1, do Grade 6, Unit 3, Lesson 7, Activity 3 to practice the concept of generating equivalent ratios.

Student Task Statement

To mix a particular shade of pink paint, white paint and red paint are mixed in the ratio $4:3$. If 12 gallons of red paint are mixed with some white paint to make this same shade of pink, how many total gallons of pink paint result?

A. 48

B. 36

C. 28

D. 16

Solution

C

3

Standards

Addressing **7.RP.A.2.c**

Narrative

The content assessed in this problem is first encountered in Lesson 1: Understanding Proportional Relationships .

In grade 7, students wrote equations to describe proportional relationships. The graphs of these equations were lines through the origin. In this unit, students will write equations for proportional relationships as well as other linear relationships.

If most students struggle with this item, plan to do Lesson 1 Activity 3. During the *Activity Synthesis* spend some extra time sharing student equations and making connections to the tick-mark diagram.

Student Task Statement

At a grocery store, 2 gallons of milk cost $7.20, and 5 gallons of milk cost $18. Which equation relates the total cost, t, to the gallons of milk purchased, m?

A. $m = 7.2t$

B. $t = 7.2m$

C. $m = 3.6t$

D. $t = 3.6m$

Solution

D

4

Standards

Addressing **8.EE.B**

Narrative

The content assessed in this problem is first encountered in Lesson 1: Understanding Proportional Relationships .

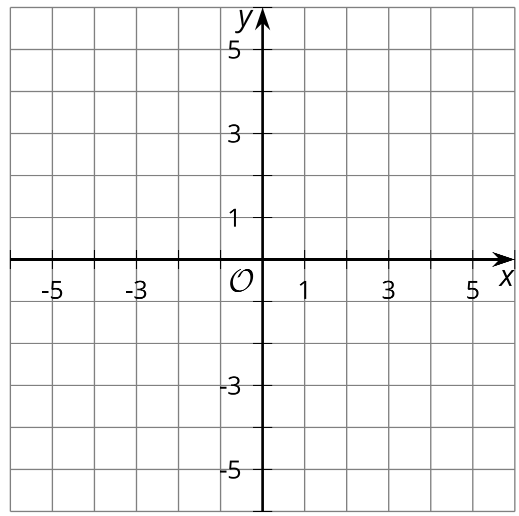
Students will need to be familiar with the coordinate plane to graph lines.

If most students struggle with this item, plan to pause students as they are working on Lesson 1, Activity 2, Question 4 to ensure that they can plot and mark points once they have identified the bug's location at the given time. If students need additional practice, refer to Grade 6, Unit 7, Lesson 11, Activity 1.

Student Task Statement

a. Plot and label 3 different points with y-coordinate -4.

b. Plot and label 3 different points with x-coordinate 2.



Solution

a. Any 3 points with y-coordinate -4 plotted and labeled. Sample responses: $(0, \text{-}4)$, $(\text{-}2, \text{-}4)$, $(1, \text{-}4)$

b. Any 3 points with x-coordinate 2 plotted and labeled. Sample responses: $(2, 0)$, $(2, \text{-}3)$, $(2, 5)$

5

Standards

Addressing **8.G.A.1, 8.G.A.1.c**

Narrative

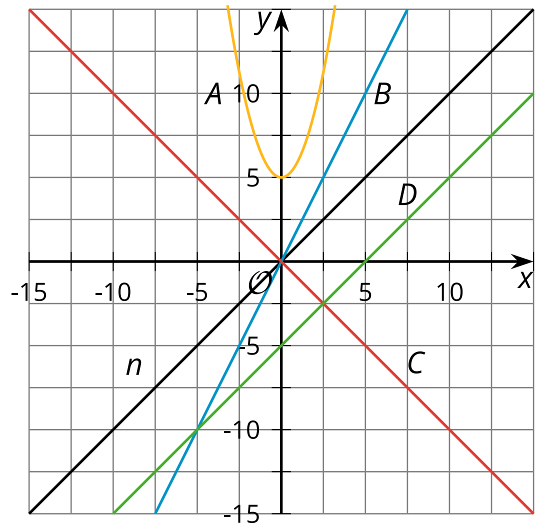
The content assessed in this problem is first encountered in Lesson 8: Translating to $y = mx + b$.

In this unit, students are presented with various forms of linear equations and various ways of thinking about those forms. One interpretation is to consider nonproportional linear equations as vertical translations of the line $y = mx$.

If most students struggle with this item, plan to use Activity 1 in Lesson 8 to review translations. If students need additional practice recalling translations, especially translations of lines, refer to Unit 1, Lesson 9, Activity 2.

Student Task Statement

Which graph is a translation of line n?



Solution

Line D

6

Standards

Addressing **7.EE.B.3**

Narrative

The content assessed in this problem is first encountered in Lesson 5: Introduction to Linear Relationships.

Another interpretation of a linear equation is to start with a given amount and thereafter increase the amount at a constant rate. Students are asked to engage in repeated reasoning in anticipation of this way of thinking.

If most students struggle with this item, plan to review it with students before beginning Lesson 6, Activity 2 and amplify vocabulary such as "constant of proportionality" and "rate of change" starting in Lesson 3, Activity 1.

Student Task Statement

At a sandwich shop, any sandwich costs $4.50, plus $0.25 for each extra topping.

a. How much does a sandwich cost with 3 extra toppings?

b. How much does a sandwich cost with 13 extra toppings?

c. If Andre's sandwich cost $2.00 more than Clare's sandwich, how many more toppings did Andre add to his sandwich?

Solution

a. $5.25

b. $7.75

c. 8 toppings

End-of-Unit Assessment (A)

1

Standards

Addressing **8.EE.B**

Narrative

Students who select B, D, or E may be reversing the x- and y-coordinates. Students who select C may be misled by the fact that the coefficients of x and y are in a 1:2 ratio. Students who do not select A or F may not know that the graph of a line is the set of all solutions to the corresponding equation.

Student Task Statement

Select **all** the points that are on the graph of the line $2x + 4y = 20$.

A. $(0, 5)$

B. $(0, 10)$

C. $(1, 2)$

D. $(4, 2)$

E. $(5, 0)$

F. $(10, 0)$

Solution

A, F

2

Standards

Addressing **8.EE.B, 8.F.B.4**

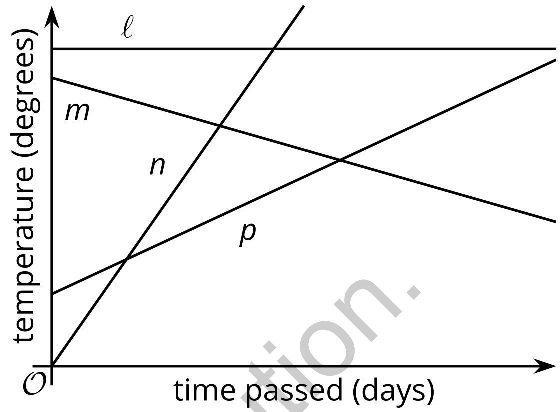
Narrative

Students identify descriptions that could match a given graph. The descriptions indicate the rate of change or slope of the linear graph, and include positive, zero, and negative slope. They also compare two positive slopes.

Students who select A may not understand that horizontal lines have a slope of 0, indicating that the temperature is neither increasing nor decreasing over time. Students who do not select B may not understand the interpretation of a negative slope as the temperature decreasing over time. Students who select C may not understand the connection between the vertical intercept and an initial amount. Students who select D may not understand that a greater slope means a faster rise in temperature.

Student Task Statement

For two weeks, the highest temperature each day was recorded in four different cities, represented by the lines ℓ, m, n, and p. Which statement is true?



A. The high temperature in the city represented by line ℓ increased as time passed.

B. The high temperature in the city represented by line m decreased steadily.

C. Initially, the high temperature was warmer in the city represented by line p than in the city represented by line m.

D. The high temperature in the city represented by line p increased faster than the high temperature in the city represented by line n.

Solution

B

3

Standards

Addressing 8.EE.B.5

Narrative

Students interpret proportional relationships from given lines. They must identify the slope of the lines as the unit rate and also quantitatively compare these unit rates in the absence of a given scale on the axes.

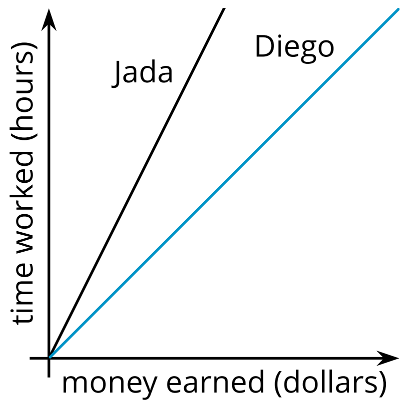
Students who select C instead of A may be associating the slope with the gallons of water, but the axes do not reflect this. Students who select B have misinterpreted the meaning of the slope of each line. Students who select

D may not understand the horizontal line represents no change over time.

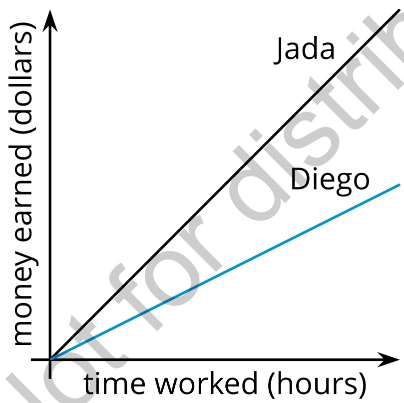
Student Task Statement

Jada earns twice as much money per hour as Diego. Which graph best represents this scenario?

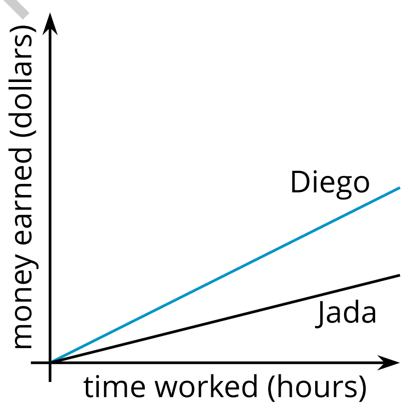
A.



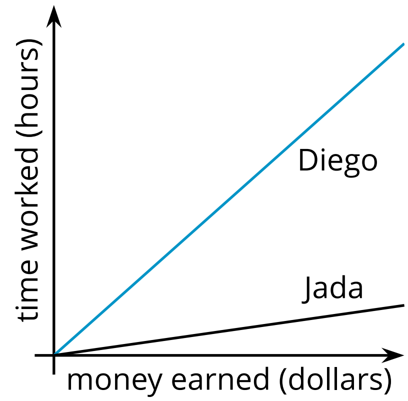
B.



C.



D.



Solution

B

4

Standards

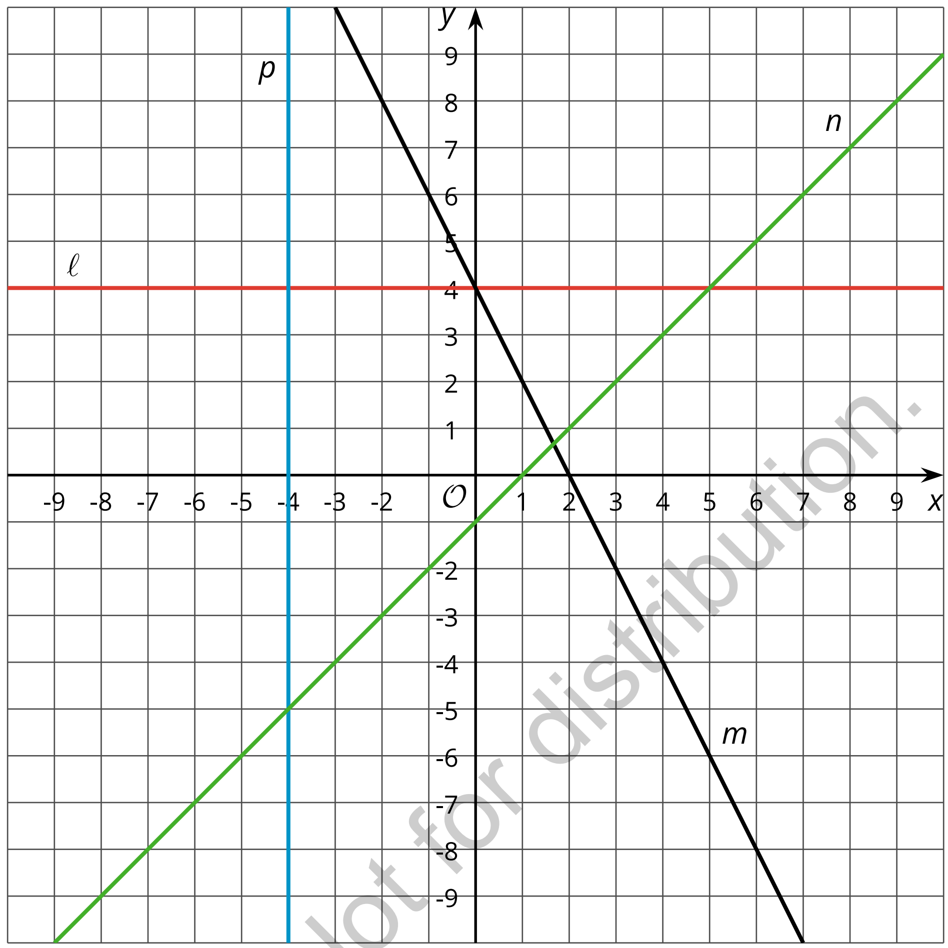
Addressing **8.EE.B**

Narrative

Students write equations for four lines given their graphs. One line is vertical, one is horizontal, one has positive slope, and one has negative slope.

Student Task Statement

Write an equation for each line.



Solution

line ℓ: $y = 4$ (or equivalent), line m: $y = 4 - 2x$ (or equivalent), line n: $y = x - 1$ (or equivalent), line p: $x = $ -4

5

Standards

Addressing **8.EE.B.5**

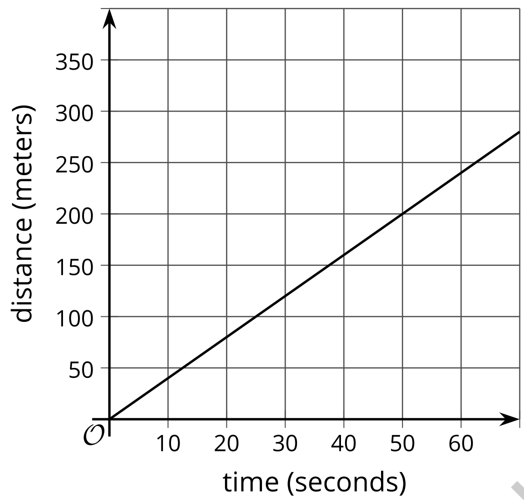
Narrative

Students compare the pace of three different runners. The proportional relationship between time and distance is represented in three different ways. There is more than one way to do this problem correctly. For example, students could determine how long it takes each runner to run 5 miles to determine the fastest runner, or could determine each runner's speed in miles per minute or miles per hour.

Student Task Statement

Three runners are training for a race. One day, they all run a lap around a track, each at their own constant speed.

- The graph shows the distance in meters that Runner #1 runs with respect to the time in seconds.



- Runner #3's information is in the table:

time (seconds)	distance (meters)
9	45
11	55
25	125
45	225
60	300

- The equation that relates Runner #2's distance (in meters) with time (in minutes) is $d = 6.5t$.

Which of the three runners runs the fastest? Explain your reasoning.

Solution

Runner #2 runs the fastest. Sample reasoning: Using the points $(0, 0)$ and $(50, 200)$ from Runner #1's graph, the slope is 4, showing that they run 4 meters every second. Runner #2's equation shows that they run 6.5 meters every second. Within the table, the unit rate for Runner #3 is 5 meters per second because $45 \div 9 = 5$.

Since Runner #2 travels farther every second, Runner #2 is the fastest.

Minimal Tier 1 response:

- Work is complete and correct.
- Sample: Runner #1 goes at 4 meters per second, Runner #2 goes at 6.5 meters per second, and Runner #3 goes at 5 meters per second. Runner #2 is the fastest because they travel the greatest distance in 1 second.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.
- Sample errors: Work contains correct unit rates for all three runners but concludes that runner #1 or #3 is the fastest or does not name a fastest runner; one unit rate is incorrect (possibly with an incorrect fastest runner identified as a consequence); insufficient explanation of work.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: Two or more incorrect unit rates; the correct runner is identified but with no justification;

response to the question is not based on unit rates or on similar methods, such as calculating which runner has gone the farthest after 10 miles.

6

Standards

Addressing **8.EE.B**

Narrative

A linear equation is described in terms of a constraint on the total cost to purchase a combination of notebooks and pencils with a given budget. The constraint gives an equation that students produce. Students then graph the solutions to the equation.

Student Task Statement

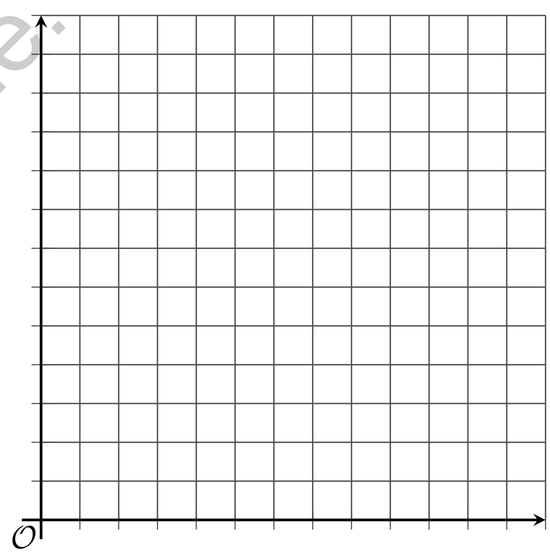
A store is selling notebooks for \$1.00 and pencils for \$0.25. Jada has \$10.00 to spend on school supplies.

a. Complete the table showing three ways Jada can spend all \$10.00 on notebooks and pencils.

number of notebooks (n)	number of pencils (p)
6	
	4
2	

b. Write an equation describing the number of notebooks n, and pencils p that Jada can buy for \$10.00.

c. Draw a graph of the solutions to your equation.



d. Notebooks still cost \$1.00 and pencils still cost \$0.25, but now Jada has \$15 to spend on supplies. How would a graph representing this new situation be the same and different from the graph representing when Jada had \$10 to spend?

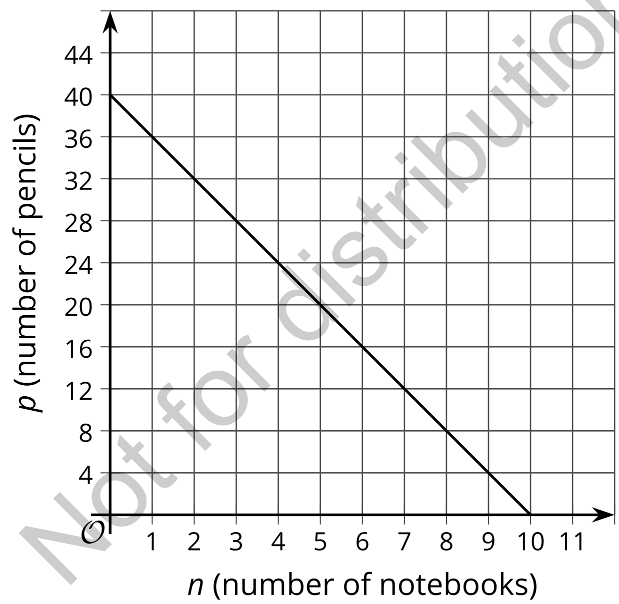
Solution

a.

number of notebooks (n)	number of pencils (p)
6	16
9	4
2	32

b. $n + 0.25p = 10$ (or equivalent)

c. Sample graph:



d. Sample responses: The two graphs would have the same slope but different vertical intercepts. The graph of the new line would be parallel to the graph of the original line.

Minimal Tier 1 response:

- Work is complete and correct.
- The graph may be a continuous line, or it may consist only of points representing whole numbers of notebooks and pencils.
- The graph may have the axes and labels reversed, where the number of pencils is on the horizontal axis, and the number of notebooks is on the vertical axis.
- Sample: See solution on graph.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.
- Sample errors: Graph contains only the three points from the table; scale chosen does not allow for all relevant data to be displayed; reasonable work in part 1 but results in an incorrect equation.
- Acceptable errors: Equation and graph are correct based on an incorrect proportional relationship in the

table; graph is correct based on an incorrect equation.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: Work does not factor in the $10 constraint; equation is not linear or does not make sense in this situation; incorrect answers in 2 or more representations.

Tier 4 response:

- Work includes major errors or omissions that demonstrate a lack of conceptual understanding and mastery.
- Sample errors: Does not come up with an equation or graph; omission of or incorrect work in three or more problem parts.

7

Standards

Addressing **8.EE.B.6**

Narrative

Students analyze the descriptions of two nonproportional linear relationships and use what they know about slopes and vertical intercepts to determine whether a given statement is true.

Student Task Statement

Han has a music playlist and each day he adds more songs to his list. The equation $y = 4x + 20$ describes Han's playlist, where x is the number of days, and y is the total number of songs.

Tyler also has a music playlist. A graph representing the number of songs on Tyler's playlist each day has a vertical intercept at $(0, 12)$ and is parallel to the graph describing Han's music playlist.

Tyler says that in 3 days he will have more songs on his playlist than Han will have on his. Do you agree or disagree with Tyler? Explain your reasoning.

Solution

I disagree with Tyler. Sample reasoning: The equation that represents Han's music playlist has a slope of 4 and a y-intercept of 20, which means that Han starts out with 20 songs on his playlist and adds 4 songs each day. The graph representing Tyler's playlist has a y-intercept of 12 and is parallel to a graph of Han's playlist, which means that Tyler starts out with 12 songs on his playlist and also adds 4 songs each day. Both Han and Tyler add the same amount of songs each day, so the rate of change for both situations is the same. However, since Han started out with more songs, Tyler will never catch up or have more songs on his playlist than Han.

Minimal Tier 1 response:

- Work is complete and correct, with complete explanation or justification.
- An explanation that compares the slopes and vertical intercepts without providing numbers, or an explanation that includes a graph or equation is also acceptable.
- Sample: See solution above.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.
- Sample errors: Work does not explicitly state agreement or disagreement with Han but can be inferred from the reasoning; work in the form of a table, graph, or completed calculations that shows Han is incorrect, but the explanation is incomplete.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: Work does not include any explanation; work shows that Han’s statement is correct.

End-of-Unit Assessment (B)

1

Standards

Addressing **8.EE.B**

Narrative

Students who select B or D may be reversing the x- and y-coordinates. Students who select C may be misled by the fact that the slope is -2 and the y-intercept is 8. Students who do not select A, E, or F may not know that the graph of a line is the set of all solutions to the corresponding equation.

Student Task Statement

Select **all** the points that are on the graph of the line $y = -2x + 8$.

A. $(0, 8)$

B. $(-4, 6)$

C. $(-2, 8)$

D. $(10, -1)$

E. $(4, 0)$

F. $(3, 2)$

Solution

A, E, F

2

Standards

Addressing **8.EE.B, 8.F.B.4**

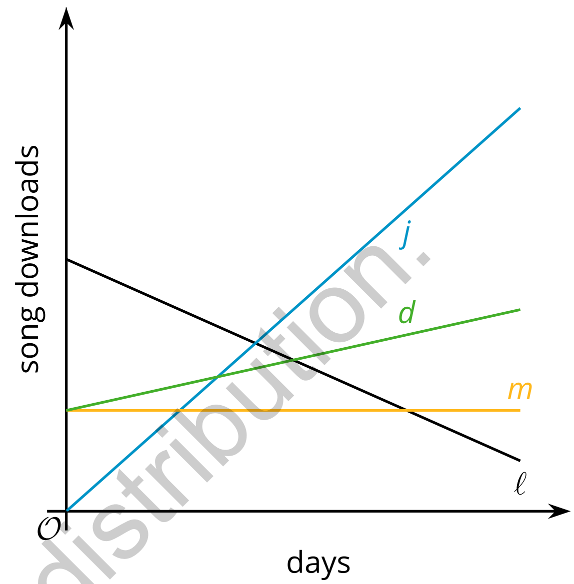
Narrative

Students identify descriptions that could match a given graph. The descriptions indicate the rate of change or slope of the linear graph, and include positive, zero, and negative slope. They also compare two positive slopes.

Students who select A may not understand the interpretation of a positive slope as the number of song downloads increasing over time. Students who select B may not understand the connection between the vertical intercept and an initial amount. Students who do not select C may not understand that horizontal lines have a slope of 0, indicating that the number of song downloads is neither increasing nor decreasing over time. Students who select D may not understand the interpretation of a negative slope as the number of song downloads decreasing over time.

Student Task Statement

For one month, a music service tracked the number of downloads each day for 4 songs, represented by the lines ℓ, j, m, and d. Which statement is true?



A. The number of song downloads represented by lines j and d both decreased over the month.

B. Initially, the number of song downloads represented by lines ℓ and d was the same.

C. The number of song downloads represented by line m remained constant throughout the month.

D. The number of song downloads represented by line ℓ steadily increased over the month.

Solution

C

3

Standards

Addressing **8.EE.B.5**

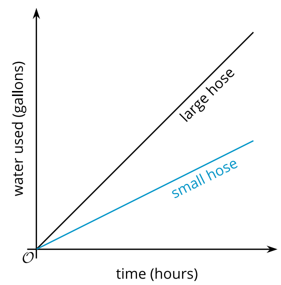
Narrative

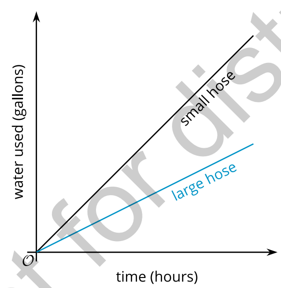
Students interpret proportional relationships from given lines. They must identify the slope of the lines as the unit rate but also quantitatively compare these unit rates in the absence of a given scale on the axes.

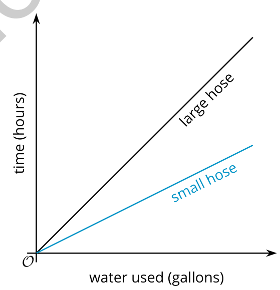
Students who select C instead of A may be associating the slope with the gallons of water, but the axes do not reflect this. Students who select B may be misinterpreting the meaning of the slope of each line. Students who select D may not understand the horizontal line represents no change over time.

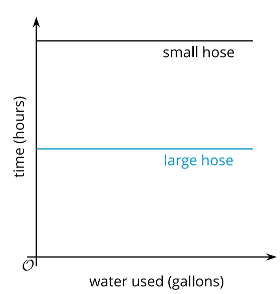
Student Task Statement

A pool holds 19,900 gallons of water. The pool can be filled using a large hose or a small hose. On an average day, it takes 5 hours to fill the pool with the large hose and 12 hours with the small hose. Which graph best represents this scenario?

A. 

B. 

C. 

D. 

Solution

A

4

Standards

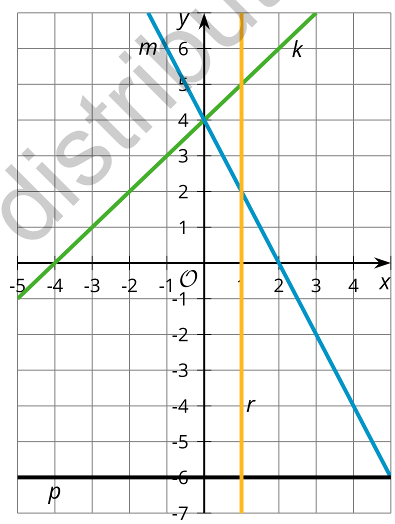
Addressing 8.EE.B

Narrative

Students write equations for four lines given their graphs. One line is vertical, one is horizontal, one has positive slope, and one has negative slope.

Student Task Statement

Write an equation for each line.



Solution

line k: $y = x + 4$ (or equivalent), line m: $y = -2x + 4$ (or equivalent), line p: $y = -6$ (or equivalent), line r: $x = 1$

5

Standards

Addressing 8.EE.B.5

Narrative

Students compare the speed of three different airplanes where the proportional relationship between distance and time is represented in three different ways.

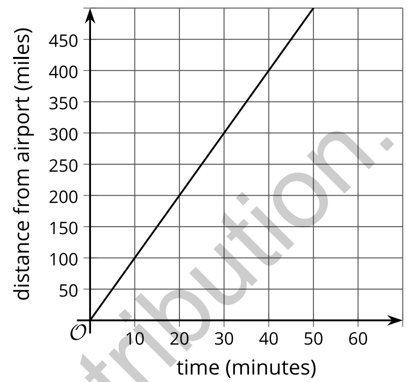
Student Task Statement

Three different airplanes take off from an airport, and each maintains a constant speed until they near their destination.

- The equation $d = 9.5t$ represents the distance of the first airplane (in miles), d, from the airport after t minutes.
- The second airplane's information is in the table:

time (minutes)	distance from airport (miles)
2	16
10	80
35	280
60	480

- The graph shows the distance from the airport (in miles) of the third plane with respect to the time in minutes.



Which airplane is flying the slowest? Explain how you know.

Solution

The second airplane is flying the slowest. Sample reasoning: The equation for the first airplane shows that it is flying at a speed of 9.5 miles per minute. Within the table, the unit rate for the second airplane is 8 miles per minute, because $16 \div 2 = 8$. Using the points $(0, 0)$ and $(30, 300)$ from the graph for the third airplane, the slope is 10, showing that it is flying at a speed of 10 miles per minute.

Since the second airplane only travels 8 miles in 1 minute, it is flying the slowest.

Minimal Tier 1 response:

- Work is complete and correct.
- Sample: The first airplane is flying at a speed of 9.5 miles per minute. The second airplane is flying at a speed of 8 miles per minute. The third airplane is flying at a speed of 10 miles per minute. The second airplane is flying the slowest because it travels the shortest distance in one minute.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.
- Sample errors: Work contains correct unit rates for all three airplanes but concludes that the first or third airplane is flying the slowest or does not name the slowest airplane; one unit rate is incorrect (possibly with an incorrect slowest airplane identified as a consequence); insufficient explanation of work.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: Two or more incorrect unit rates; the correct airplane is identified but with no justification; response to the question is not based on unit rates or on similar methods.

6

Standards

Addressing **8.EE.B.6**

Narrative

Students analyze the descriptions of two nonproportional linear relationships and use what they know about slopes and vertical intercepts to determine whether a given statement is true.

Student Task Statement

A sandwich store delivers sandwiches for a fee. The total cost to order from this store can be described by the equation $y = 10 + 4.5x$ where x is the number of sandwiches ordered and y is the total cost including the delivery fee.

A second store also delivers sandwiches for a fee. A graph representing the number of cost to order sandwiches from this store has a vertical intercept at $(0, 5)$ and is parallel to a graph describing the first sandwich store.

Jada wants to order 6 sandwiches and says that it would cost less to order from the second sandwich store. Do you agree or disagree with Jada? Explain your reasoning.

Solution

I agree with Jada. Sample reasoning: The equation that represents the first store has a slope of 4.5 and a y-intercept of 10, which means that sandwiches cost $4.50 each and there is a $10 delivery fee. The graph representing the second store has a y-intercept of 5 and is parallel to a graph of the first store, which means that the second store also charges $4.50 per sandwich but there is only a $5 delivery fee. The cost per sandwich is the same but the second store has a lower delivery fee, making it less expensive for 6 sandwiches.

Minimal Tier 1 response:

- Work is complete and correct, with complete explanation or justification.
- An explanation that compares the slopes and vertical intercepts without providing numbers, or an explanation that includes a graph or equation is also acceptable.
- Sample: See solution above.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.
- Sample errors: Work does not explicitly state agreement or disagreement with Jada but can be inferred from the reasoning; work in the form of a table, graph, or calculations are done that show Jada is correct but the explanation is incomplete.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: Work does not include any explanation; work shows that Jada's statement is incorrect.

7

Standards

Addressing **8.EE.B**

Narrative

A linear equation is described in terms of a constraint on the total weight of two different types of freight. The constraint gives an equation that students produce. Students then graph the solutions to the equation.

Student Task Statement

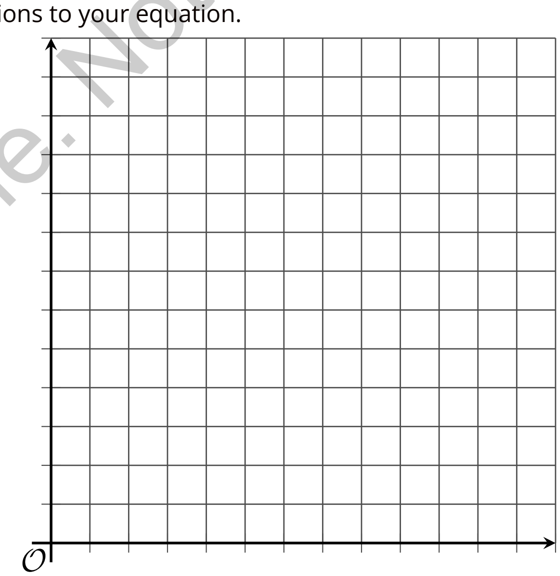
A truck is shipping jugs of drinking water and cases of paper towels. A jug of drinking water weighs 40 pounds and a case of paper towels weighs 16 pounds. The truck can carry 2,000 pounds of cargo altogether.

a. Complete the table showing three ways the truck could be packed with jugs of water and cases of paper towels so that it is carrying 2,000 pounds of cargo.

jugs of drinking water (w)	cases of paper towels (t)
10	
	50
	5

b. Write an equation describing the number of jugs of water w, and cases of paper towels t, the truck can carry.

c. Draw a graph of the solutions to your equation.



d. A different truck can carry 3,000 pounds of cargo altogether. How would a graph representing this new truck be the same and different from the graph representing the truck that could carry 2,000 pounds?

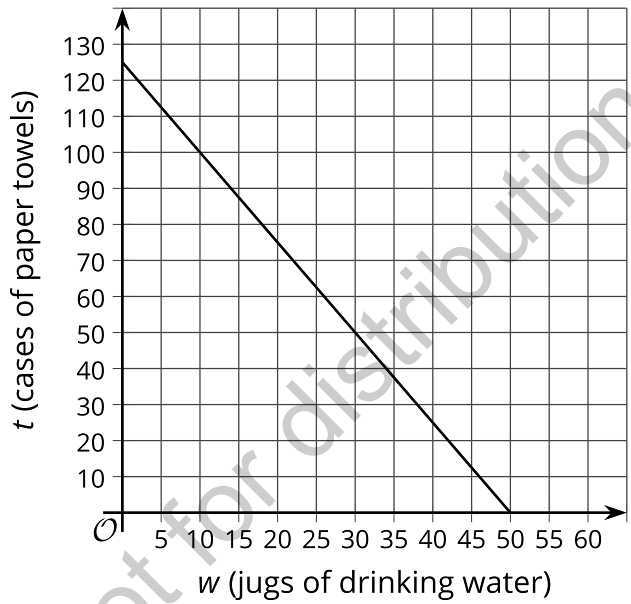
Solution

a.

jugs of drinking water (w)	cases of paper towels (t)
10	100
30	50
48	5

b. $40w + 16t = 2000$ (or equivalent)

c. Sample graph:



d. Sample responses: The two graphs would have the same slope but different vertical intercepts. The graph of the new line would be parallel to the graph of the original line.

Minimal Tier 1 response:

- Work is complete and correct.
- The graph may be a continuous line, or it may consist only of points representing whole numbers of jugs of water and cases of paper towels.
- The graph may have the axes and labels reversed, where the cases of paper towels is on the horizontal axis, and the jugs of drinking water is on the vertical axis.
- Sample: See solution above.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.
- Sample errors: Graph contains only the three points from the table; scale chosen does not allow for all relevant data to be displayed; reasonable work in part 1 but results in an incorrect equation.
- Acceptable errors: Equation and graph are correct based on an incorrect proportional relationship in the table; Graph is correct based on an incorrect equation.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: Work does not factor in the 2,000 pound constraint; equation is not linear or does not make sense in this situation; incorrect answers in 2 or more representations.

Tier 4 response:

- Work includes major errors or omissions that demonstrate a lack of conceptual understanding and mastery.
- Sample errors: Does not come up with an equation or graph; omission of or incorrect work in three or more problem parts.

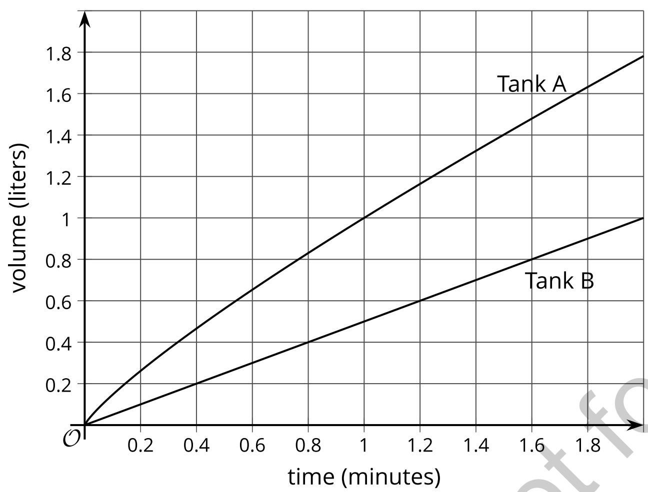
Section A: Proportional Relationships

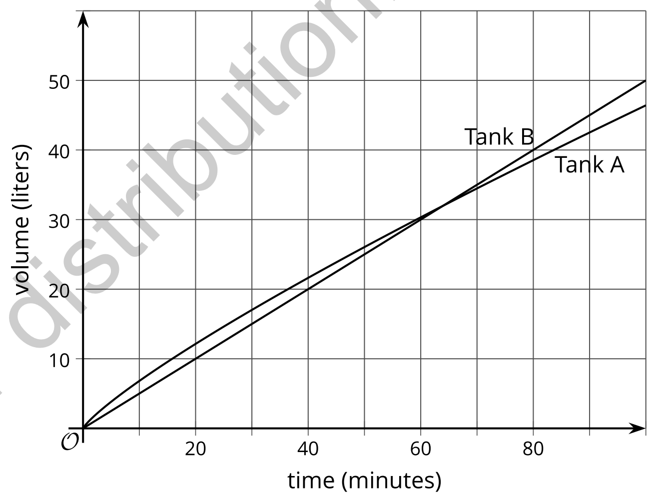
Goals

- Create an equation and a graph to represent proportional relationships, including an appropriate scale and axes.
- Interpret multiple representations of a proportional relationship in context.

Section Narrative

Work in this section takes previous learning with proportional relationships and looks at it from a grade 8 perspective in preparation for work with linear relationships. Students begin the section by observing features of graphs, such as labels and scaling of the axes, to make sense of situations. Students continue to explore the importance of scaling when studying graphs drawn using different scales. The appropriate graph to create or use will depend upon the context of what is being asked for.





When comparing graphs of two proportional relationships, students must consider the values on each axis, and not just use a visual determination when deciding which line is steeper or if the two graphs show the same relationship.

Next, students create their own graphs, strategically choosing the appropriate scaling for each axis in order to answer contextual questions about the proportional relationship. Students work flexibly between different representations of proportional relationships, sometimes using equations of the form $y = kx$ to determine the rate of change, sometimes using tables and creating graphs to show specific information about each proportional relationship.

Teacher Reflection Questions

- **Math Content and Student Thinking:** Students are introduced to proportional situations where either variable can be graphed on either axis. How does flexible thinking around the placement of a variable on a given axis affect a student's ability to interpret the meaning of slope in each situation?
- **Pedagogy:** In what ways do you determine whether students make a connection between the math work they do in the lesson and the learning goals? Which curricular resources support you in making these determinations?
- **Access and Equity:** As students worked in their small groups, whose ideas were heard, valued, and accepted? How can you adjust the group structure to ensure each student's ideas are a part of the collective learning?

Section A Checkpoint

1

Goals Assessed

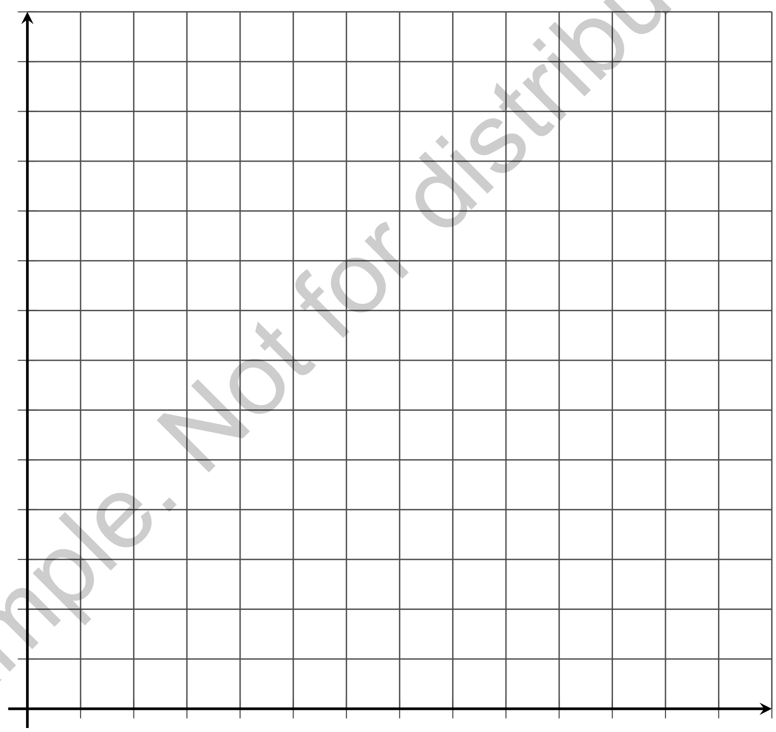
- Create an equation and a graph to represent proportional relationships, including an appropriate scale and axes.

Student Task Statement

Jada and Noah count the number of steps they take to walk a set distance. To walk the same distance, Jada takes 8 steps while Noah takes 10 steps. Then they find that when Noah takes 15 steps, Jada takes 12 steps.

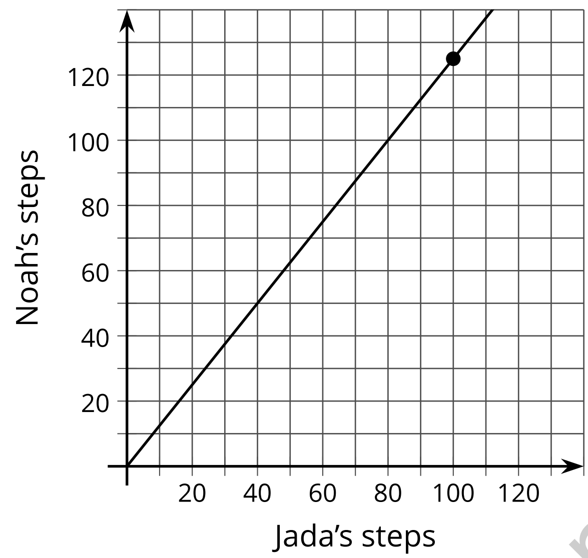
a. Write an equation that represents this situation. Use n to represent the number of steps Noah takes and j to represent the number of steps Jada takes.

b. Create a graph that represents this situation and can be used to determine how many steps Noah will take if Jada takes 100 steps.



Solution

a. $n = \frac{5}{4}j$ or $j = \frac{4}{5}n$ (or equivalent)



b. Sample graph. If Jada takes 100 steps, Noah will take 125.

Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

2

Goals Assessed

- Interpret multiple representations of a proportional relationship in context.

Student Task Statement

Diego and Priya are filling buckets with water from two different hoses.

Diego can fill 20 buckets in 5 minutes.

The equation $y = 3x$ describes how Priya can fill buckets of the same size, where x represents the time in minutes, and y represents the total number of buckets she has filled.

Who is filling buckets faster? Explain your reasoning.

Solution

Diego is filling buckets faster. Sample reasoning: Diego can fill 20 buckets in 5 minutes but Priya can only fill 15 buckets in 5 minutes.

Responding To Student Thinking

More Chances

Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Unit 3, Lesson 1

Understanding Proportional Relationships

Goals

- Comprehend that for the equation of a proportional relationship given by $y = kx$, k represents the constant of proportionality.
- Create graphs and equations of proportional relationships in context, including an appropriate scale.
- Interpret diagrams or graphs of proportional relationships in context.

Learning Targets

- I can graph a proportional relationship from a story.
- I can use the constant of proportionality to compare the pace of different animals.

Lesson Narrative

The purpose of this lesson is to get students thinking about what makes a “good” graph by considering components such as labels and scale. Students add a scale to graphs showing the pace of two bugs and graph an additional line based on a verbal description of a third bug (MP2).

This lesson includes graphs with elapsed time on the vertical axis and distance traveled on the horizontal axis. In general, a context that involves a relationship between two quantities does not dictate which quantity is the independent or dependent variable. Consider this situation where a runner is traveling one mile every 10 minutes.

- We can say the number of miles traveled, d, depends on the number of minutes that have passed, t, and write $d = 0.1t$. This way of expressing the relationship might be more useful for questions like "How far does the runner travel in 35 minutes?"
- We can also say that the number of minutes that have passed, t, depends on the number of miles traveled, d, and write $t = 10d$. This way of expressing the relationship might be more useful for questions like "How long does it take the runner to travel 2 miles?"

Both interpretations have meaning, and both could be of interest—it is up to the modeler to decide what questions they want to answer about the context and which way of expressing the relationship will be most useful in answering those questions.

Standards

Building On	**7.RP.A.2**
Addressing	**8.EE.B**
Building Towards	**8.EE.B.5**

Instructional Routines

- MLR5: Co-Craft Questions
- MLR7: Compare and Connect
- Notice and Wonder

Student Facing Learning Goals

Let’s study some graphs.

1.1 Notice and Wonder: Two Graphs

Warm-up

5 mins

Activity Narrative

The purpose of this *Warm-up* is to have students discuss which features of a graph are necessary for communicating information. While students may notice and wonder many things about these graphs, the missing labels on the second graph is an important discussion point.

When students articulate what they notice and wonder, they have an opportunity to attend to precision in the language they use to describe what they see (MP6).

Standards

Building Towards **8.EE.B.5**

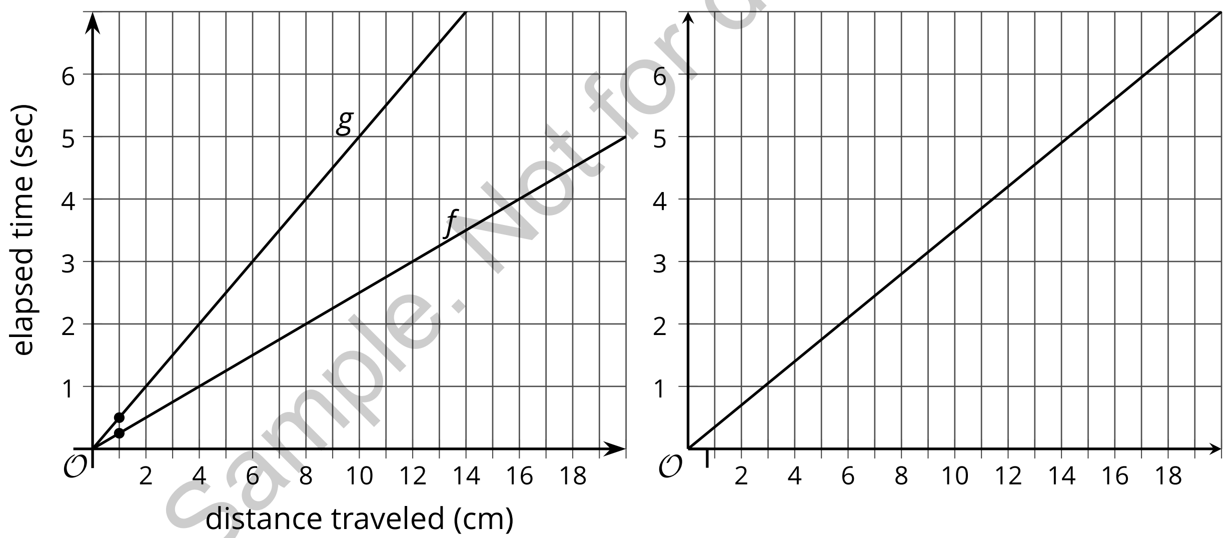
Instructional Routines

- Notice and Wonder

Launch

Arrange students in groups of 2. Display both graphs for all to see. Give students 1 minute of quiet think time and ask them to be prepared to share at least one thing they notice and one thing they wonder. Give students another minute to discuss their observations and questions.

Student Task Statement



What do you notice? What do you wonder?

Student Response

Students may notice:

- The axes on the second graph are not labeled.
- If the first graph is about speed, then f is twice as fast as g.
- Line g is something going a speed of 2 cm every sec.

- Line f is something going at a pace of about 0.25 sec per 1 cm.

Students may wonder:

- What do the two points mean?
- Why does one image show two lines while the other only has one?
- What do lines g and f represent?
- What does the line in the second graph represent?

Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses without editing or commentary for all to see. If possible, record the relevant reasoning on or near the graphs. Next, ask students, "Is there anything on this list that you are wondering about now?" Encourage students to observe what is on display and respectfully ask for clarification, point out contradicting information, or voice any disagreement.

If the fact that the second graph is missing labels does not come up during the conversation, ask students to discuss this idea.

15 mins

1.2 Moving Through Representations

Activity Narrative

In this activity, students investigate the paces of two different bugs. Students use quantitative and abstract reasoning as they use the tick-mark diagram at the start of the activity to answer questions about pace, decide on a scale for the axes, and mark and label the time needed to travel 1 centimeter for each bug (MP2).

Monitor for students who use different scales on the axes to share later. For example, some students may count by 1 second on the distance axis while others may count by 0.5 second.

Standards

Addressing	**8.EE.B**
Building Towards	**8.EE.B.5**

Instructional Routines

- MLR5: Co-Craft Questions

Launch

Arrange students in groups of 2. Before students start working, ensure that they understand that each bug's position is measured at the front of its head. For example, after 2 seconds, the ladybug has moved 4 centimeters and the ant has moved 6 centimeters.

Ask students to review the images and the first problem in the activity and give a signal when they have finished. Invite students to share their ideas about which bug is represented by line u and which bug is represented by line v. (The ladybug is u, and the ant is v.) If not mentioned by students, draw attention to how the graph shows the *pace* of the two bugs. The graph shows how much time it takes to go a certain distance, which is different than a graph of speed, which shows how much distance is traveled in a certain amount of time.

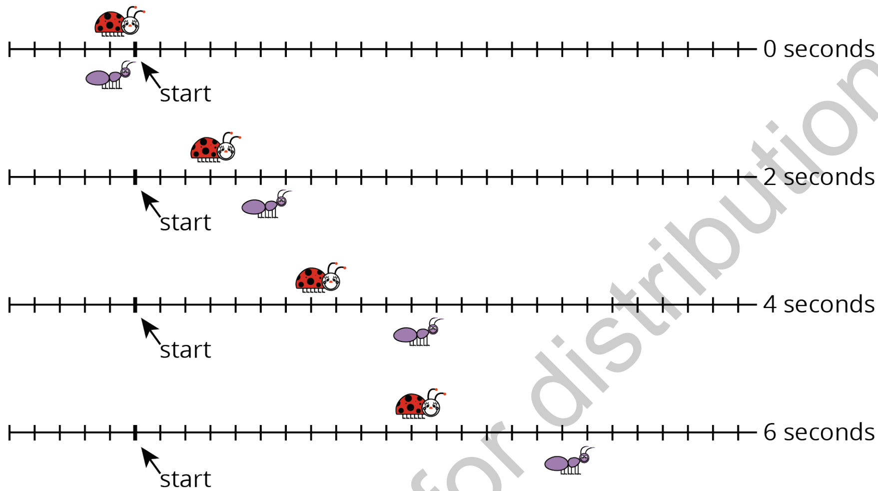
Give students work time to complete the remaining problems with their partner followed by a whole-class discussion.

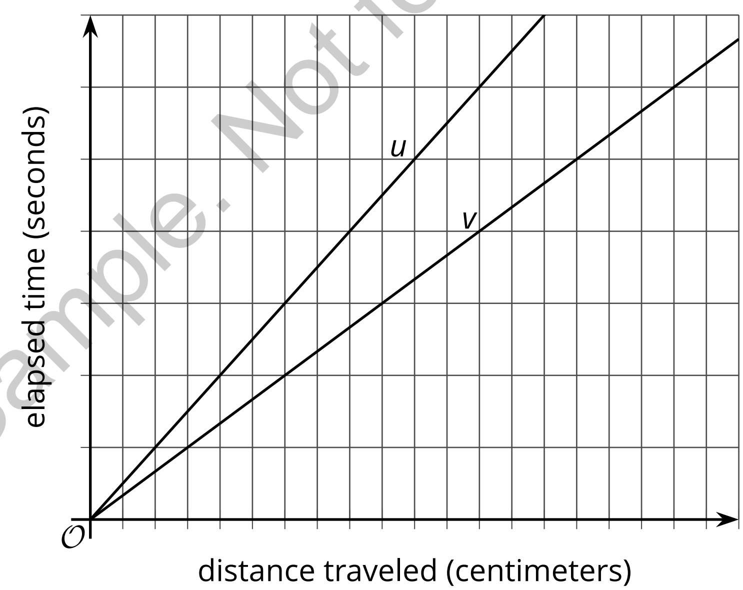
Access for English Language Learners

MLR5 Co-Craft Questions. Keep books or devices closed. Display only the task statement and first image of the bugs, without revealing the line graphs or the questions, and ask students to write down possible mathematical questions that could be asked about the situation. Invite students to compare their questions before revealing the task. Ask, “What do these questions have in common? How are they different?” Reveal the intended questions for this task and invite additional connections.
Advances: Reading, Writing

Student Task Statement

A ladybug and ant move at constant speeds. The diagrams with tick marks show their positions at different times, as measured by the front of each bug’s head. Each tick mark represents 1 centimeter.





1. Lines u and v also show the positions of the two bugs. Which line shows the ladybug’s movement? Which line shows the ant’s movement? Explain your reasoning.

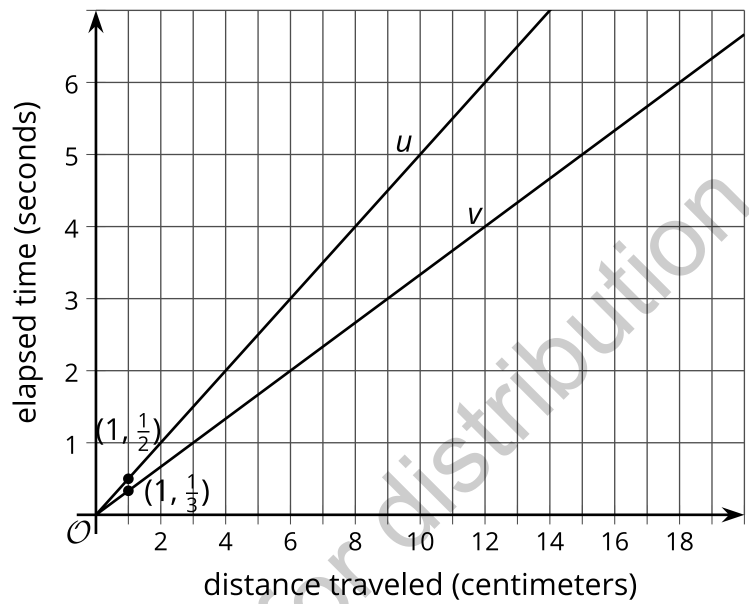
2. How long does it take the ladybug to travel 12 centimeters? The ant?

3. Scale the vertical and horizontal axes by labeling each grid line with a number. You will need to use the time

and distance information shown in the tick-mark diagrams.

4. Mark and label the point on line u and the point on line v that represent the time and position of each bug after traveling 1 centimeter.

Student Response

1. Line u shows the ladybug and line v shows the ant. Sample reasoning: After 2 seconds, the ant has gone farther than the ladybug, and looking at both lines at 2 seconds, line v shows a greater distance.
2. The ladybug travels 12 centimeters in 6 seconds, and it takes the ant 4 seconds.
3. 

4. See graph.

Building on Student Thinking

If students confuse pace with speed and interpret a steeper line to mean that the ladybug moves faster, consider:

- Asking, “How do the tick marks on either diagram help show which bug is moving faster?”
- Asking, “With distance on the x-axis and time on the y-axis, where is the ladybug’s location after 4 seconds on the graph? Which line does that point correspond with?”
- Explaining that moving twice as fast means going at half the pace.

Are You Ready for More?

1. How fast is each bug traveling?
2. Will there ever be a time when the ant is twice as far away from the start as the ladybug? Explain or show your reasoning.

Extension Student Response

1. The ladybug is traveling at 2 centimeters/second and the ant is traveling at 3 centimeters/second.
2. No, the ant is always one and a half times as far from the start as the ladybug.

Activity Synthesis

Display the images from the *Student Task Statement* for all to see. Invite students to share their solutions for how long it takes each bug to travel 12 centimeters. Encourage students to reference one or both images as they explain their thinking.

Then invite previously selected students to share their graphs and explain how they decided on what scale to use. If possible, display these graphs for all to see. There are many correct ways to choose a scale for this situation, though some scales may have made it easier to answer the last question. Highlight these graphs and encourage students to read all problems when making decisions about how to construct a graph.

1.3 Moving Twice as Fast

15 mins

Activity Narrative

In this activity, students use the tick-mark diagram and graph representations from the previous activity and add a third bug that is moving twice as fast as the ladybug. Students also write equations for all three bugs. An important aspect of this activity is students making connections between these different representations (MP2).

Monitor for students who use these different strategies to write their equations:

- Reason from the unit rates they can see on their graphs and write equations in the form of $y = kx$, where k is the constant of proportionality
- Use similar triangles to write equations in the form of $\frac{y}{x} = \frac{b}{a}$, where (a, b) is a point on the line

Access for English Language Learners

This activity uses the *Compare and Connect* math language routine to advance representing and conversing as students use mathematically precise language in discussion.

Standards

Addressing	**8.EE.B**
Building Towards	**8.EE.B.5**

Instructional Routines

- MLR7: Compare and Connect

Launch

Arrange students in groups of 2. Give 5–7 minutes work time followed by a whole-class discussion.

Select work from students with different strategies, such as those described in the activity narrative, to share later.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. To support development of organizational skills in problem-solving, chunk this task into more manageable parts. For example, show only one question at a time, pausing to check for understanding before moving on.

Supports accessibility for: Organization, Attention

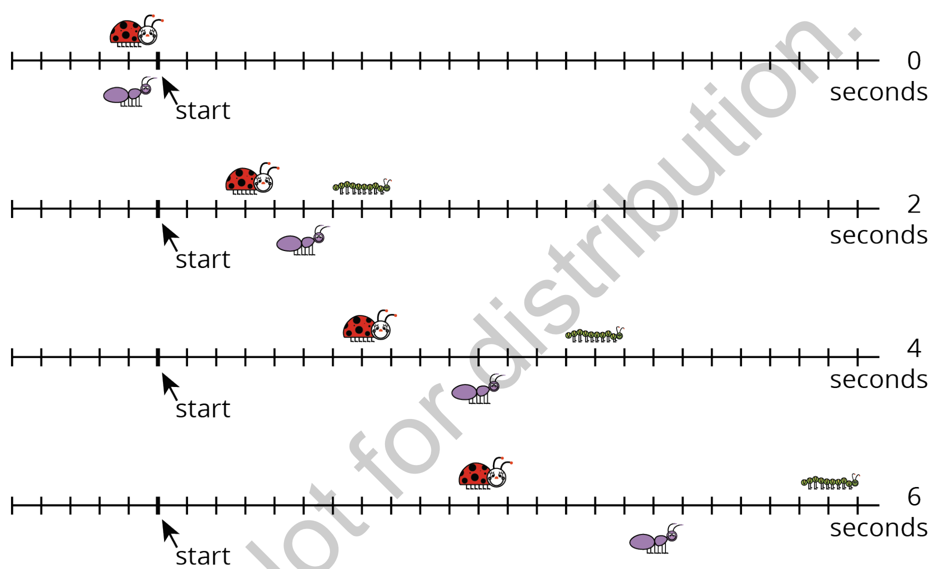
Student Task Statement

Refer to the tick-mark diagrams and graph in the earlier activity.

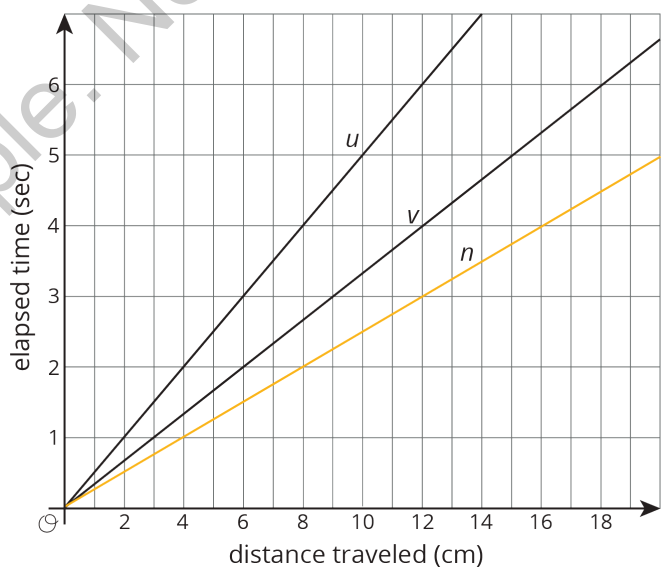
1. Imagine a bug that is moving twice as fast as the ladybug. On each tick-mark diagram, mark the position of this bug.
2. Plot this bug's positions on the coordinate axes with lines u and v, and connect them with a line.
3. Write an equation for each of the three lines where x represents the distance traveled by each bug and y represents the elapsed time.

Student Response

1.



2.



3. Sample responses:

- Ladybug: (line u) $y = \frac{1}{2}x$ or $\frac{y}{x} = \frac{1}{2}$ (or equivalent)

- Ant: (line v) $y = \frac{1}{3}x$ or $\frac{y}{x} = \frac{1}{3}$ (or equivalent)
- New bug: (line n) $y = \frac{1}{4}x$ or $\frac{y}{x} = \frac{1}{4}$ (or equivalent)

Activity Synthesis

The goal of this discussion is to connect the work of using similar triangles to write equations of a line with the work using unit rate to write equations of a line. Display both images from the previous task. Invite previously selected students to share their equations for each bug and record these for all to see.

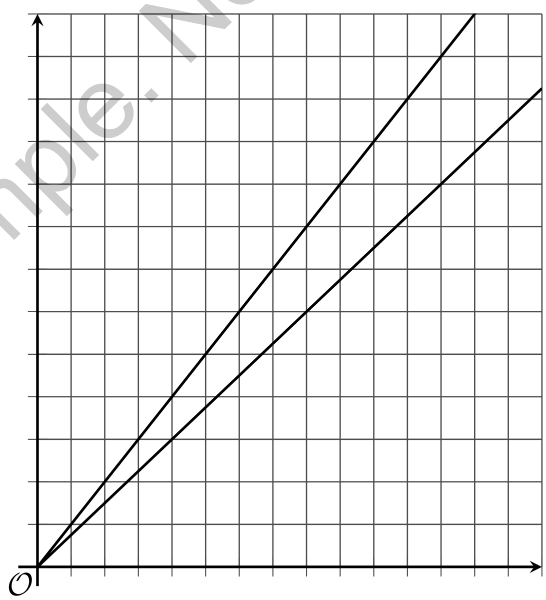
Use *Compare and Connect* to help students compare, contrast, and connect the different approaches and representations. Here are some questions for discussion:

- "Did anyone write the same equations, but would explain it differently?"
- "How does the slope of the line show up in each equation?"
- "How do these different representations show the same information?"

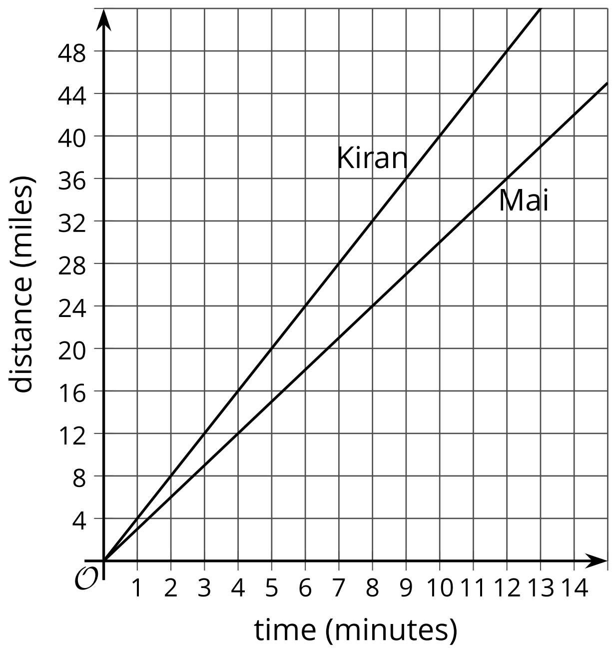
As students share their approaches for writing equations, highlight approaches where students used multiple representations to make sense of their equations. For example, ask students to identify features of the tick-mark diagrams, lines, and equations that show the same information. If time allows, demonstrate how the position of the ladybug in a tick-mark diagram can also be seen in the graph of line u, and how using the distance and elapsed time values in the corresponding equation will make it true.

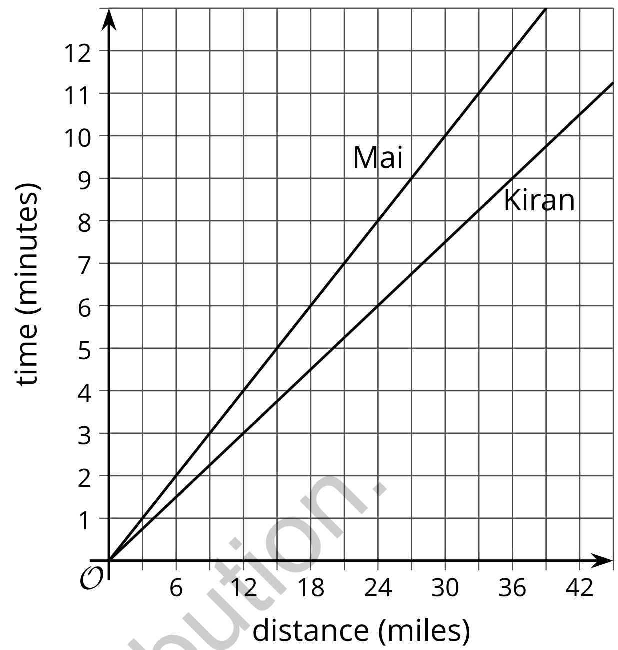
Lesson Synthesis

The goal of this discussion is for students to see how labels and an appropriate scale on a graph are necessary and can help to make sense of a relationship. For example, display this image for all to see and explain that on longer bike rides, Kiran can ride 4 miles every 16 minutes, and Mai can ride 4 miles every 12 minutes. But without labels or a scale, one can't tell which line represents Kiran and which represents Mai.



Ask students how to label the axes and add a scale, recording their work for all to see. Then ask students for at least 2 points on each line that will help determine which line is Kiran and which is Mai, and add them to the graphs. Depending on which axis students choose for time and distance, here are two possible labeled and scaled graphs.





If time allows, have students use the completed graph to answer questions such as:

- "Who rides faster?" (Mai)
- "If Kiran and Mai start a bike trip at the same time, how far apart are they after 24 minutes?" (2 miles apart)
- "How long will it take each of them to reach the end of a 12 mile bike path?" (It will take Kiran 48 minutes and it will take Mai 36 minutes.)

1.4 Turtle Race

Cool-down

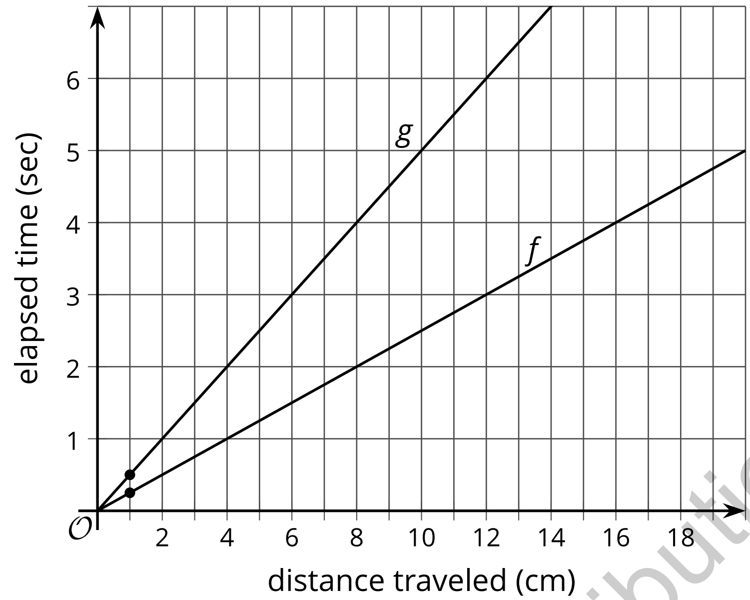
5 mins

Standards

Building On	**7.RP.A.2**
Building Towards	**8.EE.B.5**

Student Task Statement

This graph represents the positions of two turtles in a race.



1. On the same axes, draw a line for a third turtle that is going half as fast as the turtle described by line g.
2. Explain how your line shows that the turtle is going half as fast.

Student Response

1. A line through $(0, 0)$, $(1, 1)$, $(2, 2)$, etc.
2. Sample reasoning: After 2 seconds, the turtle described by line g moved 4 cm, while the third turtle moved only 2 cm. This third turtle covers half the distance in the same amount of time.

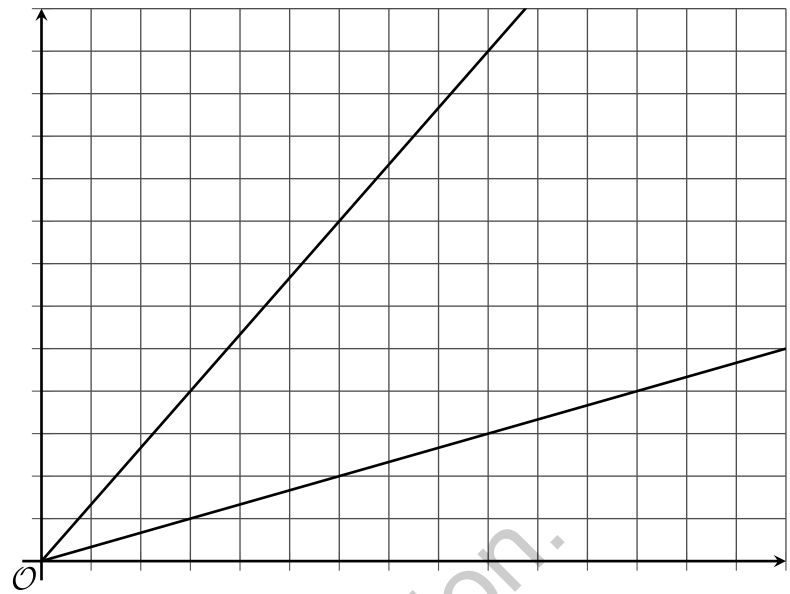
Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

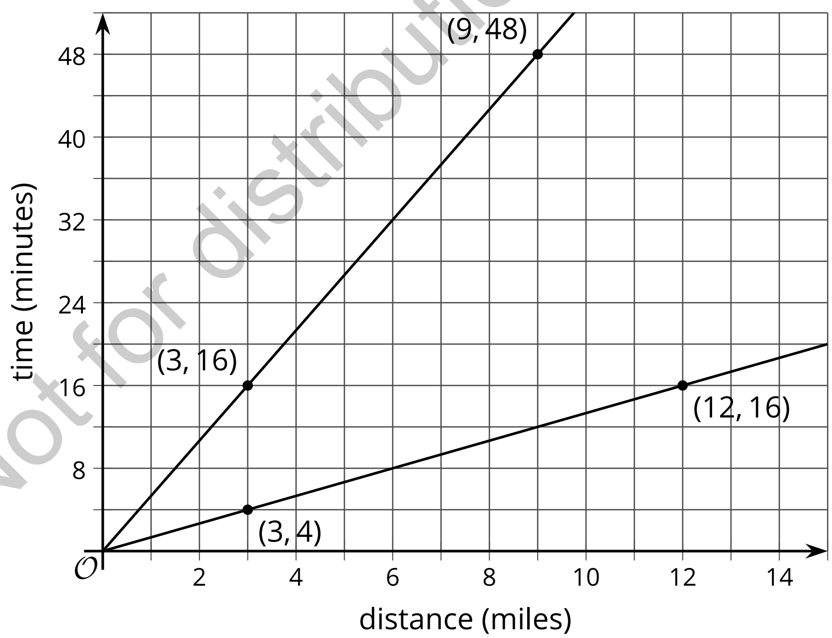
Lesson 1 Summary

Graphing is a way to help make sense of relationships.

But the graph of a line on a coordinate plane without labels or a scale isn't very helpful. Without labels, we can't tell what the graph is about or what units are being used. Without an appropriate scale, we can't tell any specific values.



Here are the same graphs, but now with labels and a scale:



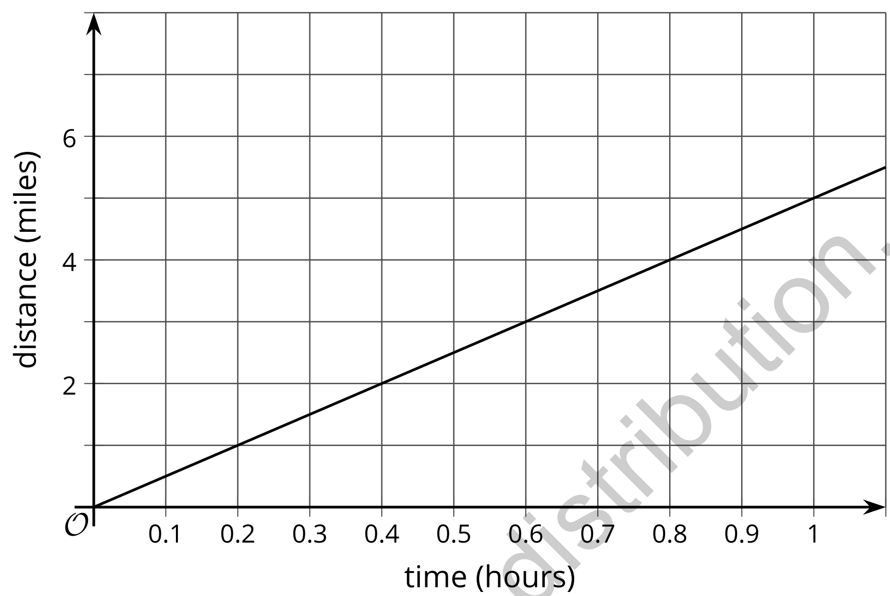
Notice how adding labels lets us know that the relationship compares time and distance and helps to understand both the speed and pace of two different items. When adding labels to axes, be sure to include units, such as minutes and miles.

Notice how adding a scale makes it possible to identify specific points and values. When adding a scale to an axis, be sure that the space between each grid line represents the same amount.

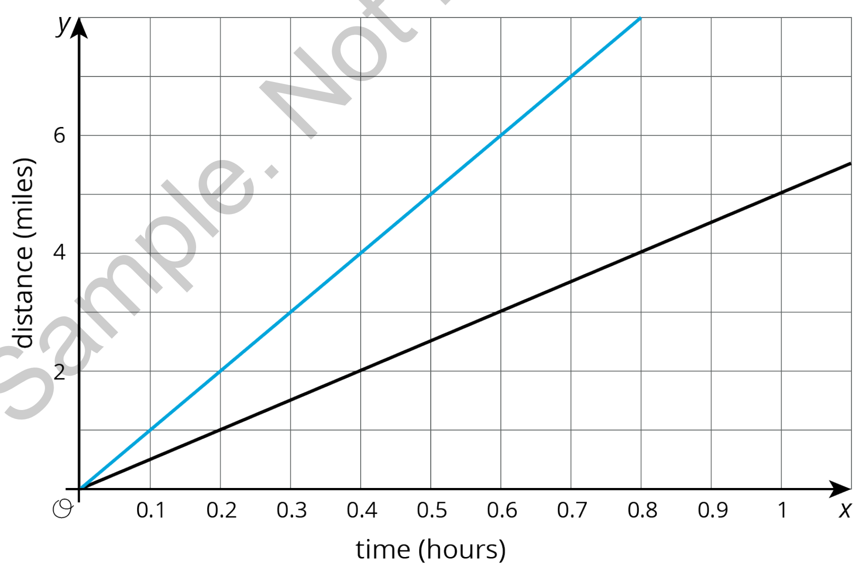
Practice Problems

1 Student Task Statement

Priya jogs at a constant speed. The relationship between her distance and time is shown on the graph. Diego bikes at a constant speed twice as fast as Priya. Sketch a graph showing the relationship between Diego's distance and time.



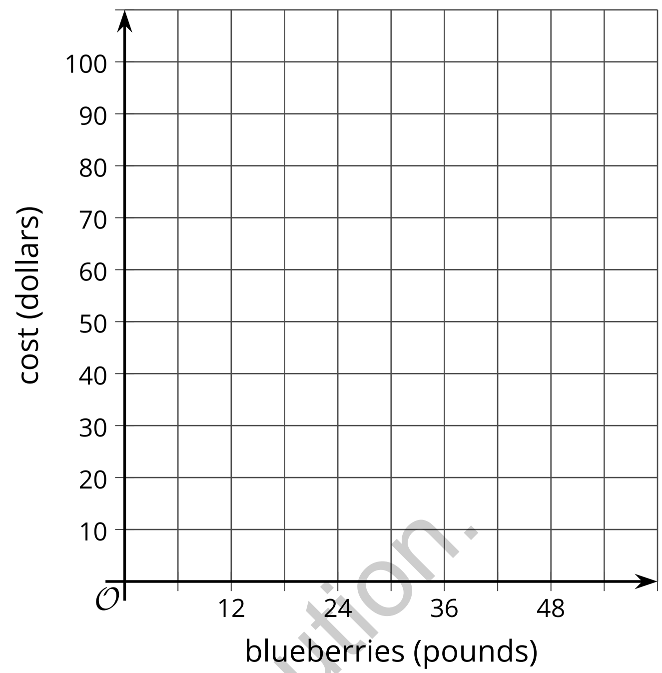
Solution



2 Student Task Statement

A blueberry farm offers 6 pounds of blueberries for $15.00.

Sketch a graph of the relationship between cost in dollars and pounds of blueberries.



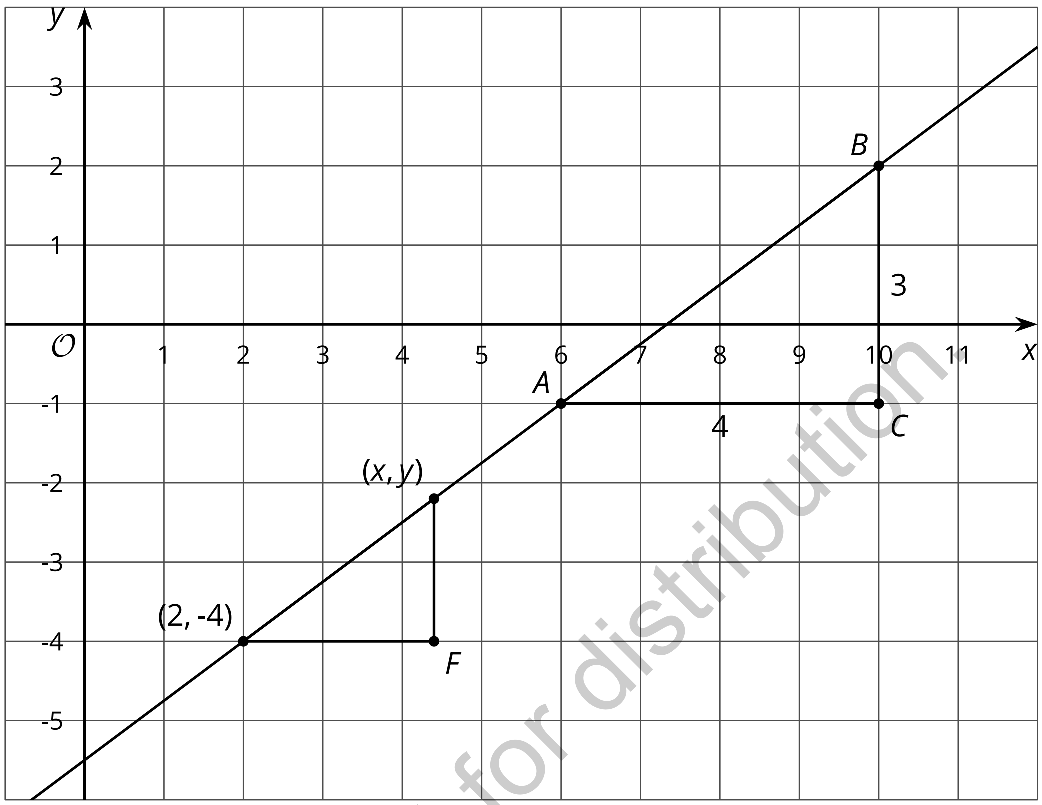
Solution

A line that passes through $(0, 0)$ and $(6, 15)$.

3 from Unit 2, Lesson 11

Student Task Statement

The points $(2, -4)$, (x, y), A, and B all lie on the line. Write an equation that describes the line.



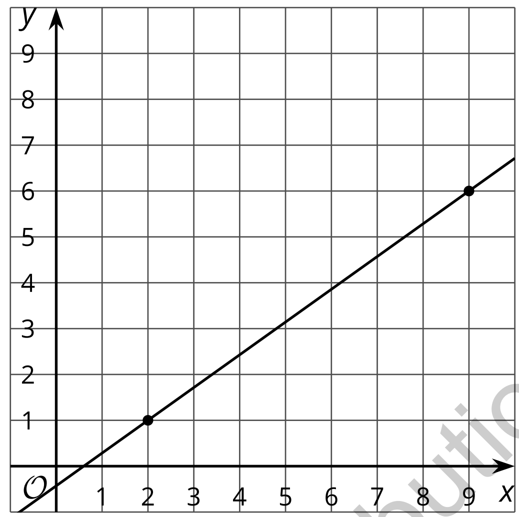
Solution

$\frac{y+4}{x-2} = \frac{3}{4}$ (or equivalent)

4 from Unit 2, Lesson 12

Student Task Statement

The graph shows a line.



Select **all** points that are on this line.

A. $(0, 3)$

B. $(4, 2)$

C. $(30, 21)$

D. $(16, 11)$

E. $(-4, -5)$

Solution

C, D

Unit 3, Lesson 2

Graphs of Proportional Relationships

Goals

- Compare graphs that represent the same proportional relationship using differently scaled axes.
- Create graphs representing the same proportional relationship using differently scaled axes, and identify which graph to use to answer specific questions.

Learning Targets

- I can graph a proportional relationship from an equation.
- I can tell when two graphs are of the same proportional relationship even if the scales are different.

Lesson Narrative

The purpose of this lesson is for students to understand that while there are many ways to scale axes when graphing a proportional relationship, certain ranges for the axes are helpful for seeing specific information.

Students begin by considering a claim that the graph of one line is steeper than the graph of a second line. While one graph looks like a steeper line, by noticing the scale of the axes of each graph, it can be determined that the two lines actually have the same slope. Next, students sort graphs on cards based on what proportional relationship they represent (MP7). Each graph has a different scale, with some scales purposefully quite different, pressing the need to pay attention to scale and rely on mathematical definitions of steepness, not just visual ones.

Then students graph a proportional relationship on two differently scaled axes, comparing it to the graph of a nonproportional relationship. By looking at the same two relationships graphed at different scales, students see how the scale of the axes affects the information that can be determined.

Standards

Building On	**7.RP.A.2**
Addressing	**8.EE.B, 8.EE.B.5**
Building Towards	**8.EE.B.5**

Instructional Routines

- Card Sort
- MLR6: Three Reads
- MLR8: Discussion Supports

Required Materials

Materials To Gather

- Straightedges: Lesson
- Straightedges: Activity 3

Materials To Copy

- Proportional Relationships Cards (1 copy for every 4 students): Activity 2

Student Facing Learning Goals

Let's think about scale.

2.1

Two Perspectives

Warm-up

5 mins

Activity Narrative

In this *Warm-up*, students work with the same proportional relationship shown on two sets of axes that are scaled differently. The purpose is to make explicit that the same proportional relationship can appear to have different steepness depending on the axes.

Standards

Building On 7.RP.A.2

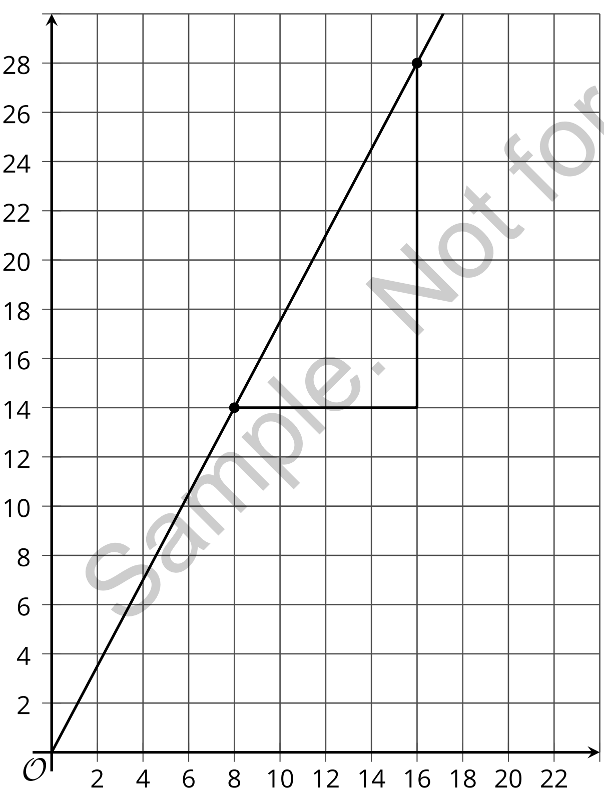
Building Towards 8.EE.B.5

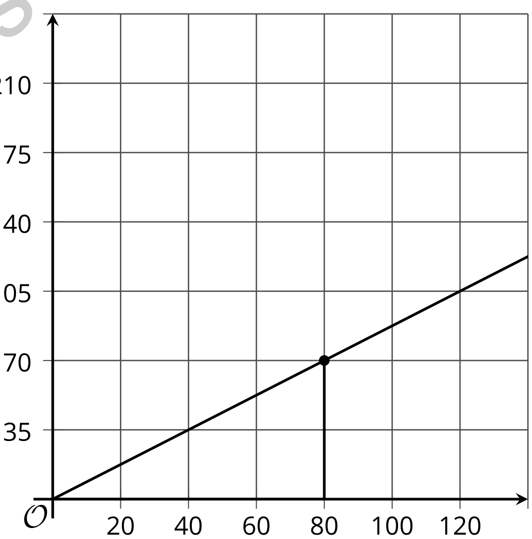
Launch

Give students 2–3 minutes of quiet work time followed by a whole-class discussion.

Student Task Statement

Here are two graphs that could represent a variety of different situations.





Andre claims that the line in the graph on the left has a greater slope because it is steeper. Do you agree with Andre? Explain your reasoning.

Student Response

No, I do not agree with Andre. Sample reasoning: Both lines have the same slope, even though the graph on the left looks steeper. Using the scales of the graphs to measure the vertical and horizontal change, the graph on the left has a slope of $\frac{14}{8} = \frac{7}{4}$ and the graph on the right has a slope of $\frac{70}{40} = \frac{7}{4}$.

Activity Synthesis

The goal of this discussion is to emphasize the importance of paying attention to scale when making sense of graphs. Display the two images from the activity for all to see. Identify 1–2 students to share their reasoning. Here are some questions for discussion:

- "How can one graph look steeper yet still have the same slope as another graph?" (The two graphs are drawn using different scales, making them look different even though the value of their slopes is equivalent.)
- "Are the two slope triangles shown similar?" (Yes. The slope triangle on the left can be dilated and translated to match the slope triangle on the right, making the two triangles similar.)
- "What would happen if these 2 lines were graphed on the same set of axes?" (They would overlap and look like the same line.)

2.2 Card Sort: Proportional Relationships

10 mins

Activity Narrative

The purpose of this activity is for students to identify the same proportional relationship graphed using different scales. Students will first sort the graphs based on what proportional relationship they represent and then write an equation representing each relationship. A sorting task gives students opportunities to analyze representations, statements, and structures closely and make connections (MP7).

Monitor for and select groups using different strategies to match graphs to share later. For example, some groups may identify the unit rate for each graph in order to match while others may choose to write equations first and use those to match their graphs.

Standards

Addressing	**8.EE.B**
Building Towards	**8.EE.B.5**

Instructional Routines

- Card Sort
- MLR8: Discussion Supports

Launch

Tell students to close their books or devices (or to keep them closed). Arrange students in groups of 4 and distribute pre-cut cards. Allow students to familiarize themselves with the representations on the cards:

- Give students 1 minute to place all the cards face up and start thinking about possible ways to sort the cards into categories.
- Pause the class and select 1–3 students to share the categories they identified.
- Discuss as many different categories as time allows.

Attend to the language that students use to describe their categories and graphs, giving them opportunities to describe their graphs more precisely. Highlight the use of terms like “slope,” “scale,” and “constant of proportionality.” If necessary, remind students that the constant of proportionality is the number that values for one quantity are each multiplied by to get the values for the other quantity. After a brief discussion, invite students to open their books or devices and continue with the activity.

Access for English Language Learners

MLR8 Discussion Supports. Students should take turns finding a match and explaining their reasoning to their partner. Display the following sentence frames for all to see: “I noticed ________ , so I matched . . . .” Encourage students to challenge each other when they disagree.
Advances: Speaking, Conversing

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Chunk this task into more manageable parts. Give students a subset of the cards to start with and introduce the remaining cards once students have completed their initial set of matches.
Supports accessibility for: Conceptual Processing, Organization, Memory

Student Task Statement

Your teacher will give you a set of cards. Each card contains a graph of a proportional relationship.

1. Sort the graphs into groups based on what proportional relationship they represent.
2. Write an equation for each *different* proportional relationship you find.

Student Response

Card sort and possible matching equation:

A: $y = 0.25x$ or $\frac{y}{x} = \frac{6}{24}$ (or equivalent)

B, E, H: $y = 3x$ or $\frac{y}{x} = \frac{6}{2}$ (or equivalent)

C, D, G, K: $y = 3.5x$ or $\frac{y}{x} = \frac{7}{2}$ (or equivalent)

I, L: $y = \frac{4}{3}x$ or $\frac{y}{x} = \frac{4}{3}$ (or equivalent)

F, J: $y = \frac{5}{2}x$ or $\frac{y}{x} = \frac{5}{2}$ (or equivalent)

Activity Synthesis

The goal of this discussion is for students to understand that looking only at the steepness of the line without paying attention to the numbers on the axes can hide the actual relationship between the two variables.

Once all groups have completed the *Card Sort*, ask previously selected groups to share their strategies for grouping the graphs. Discuss the following:

- “Which graphs were tricky to group? Explain why.”
- “Did you need to make adjustments in your groups? What might have caused an error? What adjustments were made?”

2.3 Different Scales

20 mins

Activity Narrative

In this activity, students graph a proportional relationship on two differently scaled axes and compare the proportional relationship to an already-graphed nonproportional relationship on the same axes. Students make sense of the intersections of the two graphs by reasoning about the situation and consider which scale is most helpful: zoomed in, or zoomed out. In this case, which graph is most helpful depends on the questions asked about the situation (MP4).

This is the first time *Three Reads* (Math Language Routine 6) is suggested in this course. In this routine, students are supported in reading a mathematical text, situation, or word problem three times, each with a particular focus. During the first read, students focus on comprehending the situation. During the second read, students identify important quantities. During the third read, the final prompt is revealed and students brainstorm possible strategies to answer the question. The intended question is withheld until the third read so students can make sense of the whole context before rushing to a solution. The purpose of this routine is to support students' reading comprehension as they make sense of mathematical situations and information through conversation with a partner.

Access for English Language Learners

This activity uses the Critique, Correct, Clarify math language routine to advance representing and conversing as students critique and revise mathematical arguments.

Standards

Addressing **8.EE.B.5**

Instructional Routines

- MLR6: Three Reads

Launch

Arrange students in groups of 2. Provide access to straightedges.

Use MLR6 *Three Reads* to support reading comprehension and sense-making about this problem. Display only the problem stem and the graphs, without revealing the questions.

- In the first read, students read the problem with the goal of comprehending the situation.

  Read the problem aloud while everyone else reads along, and then ask, "What is this situation about?" Allow 1 minute to discuss with a partner and then share with the whole class. A typical response may be, "Two large water tanks are filling with water. One of them is filling at a constant rate, while the other is not. Both graphs represent Tank A. Tank B only has an equation." Listen for and clarify any questions about the context.

- In the second read, students analyze the mathematical structure of the story by naming quantities.

  Invite students to read the problem aloud with their partner, or select a student to read to the class, then prompt students by asking, "What can be counted or measured in this situation?" Give students 30 seconds of quiet think time, followed by another 30 seconds to share with their partner. A typical response may be "liters of water in Tank A; liters of water in Tank B; amount of time that has passed in minutes; constant rate of 1/2 liters per minute."

- In the third read, students brainstorm possible solution strategies to answer the questions.

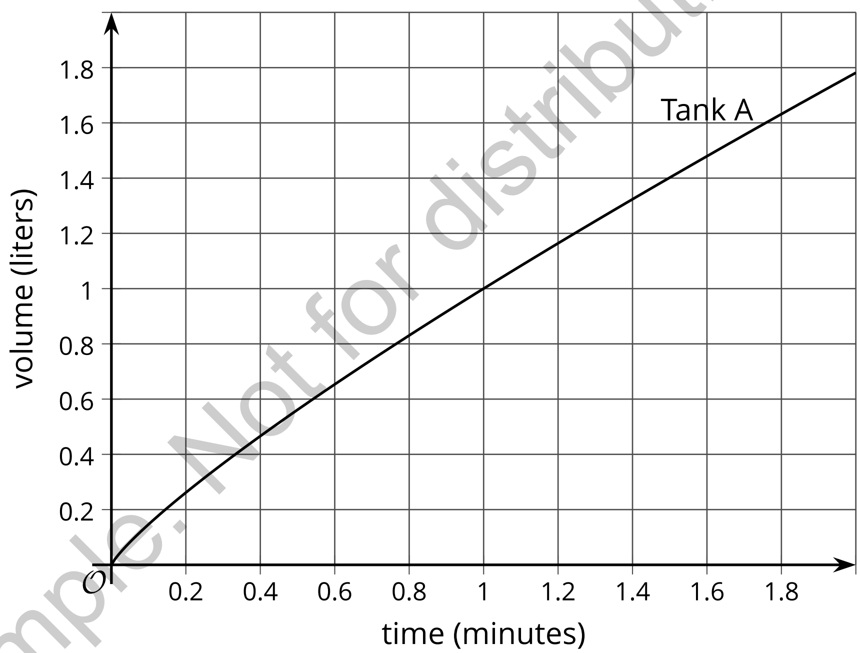
  Invite students to read the problem aloud with their partner, or select a different student to read to the class. After the third read, reveal the first question on sketching and labeling a graph for Tank B on each of the axes and ask, “What are some ways one might solve this?” Instruct students to think of ways to approach the questions without actually solving. Give students 1 minute of quiet think time followed by another minute to discuss with their partner. Invite students to name some possible strategies referencing quantities from the second read. Provide these sentence frames as partners discuss: “To draw a graph for Tank B, I would . . . .” “One way to approach the question about finding the time when the tanks have the same amount of water would be to . . . .” and “I would use the first/second graph to find . . . .”

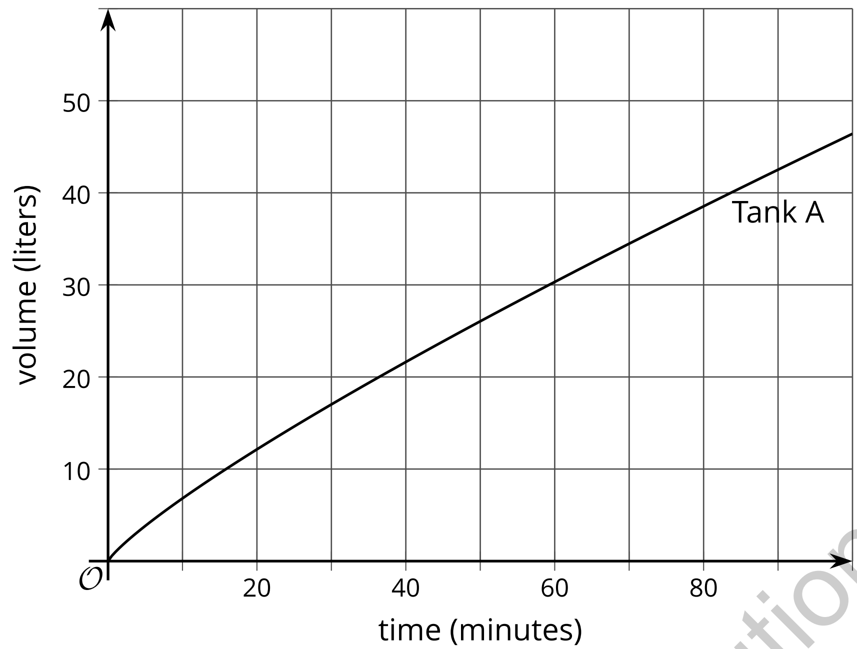
As partners are discussing their solution strategies, select 1–2 students to share their ideas with the whole class. As students are presenting their strategies to the whole class, create a display that summarizes the ideas for each question.

Give students time to complete the rest of the activity followed by a whole class discussion.

Student Task Statement

Two large water tanks are filling with water. Tank A is *not* filled at a constant rate, and the relationship between its volume of water and time is graphed on each set of axes. Tank B is filled at a constant rate of $\frac{1}{2}$ liters per minute. The relationship between its volume of water and time can be described by the equation $v = \frac{1}{2}t$, where t is the time in minutes, and v is the total volume in liters of water in the tank.

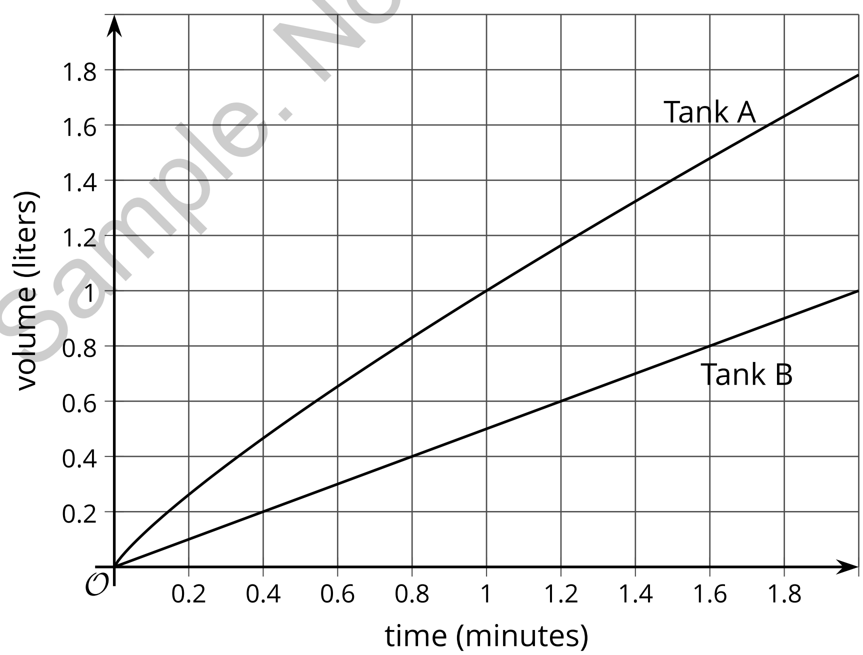


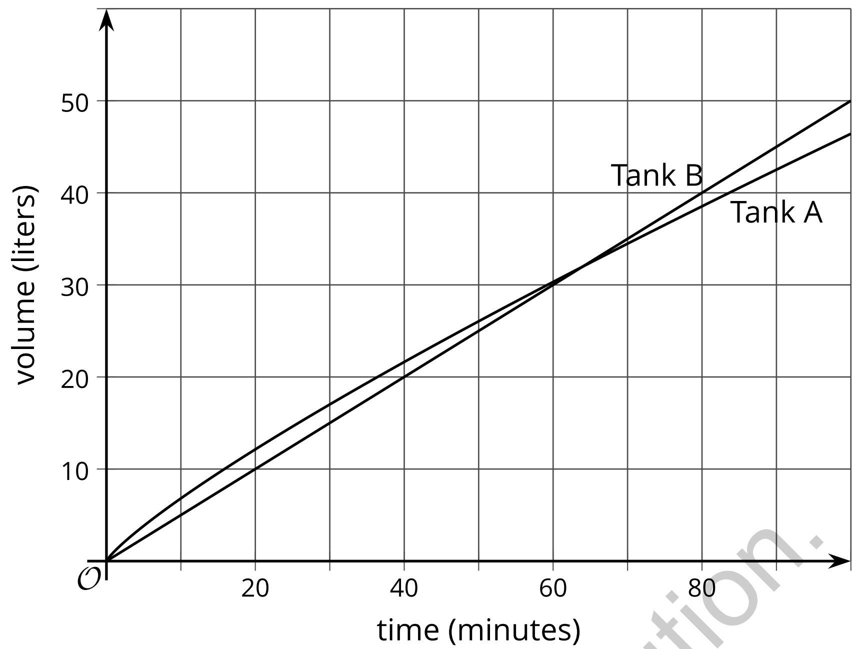


1. Sketch and label a graph of the relationship between the volume of water v and time t for Tank B on each of the coordinate planes.
2. Answer the following questions and say which graph you used to find your answer.
   a. After 30 seconds, which tank has the most water?
   b. At approximately what times do both tanks have the same amount of water?
   c. At approximately what times do both tanks contain 1 liter of water? 20 liters?

Student Response

1.





2. a. Using the first graph, Tank A has more water after 30 seconds.

   b. Using the second graph, at approximately 64 minutes both tanks have the same amount of water.

   c. Using the first graph, Tank A has 1 liter of water after 1 minute. Tank B has 1 liter of water after 2 minutes; using the second graph, Tank A has 20 liters of water at around 36 minutes while Tank B has 20 liters of water at 40 minutes.

Are You Ready for More?

A giant tortoise travels at 0.17 miles per hour and an arctic hare travels at 37 miles per hour.

1. Draw separate graphs that show the relationship between time elapsed, in hours, and distance traveled, in miles, for both the tortoise and the hare.

2. Would it be helpful to try to put both graphs on the same pair of axes? Why or why not?

3. The tortoise and the hare start out together and after half an hour the hare stops to take a rest. How long does it take the tortoise to catch up?

Extension Student Response

1. Each axes should have "time elapsed (hours)" on the horizontal axis and "distance traveled (miles)" on the vertical axis. The scale for the giant tortoise graph is likely much smaller than the scale for the "arctic hare" graph.
2. Sample response: No. Since the scales on the vertical axis are so different, it is very difficult to put both graphs on the same axes without one of the graphs being squashed up very close to an axis. This makes it difficult to read coordinate values from the graph.
3. After half an hour the hare has traveled $0.5 \cdot 37 = 18.5$ miles and the tortoise has traveled $0.5 \cdot 0.17 = 0.085$ miles, so the hare is $18.5 - 0.085 = 18.415$ miles ahead of the tortoise. Assuming the hare doesn't move, it will take the tortoise $18.415/0.17 = 108.32$ hours to catch up, or about 4.5 days.

Activity Synthesis

The goal of this discussion is to emphasize how selecting an appropriate scale is important when creating a graph from scratch. Begin the discussion by asking students:

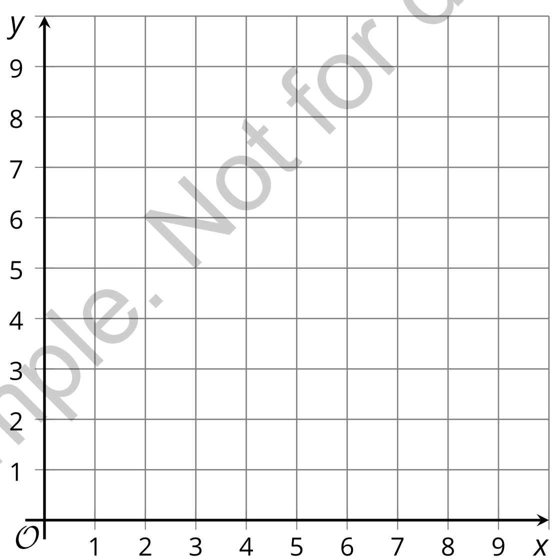
- "What information can be seen using the second graph that can't be seen with the first?" (Tank A starts out with more water than Tank B at first, and then with less water than Tank B later on. The two tanks have the same amount of water after 60 minutes.)
- "If someone only looked at the first graph, what might they think (incorrectly)?" (The first graph makes it look like Tank A will always have more water than Tank B.)
- "In what situation might the first graph be more useful?" (If someone wanted to answer a question about the tanks after only 1 or 2 minutes.)

Explain that if making a graph from scratch, it is important to first check what questions are being asked. Some things to consider are:

- Should both axes have the same scale?
- How large are the numbers in the problem? Does each axis need to extend to 10 or 100?
- What will you count by? 1s? 5s? 10s?

Lesson Synthesis

Display this blank coordinate plane for all to see and provide pairs of students with graph paper.



Ask pairs to draw a copy of the coordinate plane and give a signal when they have finished. (Students may need to be warned to leave room on their graph paper for a second graph as sometimes students like to draw graphs that fill all the space they are given.) Invite a student to propose a proportional relationship that they consider to have a "steep" line for the class to graph on the axes.

For example, say a student proposes $y = 6x$. Have students complete their graph in pairs, then add the line representing the equation to the graph on display. Next, ask students to make a second graph with the same horizontal scale, but with a vertical scale that makes $y = 6x$ not look as steep when graphed. After students have made the new graph, invite students to share and explain how they decided on their new vertical scale.

Conclude by reminding students that all these graphs of $y = 6x$ are correct since they all show a proportional relationship with a constant of proportionality equal to 6. Ask students, “Can you think of a reason someone might want to graph this relationship with such a large vertical scale?” (If they needed to also graph something like $y = 60x$, they would need a pretty big vertical scale in order to see both lines.)

2.4 Different Axes

Cool-down

5 mins

Standards

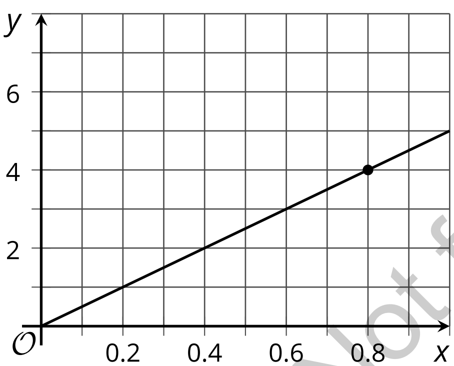
Addressing **8.EE.B**

Building Towards **8.EE.B.5**

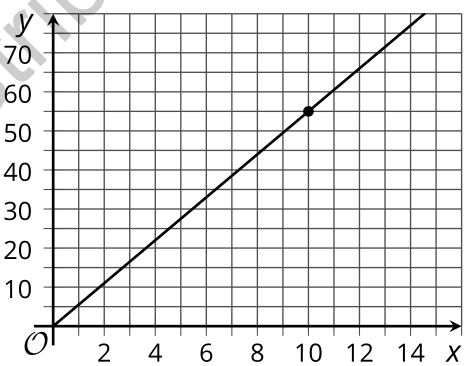
Student Task Statement

Which one of these relationships is different from the other three? Explain how you know.

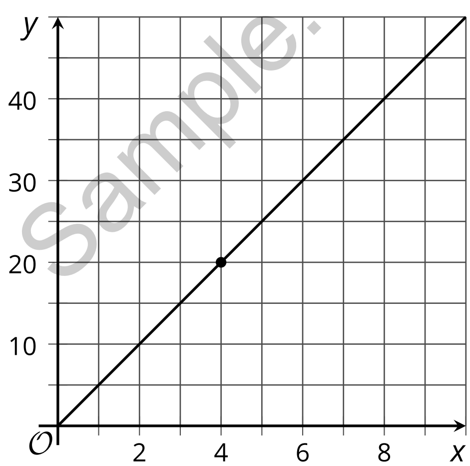
A



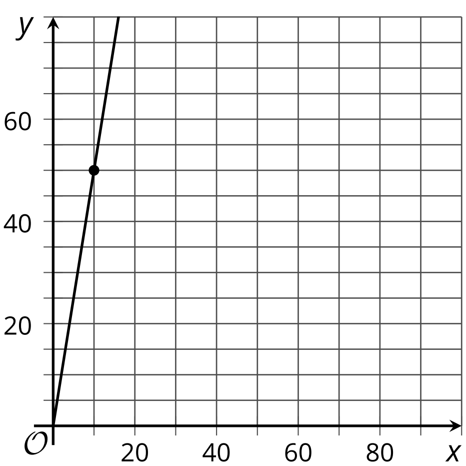
B



C



D



Student Response

Graph B is a representation of $y = 5.5x$ or $\frac{y}{x} = \frac{55}{10}$ while Graphs A, C, and D are all representations of $y = 5x$ or $\frac{y}{x} = 5$.

Responding To Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Lesson 2 Summary

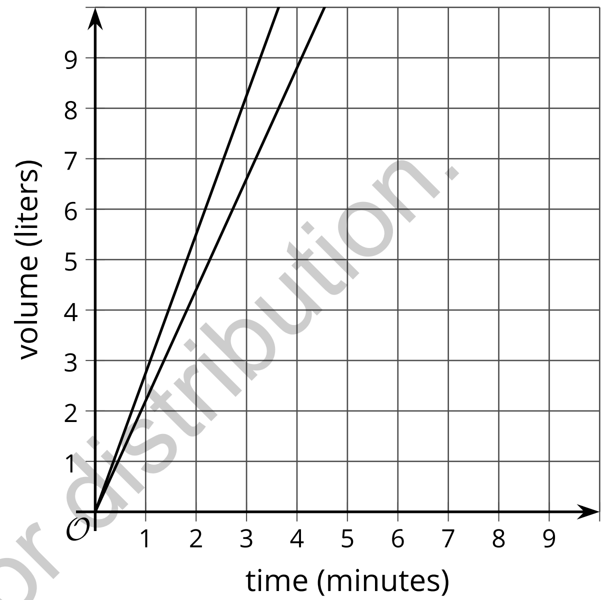
The scales we choose when graphing a relationship often depend on what information we want to know. For example, consider two water tanks filled at different constant rates.

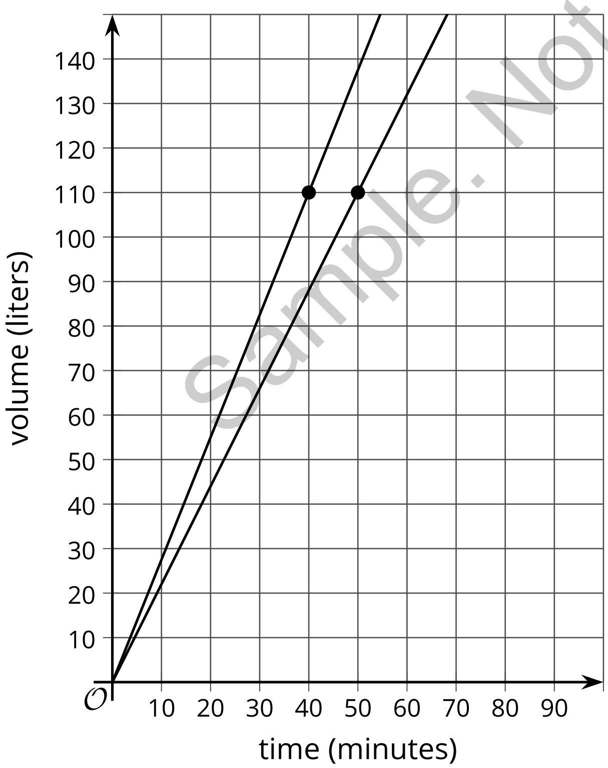
The relationship between time in minutes t and volume in liters v of Tank A can be described by the equation $v = 2.2t$.

For Tank B the relationship can be described by the equation $v = 2.75t$

These equations tell us that Tank A is being filled at a constant rate of 2.2 liters per minute and Tank B is being filled at a constant rate of 2.75 liters per minute.

If we want to use graphs to see at what times the two tanks will have 110 liters of water, then using an axis scale from 0 to 10, as shown here, isn't very helpful.





If we use a vertical scale that goes to 150 liters, a bit beyond the 110 we are looking for, and a horizontal scale that goes to 100 minutes, we get a much more useful set of axes for answering our question.

Now we can see that the two tanks will reach 110 liters 10 minutes apart—Tank B after 40 minutes of filling and Tank A after 50 minutes of filling.

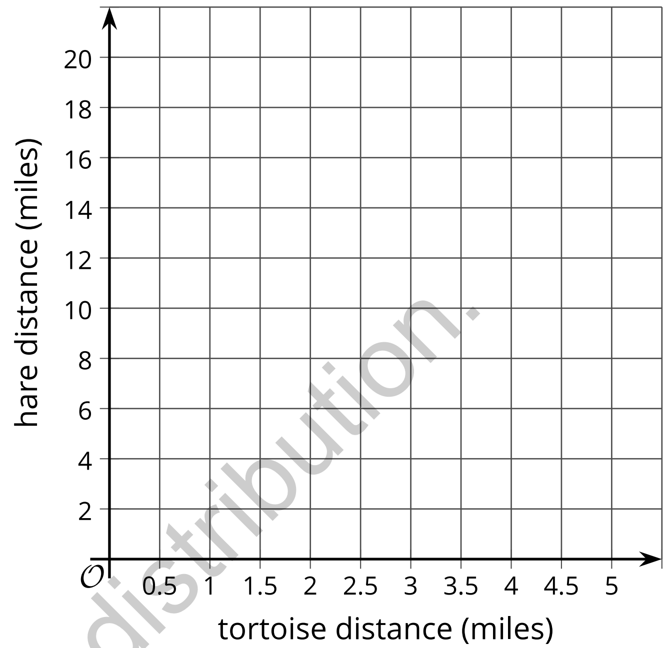
It is important to note that both of these graphs are correct, but one uses a range of values that helps answer the question. In order to always pick a helpful scale, we should consider the situation and the questions asked about it.

Practice Problems

1 Student Task Statement

The tortoise and the hare are having a race. After the hare runs 16 miles the tortoise has only run 4 miles.

This relationship can be described by the equation $y = 4x$, where x is the distance tortoise "runs" in miles, and y is the distance the hare runs in miles. Create a graph of this relationship.



Solution

A ray through $(0, 0)$ and $(2, 8)$

2 Student Task Statement

The table shows a proportional relationship between the weight on a spring scale and the distance the spring has stretched.

a. Complete the table.

b. Describe the scales you could use on the x- and y-axes of a coordinate plane that would show all the distances and weights in the table.

distance (cm)	weight (newtons)
20	28
55	
	140
1	

Solution

a.

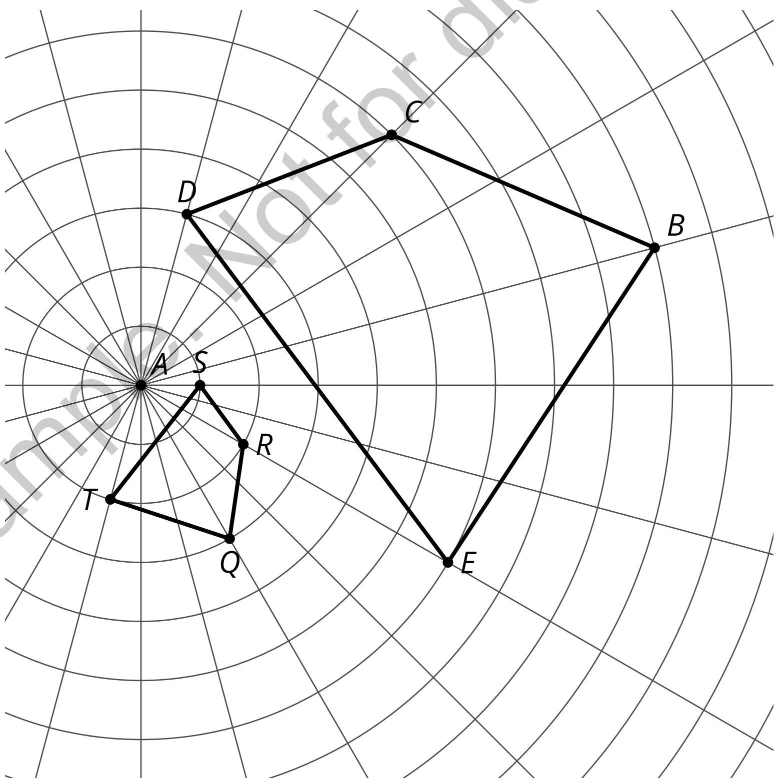
distance (cm)	weight (newtons)
20	28
55	77
100	140
1	$\frac{7}{5}$

b. Sample response: from 0 to 100 on the horizontal axis (distance) and from 0 to 140 on the vertical axis (weight)

3 from Unit 2, Lesson 6

Student Task Statement

Describe a sequence of rotations, reflections, translations, and dilations that show one figure is similar to the other. Be sure to include the distance and direction of a translation, a line of reflection, the center and angle of a rotation, and the center and scale factor of a dilation.



Solution

Sample response:

a. Begin with figure $BCDE$.

b. Dilate using A as the center of dilation with scale factor $\frac{1}{3}$.

c. Rotate using A as the center clockwise 75 degrees.

4 from Unit 2, Lesson 6

Student Task Statement

Andre said, "I found two figures that are congruent, so they can't be similar."

Diego said, "No, they are similar! The scale factor is 1."

Do you agree with either of them? Use the definition of similarity to explain your answer.

Solution

I agree with Diego. Sample reasoning: Two figures are congruent if one can be moved to the other using a sequence of rigid transformations, and two congruent figures have a scale factor of 1. They are similar if one can be moved to the other using a sequence of rigid transformations and dilations. If two figures are congruent, then they are also similar.

Representing Proportional Relationships

Goals

- Create an equation and a graph to represent proportional relationships, including an appropriate scale and axes.
- Determine what information is needed to create graphs that represent proportional relationships. Ask questions to elicit that information.

Learning Targets

- I can scale and label coordinate axes in order to graph a proportional relationship.

Lesson Narrative

In this lesson students label and choose a scale for empty pairs of axes in order to graph a proportional relationship. First, students create a graphical representation of a proportional relationship when given a table and a description to start from. They learn that the **rate of change** in a proportional relationship is the same as the constant of proportionality: the amount one variable changes by when the other variable increases by 1. Next, students use the *Info Gap* structure to graph a proportional relationship on an empty pair of axes that includes a specific point. Students will need to request information about the proportional relationship as well as calculate the specific point. The focus is on the graphs students create and their decisions on how to scale the axes in an appropriate manner for the situation (MP4).

Standards

Addressing	**8.EE.B, 8.EE.B.5**
Building Towards	**8.EE.B.5**

Instructional Routines

- MLR4: Information Gap Cards

Required Materials

Materials To Copy

- Graphing Proportional Relationships Cards (1 copy for every 2 students): Activity 2

Student Facing Learning Goals

Let's graph proportional relationships.

3.1 A Car Wash

Warm-up

10 mins

Activity Narrative

This activity gives students a chance to choose an appropriate scale when graphing a proportional relationship on a given set of blank axes (MP4). Monitor for students who create particularly clear graphs using situation appropriate scales. For example, since the problem is about a car wash, the scale for the axis showing the number of cars does not need to extend into the thousands.

Standards

Building Towards **8.EE.B.5**

Launch

Arrange students in groups of 2. Provide access to straightedges.

Ask students, "What are some different ways the communities you are a part of raise money for a cause?" (Walk-a-thon, put on an event and sell tickets, car wash, hold a raffle, sell coupon books). After a brief quiet think time, invite students to share their experiences.

Explain that an Origami Club wants to take a trip to see an origami exhibit at an art museum. Then read, or have a student read the Description in the *Student Task Statement* out loud. Explain that the same information is also shown in the table. Give students 3–4 minutes of quiet work time followed by a whole-class discussion.

Student Task Statement

Here are two ways to represent a situation.

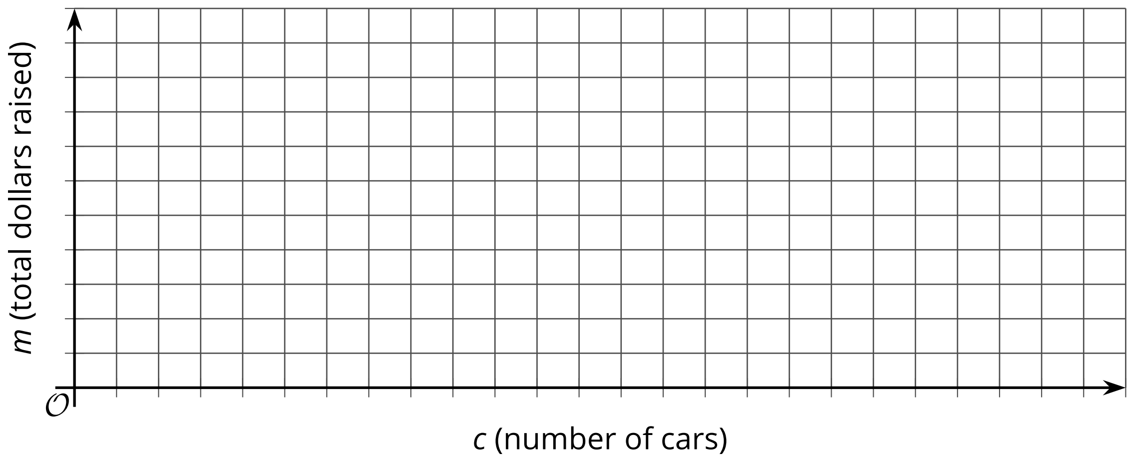
Description:

The Origami Club is doing a car wash fundraiser to raise money for a trip. They charge the same price for every car. After 11 cars, they raised a total of $93.50. After 23 cars, they raised a total of $195.50.

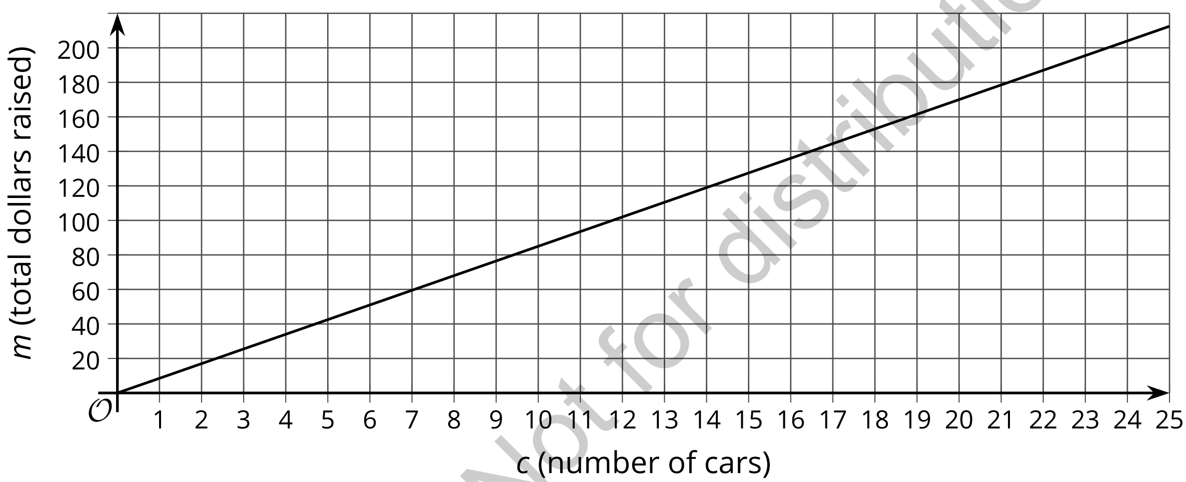
Table:

number of cars	amount raised in dollars
11	93.50
23	195.50

Create a graph that represents this situation.



Student Response



Activity Synthesis

The purpose of this discussion is to introduce students to the term "rate of change." Begin by inviting 2–3 students to share the graphs they created. Emphasize how different scales can be used, but in order to be helpful, the scale for the number of cars, c, on the horizontal axis should extend to at least 23 and the scale for the amount raised in dollars, m, on the vertical axis should extend to at least 200.

Next, tell students that an equation that represents this situation is $m = 8.5c$, where c is the number of cars, and m is the total dollars raised. Display this equation for all to see, then discuss:

- "What is the constant of proportionality and what does it mean?" (The constant of proportionality is 8.5 and it means that each car washed raised $8.50.)
- "How can you see the constant of proportionality in the graph and the table? (Graph: The slope of the line is equivalent to 8.5. Table: For any given row, the amount raised in dollars divided by the number of cars washed equals 8.5.)
- "Which representation do you think is more useful when calculating the constant of proportionality? Why?"

Explain that the constant of proportionality can be thought of as the **rate of change**: the amount one variable changes by when the other variable increases by 1. In the case of the Origami Club's car wash, the rate of change of m, the amount they raise in dollars, with respect to c, the number of cars they wash, is 8.50 dollars per car.

3.2 Info Gap: Graphing Proportional Relationships

25 mins

Activity Narrative

In this activity, students graph a proportional relationship but do not initially have enough information to do so. To bridge the gap, they need to exchange questions and ideas.

The *Info Gap* structure requires students to make sense of problems by determining what information is necessary, and then to ask for information they need to solve it. This may take several rounds of discussion if their first requests do not yield the information they need (MP1). It also allows them to refine the language they use and ask increasingly more precise questions until they get the information they need (MP6).

Access for English Language Learners

This activity uses the *Information Gap* math language routine, which facilitates meaningful interactions by positioning some students as holders of information that is needed by other students, creating a need to communicate.

Standards

Addressing **8.EE.B**

Instructional Routines

- MLR4: Information Gap Cards

Launch

Tell students they will be graphing some proportional relationships. Display the *Info Gap* graphic that illustrates a framework for the routine for all to see.

Remind students of the structure of the *Info Gap* routine, and consider demonstrating the protocol if students are unfamiliar with it.

Arrange students in groups of 2. In each group, give a problem card to 1 student and a data card to the other student. After reviewing their work on the first problem, give students the cards for a second problem and instruct them to switch roles.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Check for understanding by inviting students to rephrase directions in their own words. Keep a display of the *Info Gap* graphic visible throughout the activity or provide students with a physical copy.
Supports accessibility for: Memory, Organization

Student Task Statement

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

If your teacher gives you the problem card:

1. Silently read your card and think about what information you need to answer the question.
2. Ask your partner for the specific information that you need. “Can you tell me ________?”
3. Explain to your partner how you are using the information to solve the problem. “I need to know ________ because . . . .”
   Continue to ask questions until you have enough information to solve the problem.
4. Once you have enough information, share the problem card with your partner, and solve the problem independently.
5. Read the data card, and discuss your reasoning.

If your teacher gives you the data card:

1. Silently read your card. Wait for your partner to ask for information.
2. Before telling your partner any information, ask, “Why do you need to know ________?”
3. Listen to your partner’s reasoning and ask clarifying questions. Only give information that is on your card. Do not figure out anything for your partner!
   These steps may be repeated.
4. Once your partner says they have enough information to solve the problem, read the problem card, and solve the problem independently.
5. Share the data card, and discuss your reasoning.

Student Response

Problem Card 1: 76.5 grams of honey are needed for 17 cups of flour. Graphs vary. Possible scale: 0–28 on the cups of flour axis, 0–140 on the grams of honey axis.

Problem Card 2: 57.5 grams of salt are needed for 23 cups of flour. Graphs vary. Possible scale: 0–28 on the cups of flour axis, 0–70 on the grams of salt axis.

Building on Student Thinking

If students are unsure how to scale the axes on their graphs, consider asking:

- “What are the largest values that need to be shown on the graph?”
- “How many grid lines are there?”

Are You Ready for More?

Ten people can dig 5 holes in 3 hours. If n people digging at the same rate dig m holes in d hours:

1. Is n proportional to m when $d = 3$?
2. Is n proportional to d when $m = 5$?
3. Is m proportional to d when $n = 10$?

Extension Student Response

1. Yes, because if 10 people can dig 5 holes in 3 hours, then 2 people can dig 1 hole in 3 hours, so $n = 2m$.
2. No, because if the number of people doubles, then the time it takes to dig the holes halves, so n is not a constant times d.
3. Yes, because if 10 people can dig 5 holes in 3 hours, then they can dig $\frac{5}{3}$ holes in 1 hour, so $m = \frac{5}{3}d$.

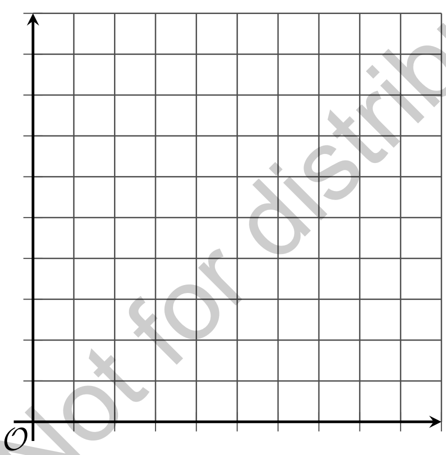
Activity Synthesis

After students have completed their work, share the correct answers and ask students to discuss the process of solving the problems. Here are some questions for discussion:

- "How did you decide what to label the two axes?"
- "How did you decide to scale the horizontal axis? The vertical axis?"
- "Where can you see the rate of change of grams of honey per cups of flour on the graph?"
- "Where can you see the rate of change of grams of salt per cups of flour on the graph?"

Lesson Synthesis

The goal of this discussion is for students to consider the importance of choosing a scale when creating graphs. Display this image or a similar blank coordinate plane and tell students that the proportional relationship $y = 5.5x$ includes the point $(18, 99)$ on its graph.



Ask what an appropriate scale might be to show this point on a pair of axes with a 10 by 10 grid. (Each horizontal grid line could represent 2 units and each vertical grid line could represent 10 or 20 units.)

Next ask students, "What are some important ideas to remember when analyzing or creating a graph in the future?" Consider creating a classroom display with their responses. Important ideas to highlight include:

- When studying a graph, pay attention to the label on each axis. For example, the placement of the variables may mean a graph is showing the pace of a bug instead of the speed of a bug.
- When studying a graph, pay attention to the scale. For example, one graph may appear steeper than another graph, but it is the actual value of the slopes that matters.
- When creating a graph, consider the question being asked and the information given when determining the scale. For example, make sure that the scale chosen extends far enough to show the necessary data.

3.3 Graph the Relationship

Cool-down

5 mins

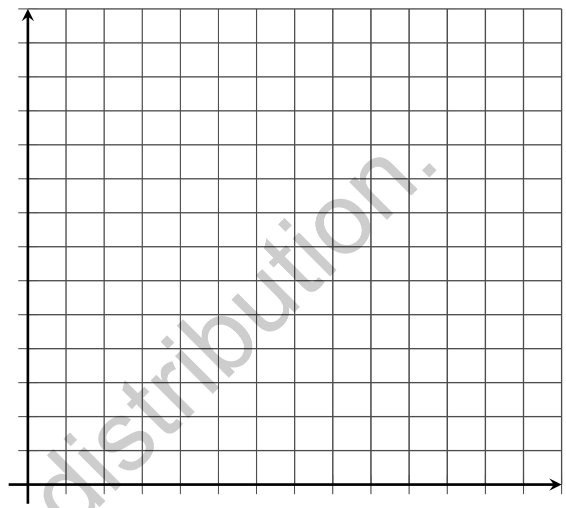
Standards

Addressing **8.EE.B.5**

Student Task Statement

Sketch a graph that shows the relationship between grams of honey and grams of salt needed for a bakery recipe. Show on the graph how much honey is needed for 70 grams of salt.

salt (grams)	honey (grams)
10	14
25	35



Student Response

Possible graph: Axes labeled from 0 to 140, with grams of salt on the horizontal axis and grams of honey on the vertical. Coordinate points may include $(0, 0)$, $(10, 14)$, and $(70, 98)$.

Responding To Student Thinking

Points to Emphasize

If most students struggle with scaling their own axes, in the second question of the activity referred to here, visit groups and support their understanding of how to determine a scale when creating a graph.

Grade 8, Unit 3, Lesson 4, Activity 1 What's the Relationship?

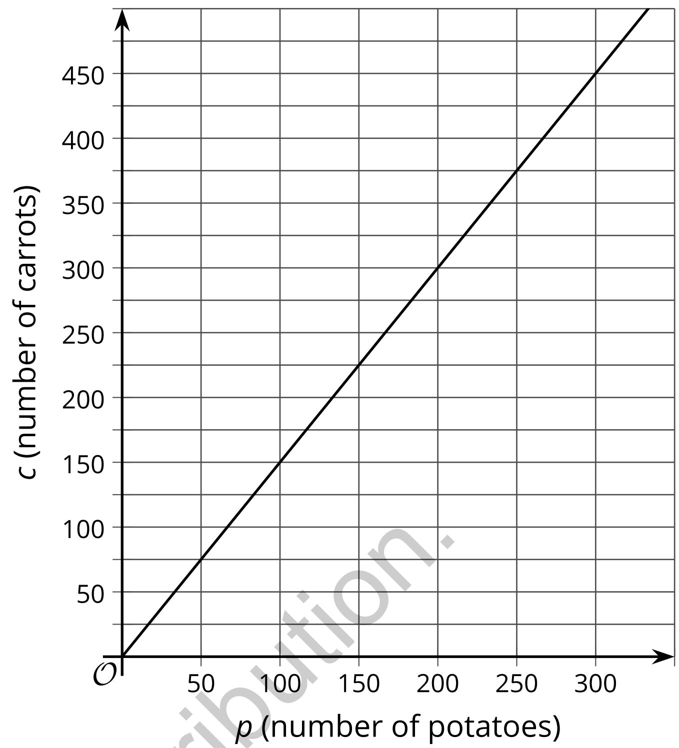
Lesson 3 Summary

Proportional relationships can be represented in multiple ways. Which representation we choose depends on the purpose. And when we create representations we can choose helpful values by paying attention to the context. For example, a stew recipe calls for 3 carrots for every 2 potatoes. One way to represent this is using an equation. If there are p potatoes and c carrots, then $c = \frac{3}{2}p$.

Suppose we want to make a large batch of this recipe for a family gathering, using 150 potatoes. To find the number of carrots, we could just use the equation: $\frac{3}{2} \cdot 150 = 225$ carrots.

Now suppose the recipe is used in a restaurant that makes the stew in large batches of different sizes depending on how busy of a day it is, using up to 300 potatoes at a time.

Then we might make a graph to show how many carrots are needed for different amounts of potatoes. We set up a pair of coordinate axes with a scale from 0 to 300 along the horizontal axis and 0 to 450 on the vertical axis, because $450 = \frac{3}{2} \cdot 300$. Then we can read how many carrots are needed for any number of potatoes up to 300.



Or if the recipe is used in a food factory that produces very large quantities and where the potatoes come in bags of 150, we might just make a table of values showing the number of carrots needed for different multiples of 150.

number of potatoes	number of carrots
150	225
300	450
450	675
600	900

No matter the representation or the scale used, the constant of proportionality, $\frac{3}{2}$, is evident in each. In the equation it is the number we multiply p by. In the graph, it is the slope. In the table, it is the number we multiply values in the left column to get numbers in the right column. We can think of the constant of proportionality as a **rate of change**: the amount one variable changes by when the other variable increases by 1. In this case, the rate of change of c with respect to p is $\frac{3}{2}$ carrots per potato.

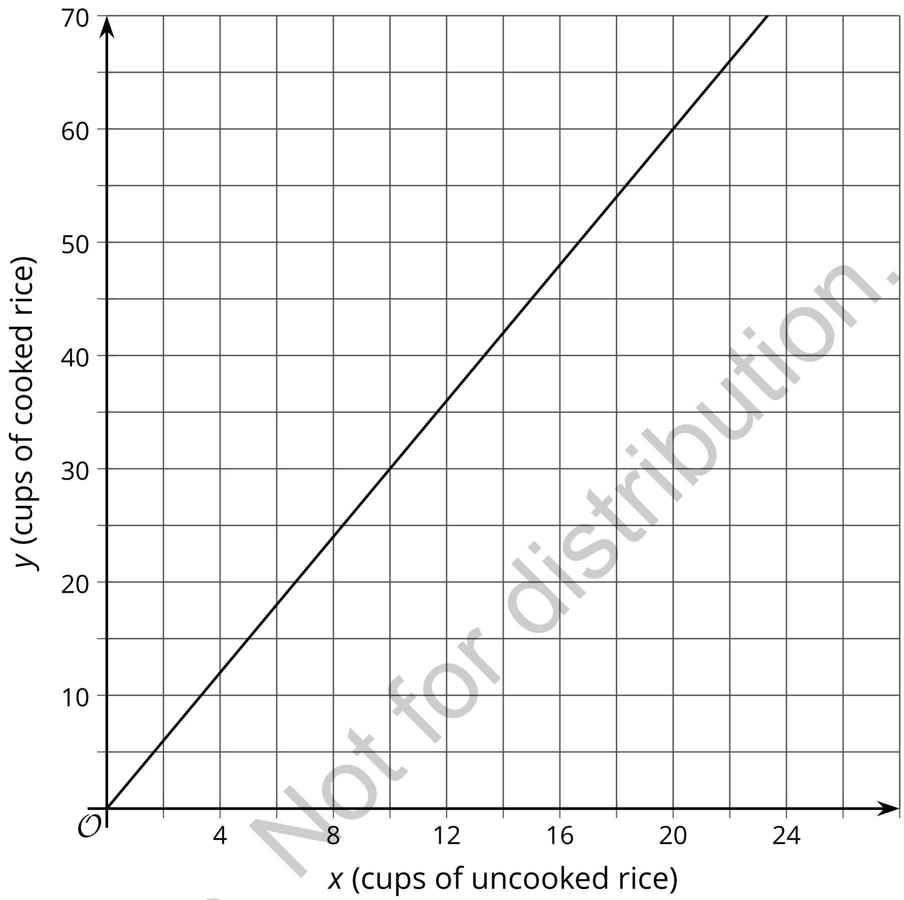
Glossary

- rate of change

Practice Problems

1 Student Task Statement

This graph describes the relationship between the volume of uncooked rice and the volume of the rice after it is cooked.



a. Write an equation that reflects this relationship where x represents the volume in cups of uncooked rice and y represents the volume in cups of the rice after it is cooked.

b. Complete the table:

cups of uncooked rice	cups of cooked rice
20	
	120
1	

Solution

a. $y = 3x$ or $\frac{y}{x} = \frac{60}{20}$ (or equivalent)

b.

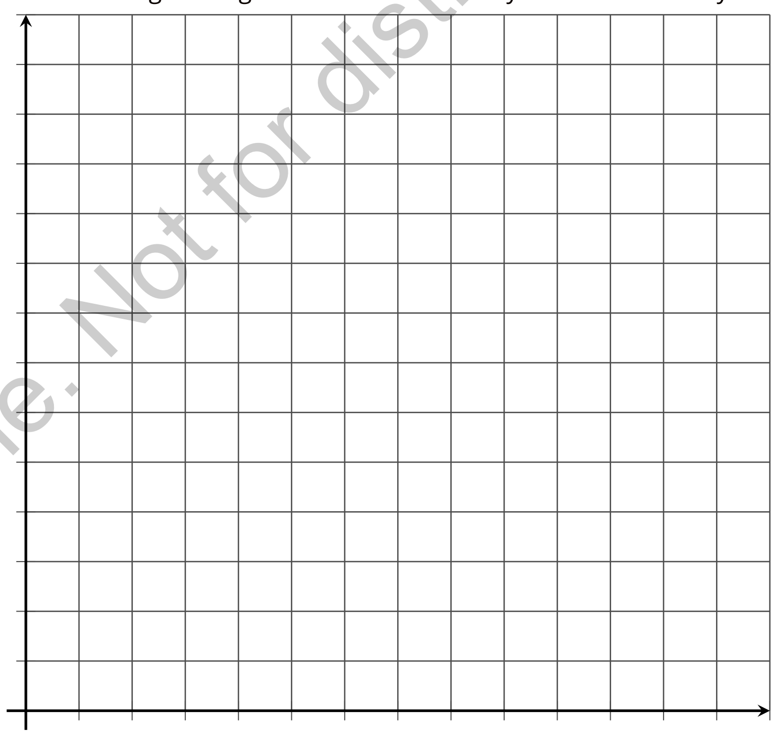
cups of uncooked rice	cups of cooked rice
20	60
$33\frac{1}{3}$	100
1	3

2 Student Task Statement

Students are selling raffle tickets for a school fundraiser. They collect $24 for every 10 raffle tickets they sell.

a. Suppose M is the amount of money the students collect for selling R raffle tickets. Write an equation that reflects the relationship between M and R.

b. Label and scale the axes and graph this situation with M on the vertical axis and R on the horizontal axis. Make sure the scale is large enough to see how much they would raise if they sell 1,000 tickets.



Solution

a. $M = \frac{12}{5}R$ (or equivalent)

b. on coordinate axes with R on the horizontal axis and M on the vertical axis, a ray through $(0, 0)$ and $(10, 24)$ (or equivalent)

3 from Unit 2, Lesson 12

Student Task Statement

A line is represented by the equation $\frac{y}{x-2} = \frac{3}{11}$. For each point, determine if it is on this line.

a. $(3, 13)$
b. $(13, 3)$
c. $(35, 9)$
d. $(40, 10)$
e. $(46, 12)$

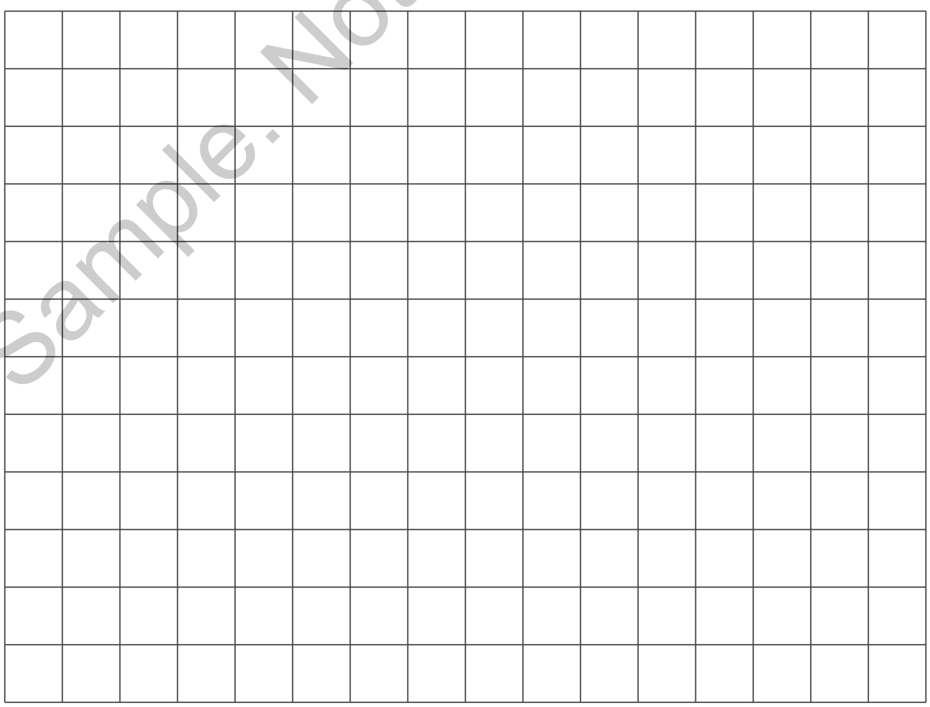
Solution

a. No
b. Yes
c. Yes
d. No
e. Yes

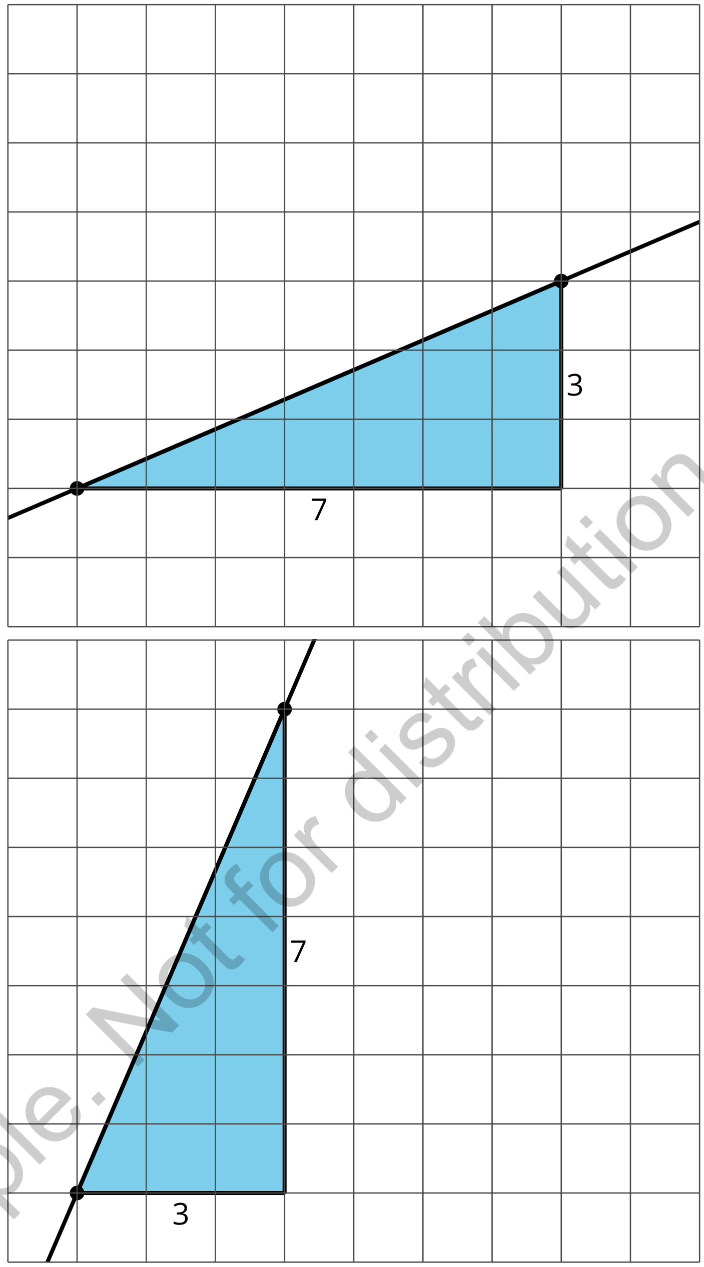
4 from Unit 2, Lesson 10

Student Task Statement

Use a straightedge to draw two lines: one with slope $\frac{3}{7}$ and one with slope $\frac{7}{3}$.



Solution



Comparing Proportional Relationships

Goals

- Compare the rates of change for two proportional relationships, given multiple representations.
- Interpret multiple representations of a proportional relationship in order to answer questions (in writing), and explain the solution method.
- Present a comparison of two proportional relationships (using words and multiple other representations).

Learning Targets

- I can compare proportional relationships represented in different ways.

Lesson Narrative

In this lesson students compare two situations that are represented in different ways. For example, one situation might specify a rate of change, while the other is represented by a table of values, a graph, or an equation. Students move flexibly between representations and consider how to find the information they need from each type. They respond to context-related questions that compare the two situations and solve problems with the information they've garnered from each representation (MP2).

Standards

Addressing **8.EE.B, 8.EE.B.5**

Instructional Routines

- MLR6: Three Reads

Required Materials

Materials To Gather

- Math Community Chart: Activity 2
- Tools for creating a visual display: Activity 2

Student Facing Learning Goals

Let's compare proportional relationships.

4.1 What's the Relationship?

Warm-up

10 mins

Activity Narrative

The purpose of this *Warm-up* is for students to describe a situation presented by an equation using other representations. Students decide on a context and then create a table and a graph, scaling the axes appropriately to the situation (MP2). Moving between representations of a proportional relationship here will be useful in a following activity where students compare proportional relationships represented in different ways.

Standards

Addressing **8.EE.B**

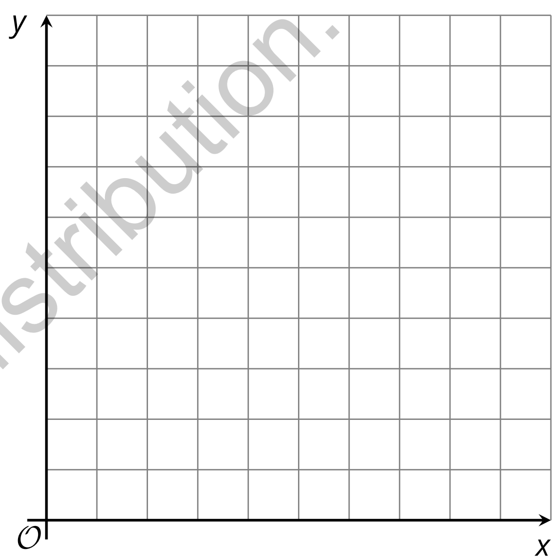
Launch

Arrange students in groups of 2. Give them 2–3 minutes of quiet work time followed by a whole-class discussion.

Student Task Statement

The equation $y = 4.2x$ could represent a variety of different situations.

1. Describe a situation that can be represented by this equation. What do the quantities x and y represent in your situation?
2. Create a table and a graph that represent the situation.



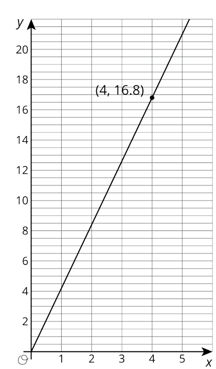
Student Response

1. Sample response: A frog jumps 16.8 feet in 4 seconds, where x represents the time in seconds and y represents how far the frog jumps in feet.
2. Sample response:

Table:

x	y
3	12.60
4	16.80

Graph:



Activity Synthesis

Invite several students to share their situations and display their graphs for all to see. Ask:

- "What does the rate of change represent in this situation?"
- "How did you decide on the scale for your axes?"

4.2 Comparing Two Different Representations

25 mins

Activity Narrative

In this activity, students consider representations of two different proportional relationships and make comparisons between them. They work in groups to compare the relationships and reason quantitatively to answer context specific questions (MP2). Groups make a visual display for their problem set to explain each of their responses and convince others of their accuracy.

Standards

Addressing **8.EE.B.5**

Instructional Routines

- MLR6: Three Reads

Launch

Math community
Display the Math Community Chart for all to see. Give students a brief quiet think time to read the norms or invite a student to read them out loud. Tell students that during this activity they are going to practice looking for their classmates putting the norms into action. At the end of the activity, students can share what norms they saw and how the norm supported the mathematical community during the activity.

Tell students that the activity today is about summer jobs. Invite students to share their experiences with a summer job or any other experience they may have earning money, such as doing chores for a neighbor or selling handmade items. If desired, update the contexts of the question sets so they are more familiar to students or to better reflect the experiences of the class.

Then arrange students in groups of 2–3. Provide each group with tools for creating a visual display. Assign each group one of the three question sets (or have groups to choose). Tell groups they will make a visual display for their responses to the questions. The display should clearly demonstrate their reasoning and use multiple representations in order to be convincing (MP3). Let them know that there will be a gallery walk when they finish for the rest of the class to inspect the accuracy of their solutions.

If time allows, consider asking groups to complete all three problems and make a visual display for just one.

Access for English Language Learners

MLR6 Three Reads. In small groups, tell students to keep books or devices closed. Display only the task statement and two descriptions, without revealing the questions. Say, "We are going to read this information 3 times."

- After the 1st read: "Tell your partner what this situation is about."
- After the 2nd read: "List the quantities. What can be counted or measured?"

- For the 3rd read: Reveal and read the questions. Ask, “What are some ways one might get started on this?”

Advances: Reading, Representing

Access for Students with Disabilities

Action and Expression: Develop Expression and Communication. Provide students with alternatives to writing on paper: students can share their learning using digital technology, such as creating a digital presentation or slideshow.
Supports accessibility for: Language, Fine Motor Skills

Student Task Statement

1. Elena babysits her neighbor’s children. Her earnings are given by the equation $y = 8.40x$, where x represents the number of hours she worked, and y represents the amount of money she earned in dollars.

   Jada earns \$7 per hour mowing her neighbors’ lawns.

   a. Who makes more money after working 12 hours? How much more do they make? Explain your reasoning by creating a graph or a table.

   b. What is the value of the **rate of change** for each situation, and what does each value mean?

   c. Using your graph or table, determine how long it would take each person to earn \$150.

2. Clare and Han have summer jobs stuffing envelopes for two different companies.

   Han earns \$15 for every 300 envelopes he finishes.

   Clare’s earnings can be seen in the table.

number of envelopes	money earned in dollars
400	40
900	90

   a. By creating a graph, show how much money each person makes after stuffing 1,500 envelopes.

   b. What is the value of the rate of change for each situation, and what does each value mean?

   c. Using your graph, determine how much more money one person makes relative to the other after stuffing 1,500 envelopes. Explain or show your reasoning.

3. Tyler plans to start a lemonade stand and is trying to perfect his recipe for lemonade. He wants to make sure the recipe doesn’t use too much lemonade mix (lemon juice and sugar) but still tastes good.

   Recipe 1 is given by the equation $y = 4x$, where x represents the amount of lemonade mix in cups, and y represents the amount of water in cups.

   Recipe 2 is given in the table.

lemonade mix (cups)	water (cups)
10	50
13	65
21	105

a. If Tyler had 16 cups of lemonade mix, how many cups of water would he need for each recipe? Explain your reasoning by creating a graph or a table.

b. What is the value of the rate of change for each recipe, and what does each value mean?

c. Tyler has 16 cups of lemonade mix to use for his lemonade stand. Which lemonade recipe should he use? Explain or show your reasoning.

Student Response

1. a. Elena will make \$16.80 more than Jada after working 12 hours. Sample reasoning: A graph shows hours worked along the x-axis and money earned in dollars along the y-axis. The graph scale is large enough to show the points on the lines representing each situation at 12 hours. The difference between the y-coordinates of these two points is \$16.80.

   b. Sample response: For Elena, the value of the rate of change is 8.4, which means that she earns \$8.40 per hour babysitting. For Jada, the value of the rate of change is 7, which means that she earns \$7.00 per hour mowing lawns.

   c. Sample response: It will take Elena almost 18 hours and Jada almost 21.5 hours.

2. a. Clare makes \$150.00 and Han earns \$75.00. Sample response: A graph shows envelopes stuffed along the x-axis and money earned in dollars along the y-axis. The graph scale is large enough to show the points on the lines representing each situation at 1,500 envelopes.

   b. Sample response: For Clare, the value of the rate of change is $\frac{1}{10}$, which means that she makes \$0.10 per stuffed envelope. For Han, the value of the rate of change is $\frac{15}{300}$, which means that he makes \$0.05 per stuffed envelope.

   c. After stuffing 1,500 envelopes, Clare makes \$75 more than Han. Sample reasoning: At 1,500 envelopes, the difference between the y-coordinates of these two points is \$75.

3. a. Recipe 1: 64 cups of water, Recipe 2: 80 cups of water. Sample reasoning: A graph shows cups of lemonade mix along the x-axis and cups of water along the y-axis. The graph scale is large enough to show the points on the lines representing each situation at 16 cups of lemonade mix.

   b. Sample response: For Recipe 1, the value of the rate of change is 4, which means that there are 4 cups of water per cup of lemonade mix. For Recipe 2, the value of the rate of change is 5, which means there are 5 cups of water per cup of lemonade mix.

   c. Sample responses:

      - Recipe 1 uses 4 cups of water for every cup of mix, while Recipe 2 uses 5 cups of water for every cup of mix. That means Recipe 2 will taste more watered down, so Tyler should use Recipe 1.
      - If Tyler uses all 16 cups of mix, he will use 64 cups of water with Recipe 1 and 80 cups of water with Recipe 2. Tyler should use Recipe 2 since Recipe 2 will make more lemonade to sell.

Building on Student Thinking

Some students may confuse the values for the rate of change of a situation. For example, Lemonade Recipe 1's equation, $y = 4x$, shows that the rate of change is 4 cups of water per cup of lemonade mix. Students may switch these values and think that the rate of change is 4 cups lemonade mix per cup of water. Consider:

- Asking “How did you find the rate of change and what does it mean?”
- Prompting students to list a few additional values or sketch a graph to see if it matches their interpretation of the rate of change.

Are You Ready for More?

Han and Clare are still stuffing envelopes. Han can stuff 20 envelopes in a minute, and Clare can stuff 10 envelopes in a minute. They start working together on a pile of 1,000 envelopes.

1. How long does it take them to finish the pile?
2. Who earns more money?

Extension Student Response

1. $33\frac{1}{3}$ minutes. Sample reasoning: Working together they can stuff 30 envelopes per minute, so it takes them $\frac{1{,}000}{30} = 33\frac{1}{3}$ minutes to finish the pile.
2. Han and Clare earn the same amount of money. Sample reasoning: Han stuffs twice as many envelopes as Clare, but he only earns half as much.

Activity Synthesis

Begin with a gallery walk for students to see how other groups answered the same set of questions they did. In small groups, invite students who created a display for the same set of problems to discuss what is the same and what is different about their work and representations on the posters. Here are some questions for discussion:

- “What representations did you choose to answer the questions? Why did you pick them?”
- “What representation did you not use? Why?”
- “How did you decide what scale to use when you made your graph?”

Math Community
Conclude the discussion by inviting 2–3 students to share a norm they identified in action. Provide this sentence frame to help students organize their thoughts in a clear, precise way:

- “I noticed our norm “________” in action today and it really helped me/my group because ________.”

Lesson Synthesis

The goal is for students to discuss strategies they can use when comparing two proportional relationships. Have students conduct a second gallery walk to see how other groups answered questions about the two contexts they did not make a display for. Here are some questions for discussion:

- “What was the same and different about the other problem sets?” (Answers vary.)
- “What do you need in order to compare two proportional relationships?” (the rate of change)
- “What type of wording in a problem statement or description of a situation tells you that you have a rate of change?” (phrases like "per," "every," and "for each")
- “How did you decide which representation to use to solve the different types of problems?” (Answers vary.)

4.3 Different Salt Mixtures

Cool-down

5 mins

Standards

Addressing **8.EE.B.5**

Student Task Statement

Here are recipes for two mixtures of salt and water that taste different.

Information about Mixture A is shown in the table.

salt (teaspoons)	water (cups)
4	5
7	$8\frac{3}{4}$
9	$11\frac{1}{4}$

Mixture B can be described by the equation $y = 2.5x$, where x is the number of teaspoons of salt, and y is the number of cups of water.

1. If you used 10 cups of water, which mixture would use more salt? How much more? Explain or show your reasoning.
2. Which mixture tastes saltier? Explain your reasoning.

Student Response

1. Mixture A uses 4 more teaspoons of salt than Mixture B. Sample reasoning: Mixture A would use 8 teaspoons of salt because I can double the row with 4 and 5 to get 8 and 10. Mixture B would use 4 teaspoons of salt because $10 = 2.5(4)$.
2. Mixture A tastes saltier because it uses more salt for the same amount of water. Sample reasoning: Mixture A uses 8 teaspoons of salt for 10 cups of water and Mixture B only uses 4 teaspoons of salt for the same amount of water.

Responding To Student Thinking

Press Pause
By this point in the unit, there should be some student mastery working with tables and equations of proportional relationships. If most students struggle, make time to revisit related work in the Grade 7 section referred to here. See the Course Guide for ideas to help students re-engage with earlier work.

Grade 7, Unit 2, Section B Representing Proportional Relationships with Equations

Lesson 4 Summary

When two proportional relationships are represented in different ways, we can compare them by finding a common piece of information.

For example, Clare's earnings are represented by the equation $y = 14.50x$, where y is her earnings in dollars for working x hours.

The table shows some information about Jada's earnings.

time worked (hours)	earnings (dollars)
7	92.75
4.5	59.63
37	490.25

If we want to know who makes more per hour, we can look at the rate of change for each situation.

In Clare's equation, we see that the rate of change is 14.50. This tells us that she earns $14.50 per hour. For Jada, we can calculate the rate of change by dividing her earnings in one row by the hours worked in the same row. For example, using the last row, the rate of change is 13.25 since $490.25 \div 37 = 13.25$. This tells us that Clare earns 1.25 more dollars per hour than Jada.

Practice Problems

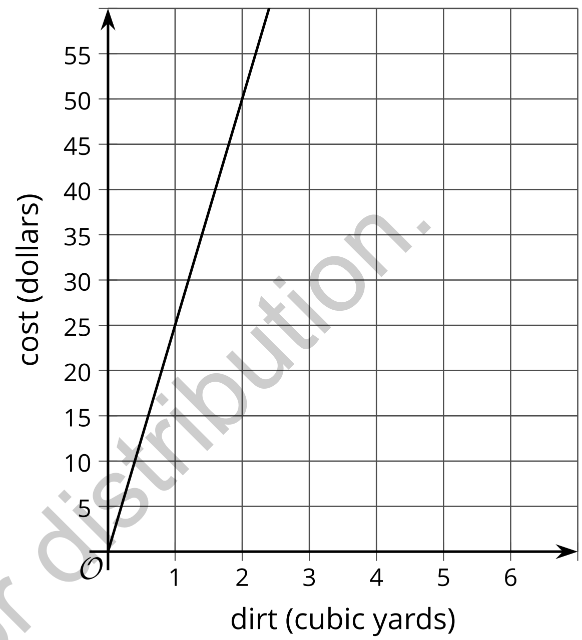
1 Student Task Statement

A contractor must haul a large amount of dirt to a worksite. She collected information from two hauling companies.

EZ Excavation gives its prices in a table.

dirt (cubic yards)	cost (dollars)
8	196
20	490
26	637

Happy Hauling Service gives its prices in a graph.



a. How much would each hauling company charge to haul 40 cubic yards of dirt? Explain or show your reasoning.

b. Calculate the rate of change for each relationship. What do they mean for each company?

c. If the contractor has 40 cubic yards of dirt to haul and a budget of $1,000, which hauling company should she hire? Explain or show your reasoning.

Solution

a. EZ Excavation: $980, Happy Hauling Service: $1,000. Sample reasoning: According to the table, EZ Excavation charges $490 for 20 cubic yards of dirt, so it would cost double that, or $980, for 40 cubic yards of dirt. According to the graph, Happy Hauling charges $25 for 1 cubic yard of dirt, so 40 cubic yards of dirt would cost $1,000 since $40\boldcdot25=1,000$.

b. EZ Excavation: 24.50 dollars per cubic yard, Happy Hauling Service: 25 dollars per cubic yard. Sample response: For each cubic yard of dirt hauled to a worksite, the company charges a set amount of dollars.

c. Either company. Sample reasoning: Both companies would meet the budget requirement, yet EZ Excavation will cost $980 for 40 cubic yards and be under budget, while Happy Hauling would cost $1,000 and use all of the budget.

2

Student Task Statement

Andre and Priya are tracking the number of steps they walk. Andre records that he can walk 6,000 steps in 50 minutes. Priya writes the equation $y = 118x$, where y is the number of steps, and x is the number of minutes she walks, to describe her step rate. This week, Andre and Priya each walk for a total of 5 hours. Who walks more steps? How many more?

Solution

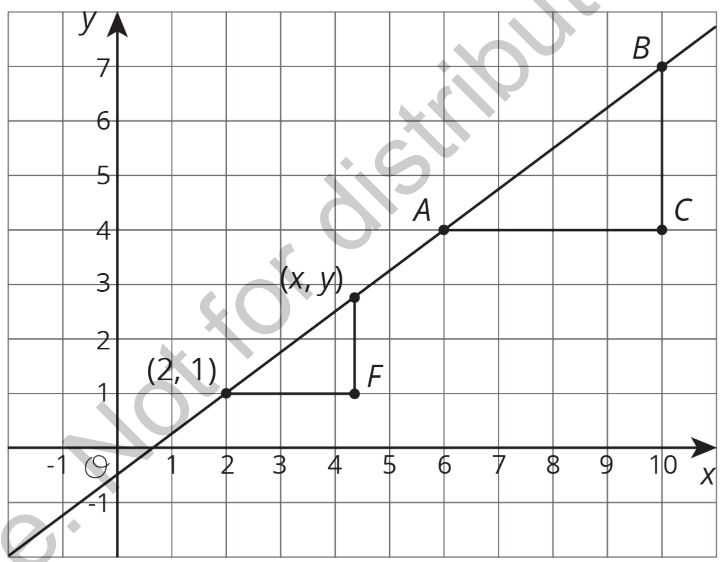
Andre walks 600 more steps than Priya.

3

from Unit 2, Lesson 11

Student Task Statement

The points $(2, 1)$, (x, y), A, and B all lie on the displayed line. Find an equation for the line.



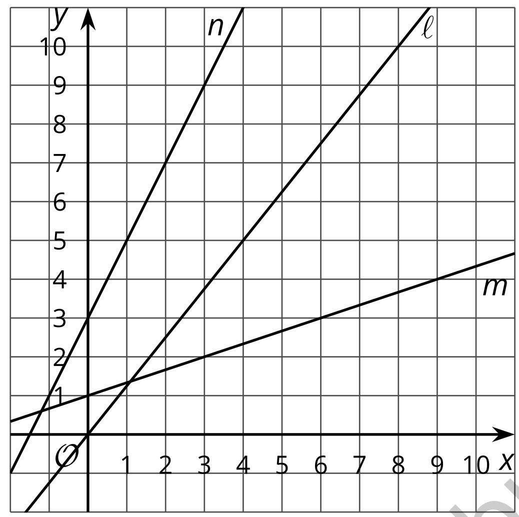
Solution

$\frac{y-1}{x-2} = \frac{3}{4}$ (or equivalent)

4 from Unit 2, Lesson 11

Student Task Statement

Calculate the slope of each line.



Solution

Line ℓ: $\frac{5}{4}$ (or equivalent)
Line m: $\frac{1}{3}$ (or equivalent)
Line n: 2 (or equivalent)